GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #3 **October 4, 2005**

Time Allowed: 45 mins. **Name:**

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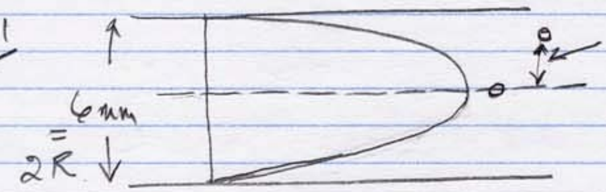
1) (3pts) Two aqueous dye droplets, 0.2mm diameter, are placed in a flowing stream of a mixture of glycerol and water in a 6mm diameter straight tube. The average flow speed of the mixture is 7cm per second and the flow is laminar. One of the droplets was placed withits centre along the centre-line of the tube and the other at a distance 2/3 of the radius from the centre-line.

Describe what you would observe for each of the droplets. What are the rates of distortion for the droplets?

2) (7pts) An empty airplane at the hangar has its engine on and the heaters for the air inside turned to maximum capacity. A temperature of 25°C was maintained inside the plane when the outside temperature was -20°C. The heat transfer coefficient for all the surfaces inside the plane was always constant at 8 W/m^2K and outside the stationary plane (in a light breeze), it was 35W/m^2K. The plane is now loaded with 225 adults all with skin surface temperatures of 30°C and an average exposed skin surface area of 0.12m^2 per person. If the plane lifts off and maintains a cruising altitude where the external temperature is -56.5°C and the external heat transfer coefficient is 450 W/m^2K, what would the steady state temperature inside the plane be? You may assume that the fusilage approximates a closed-ended cylinder with a wall thickness of 5cm, an external diameter of 3m and length of 30.98m and the thermal conductivity of the wall is 0.08 W/mK.

ENCH 501 Quiz 3 Solution Oct. 4, 2005

1



$\bar{u} = 7$ cm/s

$u_{max} = 2\bar{u} = 14$ cm/s

vel. profile

For laminar flow, $u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$

- The droplet along the centre-line will be carried downstream at 14 cm/s and it will retain its shape. There will be no deformation or distortion.

- The droplet at $r = \frac{2}{3}R$ will become elongated.

At $r = \frac{2}{3}R$, $\frac{du}{dr} = -u_{max}\frac{2r}{R^2}\Big|_{r=\frac{2}{3}R}$

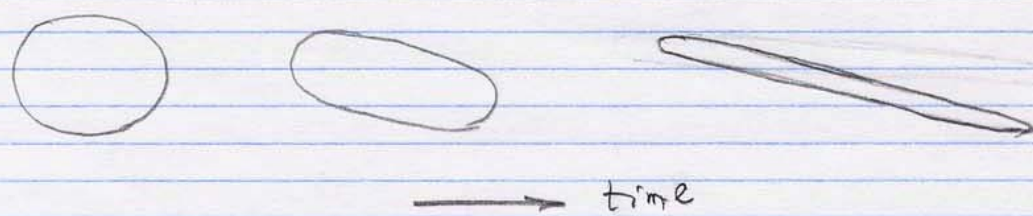
$$= -u_{max}\frac{4}{3}\frac{R}{R^2}$$

The rate of distortion $= \frac{1}{2}$ rate of angular deformation

$$\dot{\varepsilon}_{zr} = \dot{\varepsilon}_{rz} = \frac{1}{2}\left\{-u_{max}\frac{4}{3R}\right\}$$

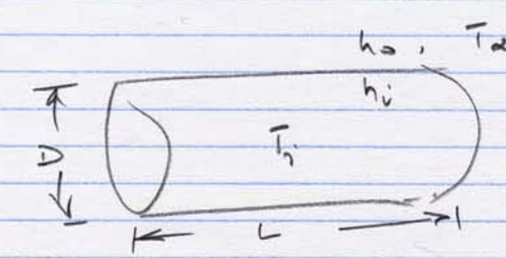
$$= -\frac{1}{2}\left\{\frac{(0.14)\,4}{3(3)(10^{-3})}\right\}$$

$$= -31.1 \text{ radians/s}$$



The dye will become smeared out with diffusion.

