The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Mathematical Methods in Chemical Engineering
Quiz #3

Time Allowed: 50 mins. **October 10, 2000**

Student's Name: ______________________

A ceramic container is a cylinder which has an external diameter of 16 cm and a wall thickness of 1 cm. The height (measured from the outside) is 20 cm. This open-top cylinder is filled with an ethylene glycol-water mixture and chilled to -25°C.

The container is then suddenly placed on an insulating surface in a large reservoir of air free of moisture but at a temperature of 18°C. You may assume that the glycol mixture is constantly well stirred and, at any instant, the mixture and the container wall are at exactly the same temperature. Also assume that the mixture is not volatile. The heat transfer coefficient for air around the vertical surface of the ceramic is 60 W/mK. The coefficient above the horizontal surface is 38 W/mK.

Given the following properties:

Ceramic :	ρ = 2300 kg/m^3 ;	C_p = 0.96 kJ/kg K
Glycol Solution:	ρ = 1070 kg/m^3 ;	C_p = 2.54 kJ/kg K
Water:	ρ = 999.8 kg/m^3 ;	C_p = 4.22 kJ/kg K
Ice:	ρ = 916.8 kg/m^3 ;	C_p = 2.1 kJ/kg K
Heat of fusion of water	= 333.4 kJ/kg	

(a) Derive a relationship between the temperature of the glycol solution and time.

(b) After how long will the ceramic container attain a temperature of 10°C?

(c) How would your answer to part (b) change if the container had been filled with ice at -25°C rather than glycol solution?

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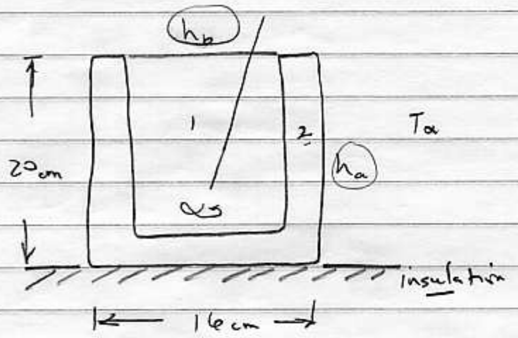

This is a lumped analysis problem.
Let the temperature of the liquid / ceramic at any instant be T and the ambient air at T_α.

The volume of the glycol mixture $= \frac{\pi D^2}{4} \cdot H$; $D = 14$ cm, $H = 19$ cm

i.e. $V_L = \frac{\pi (0.14)^2}{4} \cdot 0.19 = 2.925 (10^{-3})\, m^3$

The volume of the ceramic container $= \frac{\pi d^2}{4} \cdot h - V_L$; $d = 16$ cm, $h = 20$ cm.

$$V_s = \frac{\pi (0.16)^2}{4} \cdot 0.2 - 2.925 (10^{-3})$$
$$= 1.096 (10^{-3})\, m^3$$

(a)

Energy Balance on the ceramic container and solution

input + Gen = Output + Accum.

(Gen ≈ 0, Output ≈ 0)

$h_a (\pi d h)(T_\alpha - T)$ for side
$+$
$h_b \left(\pi \frac{d^2}{4}\right)(T_\alpha - T)$ for top

$(m_1 C_{p_1} + m_2 C_{p_2}) \frac{dT}{dt}$

where
$m_1 = V_L \rho_L$
$m_2 = V_s \rho_s$

(note that bottom is insulated)

$$(m_1 C_{p_1} + m_2 C_{p_2}) \frac{dT}{dt} = (h_a A_a + h_b A_b)(T_a - T)$$

$$\frac{dT}{T - T_a} = -\left[\frac{h_a A_a + h_b A_b}{m_1 C_{p_1} + m_2 C_{p_2}}\right] dt$$

Integrate subject to : $t = 0$, $T = T_o$ (initial condition)

$$\frac{T - T_a}{T_o - T_a} = \exp\left[-\frac{h_a A_a + h_b A_b}{m_1 C_{p_1} + m_2 C_{p_2}}\right] t$$

Substitute values, $T_a = 18°C$; $T_o = -25°C$

$h_a = 60$ W/m K , $h_b = 38$ W/m K

$A_a = \pi d h = \pi(0.16)(0.2) = 0.1005$ m²

$A_b = \pi d^2/4 = \pi(0.16)^2/4 = 0.0201$ m²

$m_1 = V_L \rho_L = 2.925(10^{-3})(1070) = 3.1298$ kg

$m_2 = V_s \rho_s = 1.096(10^{-3})(2300) = 2.5208$ kg

$C_{p_1} = 2540$ J/kg K

$C_{p_2} = 960$ J/kg K.

$$\frac{T - 18}{-25 - 18} = \exp\left[-\frac{6.7938}{10369.66}\right] t \quad ; \quad t \text{ in secs.}$$

or $$\frac{18 - T}{43} = \exp\left[-6.5516(10^{-4})\, t\right]$$ →

(b) When $T = 10$ °C,

$t = 2{,}567$ s or 42.78 minutes. →

(c) Heat transfer to the ice occurs in 3 phases, – sensible heat to the ice as solid, to melt the ice at a constant temperature and sensible heat to the water.

Phase 1 – ice – assume no temperature gradients
from $T = -25°C$ to $T = 0°C$

$$\frac{0-18}{-25-18} = \exp\left[-\frac{6.7938}{(2.925)(10^{-3})(916.8)(2100) + 2419.968}\right] t_1$$

$$= \exp\left[-\frac{6.7938}{8051.4} t_1\right] = \exp\left[-8.438(10^{-4})t\right]$$

$$t_1 = 1032.03\,s$$

Phase 2 – melting at constant temp. – 0°C

$$\frac{1}{t_2}\left[2.925(10^{-3})(916.8)(333.4)(10^3)\right] = (h_a A_b + h_b A_b)\Delta T$$

$$= 6.7938(18)$$

$$t_2 = 7311.07\,s$$

Phase 3 – sensible heat into agitated water, $T_0 = 0°C$!
Also mass ice = mass water!

$$\frac{10-18}{0-18} = \exp\left[-\frac{6.7938}{(2.925)(10^{-3})(916.8)(4220) + 2419.968}\right] t_3$$

mass ice = mass water

$$t_3 = 1639.63\,s$$

Total time $= t_1 + t_2 + t_3 = 9982.73$, or 166.38 mins.