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ENCH 501: Transport Processes Quiz #3 **September 30, 2003**

Time Allowed: 50 mins. **Name:**

Problem #1 (6 points)

Compound C is produced from a reaction involving compounds A and B. The rate of production of C (in kg/hr) is proportional to the product of the instantaneous masses of A and B present. The ratio of the molar masses of A : B are 3 : 2. If 60 kg each of A and B are charged into a vessel and 15 kg of C is produced in 1 hour,

(a) obtain an expression for the amount of C present in the vessel (in kg) as a function of time.
(b) How much C is present in 2 hours?
(c) What is the maximum amount of C that can be produced?

Problem #2 (4 points)

A cavern in the shape of a torus is at a depth of 10m below the surface of a flat expanse of sand. The circular cavern houses a particle accelerator. Its cross-section has a diameter of 2m. The diameter of the external surface of the torus is 18m. The space is air-conditioned and maintained at a constant temperature of 18°C. At this temperature the cooling system consumed 12.176kW of power (the load or duty). If the effective thermal conductivity of the soil is 1.3 W/mK, estimate the temperature of the sand at the surface.

If the temperature in the cavern rapidly rose to and remained constant at 27°C, by what fraction has the cooling duty been reduced?

ENCH 501 Solution Quiz 3, Sept. 30, 2003

#1 The reaction (stoichiometric relation) is

$$A + B \longrightarrow C$$

	A	B	C
Molar Masses	3M	2M	5M
Mass of C formed (kg)			x
Moles formed or consumed	$\frac{x}{5M}$	$\frac{x}{5M}$	$\frac{x}{5M}$
Mass formed or consumed	$\frac{3x}{5}$	$\frac{2x}{5}$	x
Mass in Vessel	$60-\frac{3}{5}x$	$60-\frac{2x}{5}$	x

Given: Rate of production of C $\propto$ (mass A)(mass B)

i.e.

$$\frac{dx}{dt} = k\left(60-\frac{3x}{5}\right)\left(60-\frac{2x}{5}\right) \quad ; \text{ k unknown}$$

$$= k\frac{6}{25}(100-x)(150-x)$$

$$\frac{dx}{dt} = K_1(100-x)(150-x) \quad ; \quad K_1 = \frac{6}{25}k$$

This is a first order ODE which requires 2 specified conditions — one for the integration constant and the second to evaluate K_1.

These are $t=0$ $x=0$ condition 1

$t = 1\text{ hr}$ $x = 15\text{ kg}$ condition 2

Solve equation

$$\int_0^x \frac{dx}{(100-x)(150-x)} = K_1\int_0^t dt$$

used condition 1

Solve L.H.S. by method of partial fractions.

$$\frac{1}{(100-x)(150-x)} = \frac{A}{100-x} + \frac{B}{150-x}$$

$$= \frac{1}{50}\left[\frac{1}{100-x} - \frac{1}{150-x}\right]$$

Substitute and integrate

(using $\ln A - \ln B = \ln(A/B)$)

$$\ln\left[\frac{2}{3}\frac{(150-x)}{(100-x)}\right] = 50kt$$

Use condition 2 to obtain k

$$\ln\left[\frac{2}{3}\frac{(150-15)}{(100-15)}\right] = 50k \cdot (t = 1\text{ hr})$$

∴ we obtain

$$\ln\left[\frac{2}{3}\frac{(150-x)}{(100-x)}\right] = \ln\left[\frac{2}{3}\frac{135}{85}\right] \cdot t$$

$$= \ln\left[\frac{2}{3}\frac{135}{85}\right]^t$$

or

$$\frac{2}{3}\cdot\frac{150-x}{100-x} = \left[\frac{2}{3}\frac{135}{85}\right]^t$$

(a) $$\frac{150-x}{100-x} = \frac{3}{2}\left[\frac{270}{255}\right]^t = \frac{3}{2}\left[\frac{18}{17}\right]^t$$

Check $t = 1$, $x = 15$

LHS $= 135/85 = 1.588235$

RHS $= \frac{3}{2}\cdot\frac{18}{17} = 1.588235$

(b) $t = 2$ hrs

$$\frac{150 - x}{100 - x} = \frac{3}{2}\left[\frac{18}{17}\right]^2 = 1.68164$$

$$x = 26.65 \text{ kg/s} \longrightarrow$$

(c) The maximum amount of C is produced when all A is consumed. A is the limiting compound since it disappears faster than B. In this case

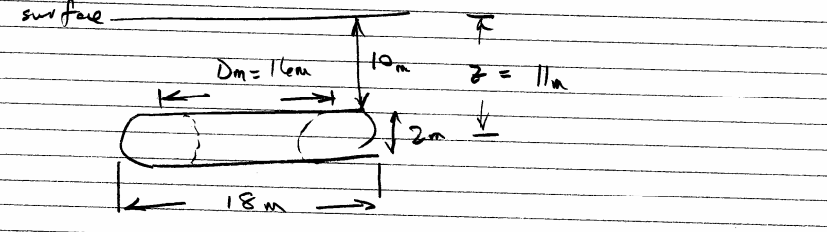
$$60 - \frac{3}{5}x = 0$$

$$x = 100 \text{ kg/s} \longrightarrow$$

This will require infinite time to occur!

#2

Side view



From provided tables, the shape factor

$$S = \frac{2\pi^2 D_m}{\ln\left(\frac{4z}{D_m}\right)}$$

where $D_m = 16$m, $z = 11$m

$$= 312.2 \text{ m}$$

The heat load is

$$Q = kS\Delta T$$

$$12176\ W = 1.3(312.2)(T_s - 18)$$

$$T_s = 48\ ^\circ C$$

When temp in chamber rises to 27°C

$$Q = 1.3(312.2)(48 - 27)$$

$$= 8523\ W$$

∴ Fractional Reduction in duty $= \frac{12176 - 8523}{12176}$

$$= 0.3$$

Load has been reduced by 30%.