2

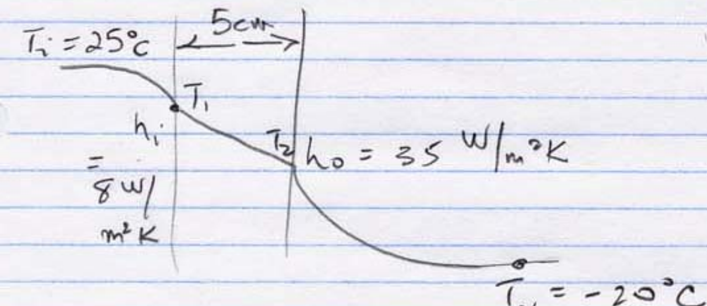


Assume the fusilage or plane body is a cylinder closed-ended by flat plates.

The problem should be solved in 2 stages – stage 1 while the plane is on the ground and stage 2 while it is in air in the isothermal zone of the "standard air".

Stage 1

The heat generated inside the plane is at maximum level. At steady state, this heat is lost through the walls.



From the curved wall, the heat transfer rate is given by (see sketch)

r_o = outside radius, r_i = inside radius } of fusilage

$$Q_c = h_o A_o (T_2 - T_\alpha)$$

$$Q_c = \frac{2\pi k L (T_1 - T_2)}{\ln(r_o/r_i)}$$

$$Q_c = h_i A_i (T_i - T_1)$$

Re-arrange the equations

$$\frac{Q_c}{h_o A_o} = T_2 - T_\alpha$$

$$Q_c \frac{\ln(r_o/r_i)}{2\pi k L} = T_1 - T_2$$

$$Q_c / h_i A_i = T_i - T_1$$

Sum both sides

$$Q_c \left\{ \frac{1}{h_o A_o} + \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_i A_i} \right\} = T_i - T_\alpha$$

From this relationship Q_c can be evaluated.

$$\begin{cases} A_o = \pi D_o L_o = \pi(3)(30.98) = 291.98\ m^2 \\ h_o = 35\ W/m^2K \quad \text{(given)} \end{cases}$$

$$\begin{cases} r_o = (3/2)\,m, \quad r_i = (2.9/2)\,m, \quad L = 30.88\,m \\ k = 0.08\ W/mK \end{cases}$$

$$\begin{cases} A_i = \pi D_i L_i = \pi(2.90)(30.88) = 281.34\ m^2 \\ h_i = 8\ W/m^2K \end{cases}$$

$$Q_c \left\{ \frac{1}{35(291.98)} + \frac{\ln\left(\frac{3}{2.9}\right)}{2\pi(0.08)(30.88)} + \frac{1}{8(281.34)} \right\} = 25-(-20)$$

$$Q_c\,(0.000098 + 0.002184 + 0.000444) = 45$$

$$Q_c = \frac{45}{0.002726} = 16,506.1\ W$$

For the 2 end walls, the heat rate thro each may be evaluated from —

$$Q_p \left\{ \frac{1}{h_o A_o} + \frac{S}{kA} + \frac{1}{h_i A_i} \right\} = T_i - T_\alpha$$

where $A_o = \frac{\pi D_o^2}{4} = 7.069\ m^2$; $D_o = 3\,m$

$A_i = \frac{\pi D_i^2}{4} = 6.605\ m^2$; $D_i = 2.9\,m$

Let $A \doteq \frac{1}{2}(A_o + A_i) = 6.837\ m^2$

$$Q_p \left\{ \frac{1}{35(7.069)} + \frac{0.05}{0.08(6.837)} + \frac{1}{8(6.605)} \right\} = 45$$

$$Q_p \{ 0.00404 + 0.0914 + 0.0189 \} = 45$$

$$Q_p = \frac{45}{0.1144} = 393.4\ W$$

$\therefore$ for the 2 end plates $2Q_p = 786.83\ W$

• Hence, on the ground, with the heaters fully on, the heat loss $= Q_c + 2Q_p = 17,292.95$ W

$\longrightarrow$

Stage 2 — When the plane is in the air, the heat produced by the heaters and by the passengers will be lost to the ambient. There is another difference — the external heat transfer coefficient is increased substantially and $T_\infty = -56.5°C$. Calculations for stage 1 is repeated with these changes.

