The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #3 **September 28, 2004**

Time Allowed: 35 mins. **Name:**

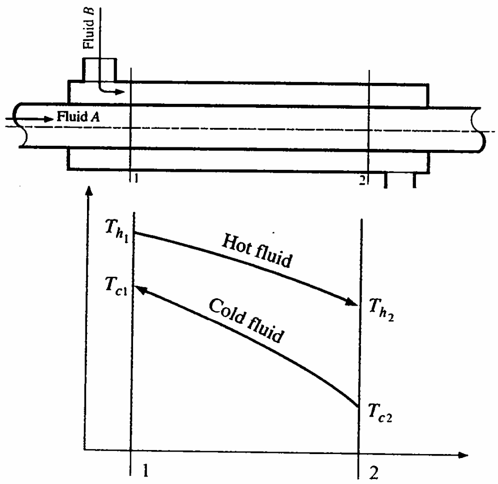
Problem #1 (4 points)

It is given that a solution of 6% by weight salt (NaCl) in water at 20°C has a density of 1041.3 kg/m^3. Calculate the mass concentrations for salt and water in the solution.

Problem #2 (6 points)

The temperature differences between the fluids at the ends of a double-pipe heat exchanger are determined to be ΔT1 and ΔT2. The log-mean temperature difference ΔTm is defined as:

$$\Delta Tm = \frac{\Delta T1 - \Delta T2}{\ln \frac{\Delta T1}{\Delta T2}}$$



What is ΔTm when ΔT1 ≈ΔT2? Show all your steps.

Extended Mean Value Theorem (Taylor series)

$f(b) = f(a) + f'(a)\,(b-a) + f''(a)\,(b-a)^2/2! + \dots f^n(a)\,(b-a)^n/n! \dots$

$\ln(1+x) = x - x^2/2 + \dots x^3/3 \dots\dots\dots (-1)^{n-1} x^n/n \dots$

$$\therefore \quad \ln \Delta T_1 = \ln \Delta T_2 + \frac{1}{\Delta T_2}(\Delta T_1 - \Delta T_2) - \frac{1}{(\Delta T_2)^2}(\Delta T_1 - \Delta T_2)^2 + \ldots$$

As $\Delta T_1 \to \Delta T_2$

$$\ln \Delta T_1 - \ln \Delta T_2 \simeq \frac{\Delta T_1 - \Delta T_2}{\Delta T_2}$$

$$\therefore \quad \Delta T_m \to \Delta T_2 \to \Delta T_1$$

$\longrightarrow$

□ Alternately, start from the series

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots (-1)^{n-1}\frac{x^n}{n} + \ldots$$

When $\Delta T_1 \to \Delta T_2$

$$\ln \frac{\Delta T_1}{\Delta T_2} = \ln\left(1 + \frac{\Delta T_1 - \Delta T_2}{\Delta T_2}\right)$$

or $$x \sim \frac{\Delta T_1 - \Delta T_2}{\Delta T_2}$$

$\therefore$ As $x \to 0$, neglect higher order terms

and $$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) \simeq \frac{\Delta T_1 - \Delta T_2}{\Delta T_2}$$

Substitute in ΔT_m definition to give

$$\Delta T_m = \Delta T_2 \doteq \Delta T_1$$

$\longrightarrow$

□ A third method is to use L'Hôpitals rule.

$$\lim_{\Delta T_1 \to \Delta T_2} \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1/\Delta T_2} \quad \text{(treat } \Delta T_2 \text{ as a constant.)} = \lim \frac{d\Delta T_1/d\Delta T_1}{\frac{d}{d\Delta T_1}\ln(\Delta T_1/\Delta T_2)} = \Delta T_1$$

$\longrightarrow$

ENCH 501 Quiz #3 Sept. 28, 2004

Problem #1

6% salt by wt ⇒ 6 kg salt in 100 kg solution

∴ Volume of 100 kg solution, $V = \frac{100}{1041.3}$

$= 0.096\ m^3$

∴ Mass conc. of salt, $\rho_i = \frac{6}{0.096}$

$= 62.478\ \frac{kg\ i}{m^3\ solution}$ →

Mass conc. of water, $\rho_w = \frac{94}{0.096}$

$= 979.167\ \frac{kg\ w}{m^3\ solution}$ →

$\rho_i + \rho_w = 62.478 + 979.167$

$= 1041.64\ kg/m^3$ solution

Problem #2

Given $\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$

$\ln \frac{\Delta T_1}{\Delta T_2} = \ln \Delta T_1 - \ln \Delta T_2$

Use Taylor series (or Extended Mean Value Theorem)

$f(b) = f(a) + f'(a)(b-a) + \frac{f''(a)}{2!}(b-a^2) + \ldots$

$\frac{f^{n-1}(a)}{(n-1)!}(b-a)^{n-1} \ldots$; $b \simeq a$

Lets $f(b) = \ln \Delta T_1$ and $f(a) = \ln \Delta T_2$

$f'(a) = \frac{1}{\Delta T_2}$, $f''(a) = -\frac{1}{(\Delta T_2)^2}$ ...