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Department of Chemical & Petroleum Engineering

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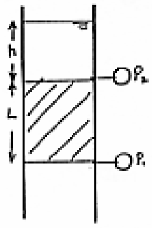
ENCH 501: Transport Processes Quiz #3 **October 8, 2002**

Time Allowed: 50 mins. **Name:**

Problem #1 (5 points)

A crude filter is made to remove organics and micro-organisms from tap water while passing through a bed of activated charcoal. The device is a straight tube, 8cm i.d., with a 18 cm long porous bed of charcoal particles at its mid section. The tube is held vertically and the charcoal bed was fully saturated with water.

If 1 litre of water is poured above the bed, how long will it take for this amount of water to pass through the bed?



Data:
Properties of water: Density, ρ = 998 kg/m^3; Viscosity, μ = 1.3 mPa.s
Property of Bed: Permeability, $\kappa = 2.32\ (10^{-11})\ m^2$

Useful Information:
Darcy's Law: Superficial velocity, $u = Q/A = -(\kappa/\mu)((p_2 - p_1 + \rho g L)/L)$

Problem #2 (5 points)

Many homes are equipped with thermostats which control the temperature through out the day and night. Overnight, a house is allowed to cool down to 12°C. At 5.30am, the furnace "kicks in" and a steady stream of warm air is blown into the house. By 6.00am, the temperature of the air in the house has reached 22°C and its maintained at this level.

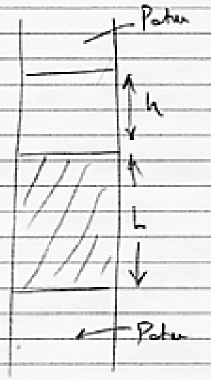
(a) If a covered cylindrical container full of milk (temp. 3°C) was taken out of the fridge at 5am and set on the counter in the kitchen, what will the milk temperature be at 6.30am? Use lumped analysis and neglect the mass of the container itself.

(b) If the milk starts to spoil when it temperature reaches 18°C, is the milk now unsafe to drink?

You are given the density of milk to be 1010 kg/m^3, the heat capacity of milk is 4.193 kJ/kg K, the heat transfer coefficient around the container was 54 W/m^2 K and the cylinder is 7 cm diameter and 15 cm tall.

ENCH 405   Quiz 3 Solution   Oct 8, 2002

Problem #1



1 litre of water added, ∴ h at t=0 is given by

$$h_o = \frac{10^{-3}\,m^3}{\pi (0.04)^2\,m^2} = 0.1989\ m$$

Darcy's equation

$$u = \frac{Q}{A} = -\frac{k}{\mu}(\rho g)\left(\frac{h+L}{L}\right)$$

But $u = \frac{dh}{dt}$

$$\therefore \quad \frac{dh}{dt} = -\beta(h+L) \quad ; \quad \beta = \frac{k(\rho g)}{\mu L}$$

with $t=0$ $h=h_o$

Solve.
$$\int_{h=h_o}^{0} dh\,(h+L) = -\beta\int_0^t dt$$

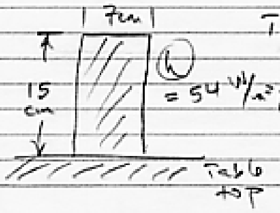
or
$$\beta t = \ln\left(\frac{h_o+L}{L}\right)$$

But
$$\beta = \frac{998(9.81)(2.32)(10^{-11})}{1.3(10^{-3})(0.18)} = 9.7067(10^{-4})$$

Substitute for β, h_o and L

$t = 766.8$ s or 12 min 46.8 s →

Problem #2



There are 3 stages

5am - 5.30am T_∞ = 12 °C, const

5.30 - 6.00am T_∞ changes 12 - 22 °C

6.00 - 6.30am T_∞ = 22 °C, const.

Stage 1 $T_\alpha = 12°C$

Energy Balance on Container (Lumped Analysis)

$$\rho V C_p \frac{dT}{dt} = hA(T_\alpha - T)$$

with T_α = const

Sol^n

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = \exp\left[-\frac{hA}{\rho V C_p} t\right]$$

where

$$\frac{A}{V} = \frac{\pi R^2 + 2\pi R L}{\pi R^2 L}$$ (bottom surface insulated!)

$$= \frac{R + 2L}{RL} = \frac{0.035 + 2(0.15)}{0.035(0.15)} = 63.8095 \text{ m}^{-1}$$

$T_\alpha = 12°C$ & $T_0 = 3°C$

$\beta = \frac{hA}{\rho V C_p} = 8.1364 (10^{-4})$

substitute when $t = 1800$ s

$$\frac{T - T_\alpha}{T_0 - T_\alpha} = 0.2512 \quad \therefore T = 9.919 °C$$

This is the initial temp for stage 2.

Stage 2 Ambient temp. varies

$$T_\alpha = 12 + \frac{1}{180} t$$ where t is in sec.

In energy balance eq. above, subst $T_\alpha(t)$

$$\frac{dT}{dt} = -\beta(T - T_\alpha) \quad \text{where } \beta = \frac{hA}{\rho V C_p}$$

$$\frac{dT}{dt} + \beta T = \beta\left(12 + \frac{1}{180}t\right)$$

Use integrating factor $e^{\int \beta dt} = e^{\beta t}$

$$e^{\beta t}\frac{dT}{dt} + \beta e^{\beta t} T = \beta e^{\beta t}\left(12 + \frac{1}{180}t\right)$$

L.H.S. $\frac{d(Te^{\beta t})}{dt} = \beta e^{\beta t}\left(12 + \frac{t}{180}\right)$

Integrate $T e^{\beta t} = \frac{12\beta}{\beta} e^{\beta t} + \frac{1}{180}\beta \frac{e^{\beta t}}{\beta}\left(t - \frac{1}{\beta}\right)$

$T = 12 + \frac{1}{180}\left(t - \frac{1}{\beta}\right) + a_o$ const. of integration

Given $t=0$, $T = 9.92\ °C$, $\beta = 8.3164\,(10^{-4})$

$a_o = 4.7474$

∴ $T = 12 + \frac{1}{180}\left(t - \frac{1}{\beta}\right) + 4.7474$

when $t = 30$ min or 1800s, $T = 20.07\ °C$

This is initial temp. for stage 3.

Stage 3 — Similar to stage 1

$\frac{T - T_\infty}{T_o - T_\infty} = 0.2312$

with $T_o = 20.07\ °C$, $T_\infty = 22\ °C$

$T = 21.55\ °C$ ⟶

(b) The milk would be spoiled. It is <u>unsafe</u> to drink.