$$Q_c' \left\{ \frac{1}{450(291.98)} + \frac{\ln(3/2.9)}{2\pi(0.08)(30.88)} + \frac{1}{8(281.34)} \right\} = T_i - (-56.5)$$

where T_i is the steady inside temp. and Q_c' is heat loss through the curved wall.

$$Q_c' \left\{ 7.61(10^{-6}) + 0.002184 + 0.000444 \right\} = T_i + 56.5$$

$$Q_c' = \frac{T_i + 56.5}{2.6356(10^{-3})}$$

For the plane walls, for each

$$Q_p' \left\{ \frac{1}{450(7.069)} + \frac{0.05}{0.08(6.837)} + \frac{1}{8(6.605)} \right\} = T_i - (-56.5)$$

$$Q_p' \left\{ 0.000314 + 0.0914 + 0.0189 \right\} = T_i + 56.5$$

$$Q_p' = \frac{T_i + 56.5}{0.1106}; \quad \text{For 2 walls} \quad 2Q_p' = \frac{2(T_i + 56.5)}{0.1106}$$

The total heat loss from the plane equals the heat produced by heaters at max. capacity and released by the passengers.

Hence

$$225(0.12)(8)(30-T_i) + 17292.95$$
$$= T_i + 56.5 \left\{ \frac{1}{2.6356(10^{-3})} + \frac{2}{0.1106} \right\}$$

$$216(30-T_i) + 17292.95 = (T_i + 56.5)(397.5)$$

$$T_i = \frac{1314.2}{613.5} \approx 2.14\,^{\circ}C$$

The cabin will be at ≈ 2 °C. Cold!

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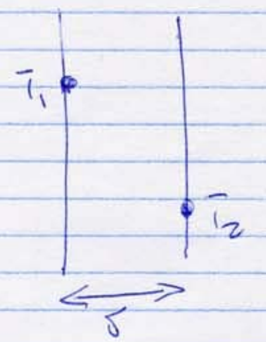
☐ Steady Heat Transfer Rates. (From Holman, Heat Transfer 9th ed. p. 25 and 29.)

Wall

1/



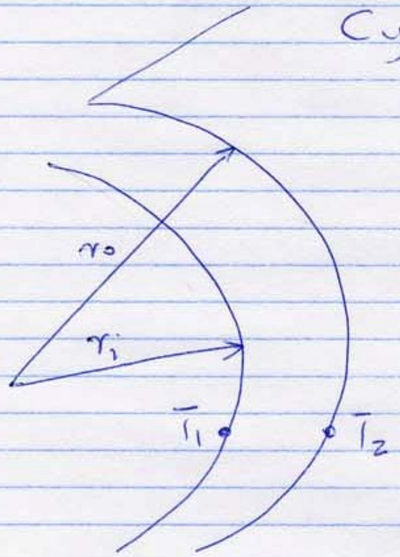
$$Q = kA\frac{(T_1 - T_2)}{\delta}$$

k = thermal conductivity of wall

A = Area

Cylinder.

2.

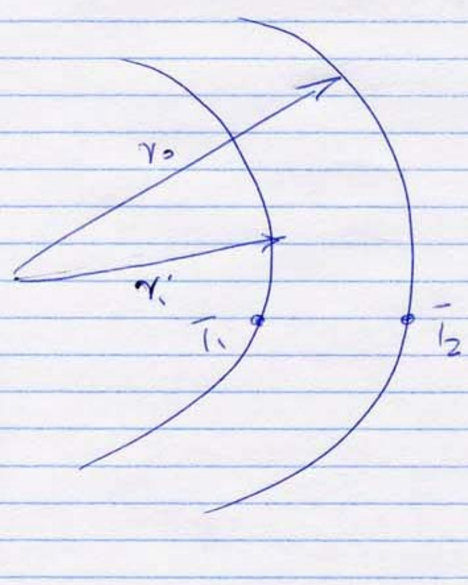


$$Q = \frac{2\pi k L (T_1 - T_2)}{\ln(r_o/r_i)}$$

where L is length of cylinder

Sphere

3



$$Q = \frac{4\pi k (T_1 - T_2)}{\frac{1}{r_i} - \frac{1}{r_o}}$$