The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Mathematical Methods in Chemical Engineering
Quiz #3

Time Allowed: 45 mins. **October 9, 2001**

Instructions for roasting a fresh turkey at 20°C in a natural convection oven maintained at 325°F include that 15 minutes be allowed for every pound of the bird. The turkey, for analysis purposes, is assumed to be a prolate ellipsoid (like a football). The meat and bone are also assumed to have the same thermophysical properties. The turkey is hollow inside but the openings are sealed off. The meat is assumed cooked when a temperature probe stuck into the thigh registers 185°F.

(A) Given a turkey that has semi-axes dimensions a = 13 inches, b = 10 inches (and the ellipse is rotated about the axis parallel to a), and a turkey weighing 5.1 kg, use the lumped analysis method to estimate the heat transfer coefficient (h) around the roasting bird.

(B) If the turkey just met the criterion that lumped analysis method is valid, estimate the thermal conductivity of the meat.

(C) How much longer would be required to cook another turkey if it were of the same external dimension as above but is not hollow, i.e. completely full of meat?

Data and Formulas:

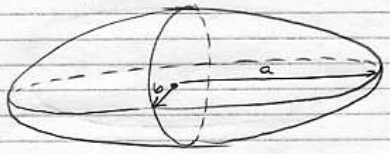
Density of turkey meat	=	120 kg/m^3
Heat Capacity, C_p	=	3.2 kJ/kg K
Volume of Ellipsoid	=	4/3 π ab^2
Surface area of ellipsoid	=	$2\pi b^2 + \frac{2\pi ab}{\varepsilon}\sin^{-1}\varepsilon$

where $\sin^{-1}\varepsilon$ *is in radians,*

eccentricity, $\varepsilon = \frac{c}{a}$ *and* $c^2 = a^2 - b^2$

ENCH 501 Solution Quiz #3 Oct. 9, 2001

Turkey



$a = 13''$ or $0.3302\ m$

$b = 10''$ or $0.254\ m$

$T_\infty = 325\ ^\circ F$ or $162.78^\circ C$; $T_0 = 20^\circ C$

Final temp, $\bar{T} = 185\ ^\circ F$ or $85^\circ C$

a) Using lumped analysis : Control volume = turkey

Input + Gen = Output + Accum.

$$h A (\bar{T}_\infty - \bar{T}) = \rho \forall C_p \frac{d\bar{T}}{dt}$$

where $\rho \forall$ = mass of turkey

Solve subject to i.c. $t = 0$, $\bar{T} = \bar{T}_0$

$$\frac{\bar{T} - \bar{T}_\infty}{\bar{T}_0 - \bar{T}_\infty} = \exp\left[- \frac{hA}{\rho \forall C_p} t \right]$$

Time allowed for cooking is $5.1\ kg \times 2.2\ lbs/kg \times 15\ min \times 60\ s$

$$t = 10098\ s$$

Volume of turkey, $\forall = \frac{4}{3}\pi a b^2 = 0.089235\ m^3$

Area of surface, $A = 2\pi b^2 + \frac{2\pi a b}{\varepsilon} \sin^{-1}\varepsilon$

Eccentricity, $\varepsilon = c/a$ where $c^2 = (0.3302)^2 - (0.254)^2$

$c = 0.211$, $\varepsilon = c/a = 0.639$

$\sin^{-1}\varepsilon = 39.715°$ or 0.6932 radians

$$A = 2\pi(0.254)^2 + 2\pi\left(\frac{(0.3302)(0.254)}{0.639}\right) \times 0.6932$$

$$= 0.977\ m^2$$

Substitute into temp – time equation

$$\frac{85 - 162.78}{20 - 162.78} = \exp\left[-\frac{h(0.977)}{5.1\,(3200)} \cdot 1.098\right]$$

$$0.5447 = \exp\left[-0.6045\,h\right]$$

$$h = 1.005 \quad W/m^2K. \longrightarrow$$

(b) Criterion is $\frac{h(V/A)}{k} \leq 0.1$; p 70 Notes

$$\therefore \frac{1.005\left(\frac{0.089235}{0.977}\right)}{k} = 0.1$$

$$\therefore k = 0.918 \quad W/m\,K \longrightarrow$$

(c) If the turkey were solid, mass $= \rho . V$

$= (120)(0.089235) = 10.708$ kg

Total time required $= 10.708(2.2)(15) = 353.37$ min →

Extra time required $= 185.06$ minutes →