ENCH 501 F2017 **Quiz #2** Name:______________________________

September 26, 2017 Time Allowed: 45 minutes aJ

1. The Archimedes number $Ar = \frac{gL^3\rho_L(\rho-\rho_L)}{\mu^2}$ relates the relative effects, during heat transfer, of free and forced convection of a fluid with density ρ_L and viscosity μ on a solid body with characteristic dimension L and density ρ.

   If g = 9.81 ± 0.0001 m/s^2, L = 0.42 ± 0.02 m, ρ_L = 998 ± 5 kg/m^3, ρ= 2400 ± 8 kg/m^3 and μ= 1.5 ± 0.1 mPa s, estimate Ar and its errors if the data are correlated and if they are independent.

2. In some fluidized bed reactors, particles are suspended by fluids flowing, from the bottom, through vertical cylinders. Consider a system where the particles are uniform spheres, each of diameter **d** mm, at a temperature that is higher than the temperature of the suspending fluid. If the heat transfer coefficient around each sphere is **h** W/m^{2}K, its thermal conductivity and heat capacity are **k** W/m K and $\mathbf{C_p}$ kJ/kg K, the average velocity of the fluid is **U** m/s, its density and viscosity are $\boldsymbol{\rho}$ kg/m^3 and $\boldsymbol{\mu}$ mPa s, find the dimensionless groups that describe heat exchange between the spheres and the fluid.

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1. $Ar = f = \dfrac{gL^3\rho_L(\rho-\rho_L)}{\mu^2}$; let $x = \rho - \rho_L$

Substitute values, all in SI units.

$\mu = (1.5 \pm 0.1)(10^{-3})$ Pa.s, all others are SI

$$Ar = \frac{(9.81)(0.42)^3(998)(2400-998)}{(1.5(10^{-3}))^2}$$

$$= 4.5197\,(10^{11}) \quad >> 1$$

∴ Free convection is much more important than forced for this system.

Because f is expressed in terms of the other variables in an equation, suggest the use of the method of propagation of errors.

For <u>correlated</u> data, with $x = \rho - \rho_L$

$$\Delta f = \left|\frac{\partial f}{\partial L}\Delta L\right| + \left|\frac{\partial f}{\partial \rho_L}\Delta\rho_L\right| + \left|\frac{\partial f}{\partial x}\Delta x\right| + \left|\frac{\partial f}{\partial \mu}\Delta\mu\right|$$

The error for g is neglected as $\Delta g/g <<< 1$.

The expected values of all the variables are given, except for $x = (2400-998) \pm (5+8)$ kg/m^3

$x = 1402 \pm 13$ kg/m^3

Now $\dfrac{\partial f}{\partial L} = 3\left(\dfrac{gL^2\rho_L x}{\mu^2}\right) = \dfrac{3f}{L}$; $\dfrac{\partial f}{\partial \rho_L} = \dfrac{gL^3 x}{\mu^2} = \dfrac{1}{\rho_L}f$

$\dfrac{\partial f}{\partial x} = \dfrac{gL^3\rho_L}{\mu^2} = \dfrac{1}{x}f$; $\dfrac{\partial f}{\partial \mu} = -\dfrac{2}{\mu^3}(gL^3\rho_L x) = -\dfrac{2}{\mu}f$

$$\therefore \quad \Delta f = 3f\left(\frac{\Delta L}{L}\right) + f\left(\frac{\Delta \rho_L}{\rho_L}\right) + f\left(\frac{\Delta x}{x}\right) + 2f\frac{\Delta \mu}{\mu}$$

$$\frac{\Delta f}{f} = 3\left(\frac{0.02}{0.42}\right) + \frac{5}{998} + \frac{13}{1402} + 2\left(\frac{0.1}{0.5}\right)$$

$$= \quad 0.1429 \quad 0.00501 \quad 0.009272 \quad 0.13333$$

$$= \pm\ 0.29047$$

Errors for ρ_L and x are small.

Check for the neglect of g. The relative error is

$$\frac{10^{-4}}{9.81} \simeq 10^{-5}.$$ This is much smaller than the rest.

The absolute error

$$\Delta f = \pm\ 1.3129\,(10^{11})$$

For independent data

$$\Delta x = \pm\sqrt{5^2+8^2} = \pm\ 9.434$$

and

$$\Delta f = \pm\sqrt{\left(\frac{\partial f}{\partial L}\Delta L\right)^2 + \left(\frac{\partial f}{\partial \rho_L}\Delta \rho_L\right)^2 + \left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial \mu}\Delta \mu\right)^2}$$

Substitute numbers as above

$$\frac{\Delta f}{f} = \pm\ 0.1956$$

2. The variables are $h, k, C_p, u, \rho, \mu, d$

with units $\frac{W}{m^2 K}$, $\frac{W}{mK}$, $kJ/kg\,K$, m/s, kg/m^3, $Pa.s$, m

These dimensions are obvious $\longrightarrow$ L/t, M/L^3, L

The rest have to be derived.

$W = J/s$; $J = F.L = M.a.L = \frac{ML^2}{t^2}$ $\therefore W \equiv \frac{ML^2}{t^3}$

$Pa = F/A = \frac{m.a}{A} = \frac{ML}{L^2 t^2} = \frac{M}{Lt^2}$

Dimensions

h	k	C_p	u	ρ	μ	d
$\frac{ML^2}{t^3}\frac{1}{L^2 T}$	$\frac{ML^2}{t^3}\frac{1}{LT}$	$\frac{ML^2}{t^2}\frac{1}{MT}$	$\frac{L}{t}$	$\frac{M}{L^3}$	$\frac{M}{Lt^2}.t$	L
$\frac{M}{Tt^3}$	$\frac{ML}{Tt^3}$	$\frac{L^2}{Tt^2}$	$\frac{L}{t}$	$\frac{M}{L^3}$	$\frac{M}{Lt}$	L

There are 4 fundamental dimensions — M, L, t, T

$\therefore$ There are $7 - 4 = 3$ dimensionless groups.

One can immediately be found by inspection —

$$\Pi_1 = \frac{hd}{k}$$ $\therefore$ drop one variable, say h.

Choose any 4 repeating variables — e.g. C_p, μ, ρ, d

$$\Pi_2 = C_p^a \mu^b \rho^c d^e k^f$$

$$\Pi_3 = C_p^{a'} \mu^{b'} \rho^{c'} d^{e'} u^{f'}$$

Apply the Buckingham Pi Theorem

$$\Pi_2 = \left[\frac{L^2}{Tt^2}\right]^a \left[\frac{M}{Lt}\right]^b \left[\frac{M}{L^3}\right]^c \left[L\right]^e \left[\frac{ML}{Tt^3}\right]^f$$

$$\left.\begin{array}{lll} M & 0 = & b + c + f \\ L & 0 = & 2a - b - 3c + e + f \\ t & 0 = & -2a - b - 3f \\ T & 0 = & -a - f \end{array}\right\} \quad \text{solve} \quad \begin{array}{l} a = -f \\ b = -f \\ c = 0 \\ e = 0 \end{array}$$

$$\Pi_2 = c_p^{-f} \mu^{-f} k^f = \left(\frac{k}{\mu c_p}\right)^f$$

Let $f = -1$ $\quad \Pi_2 = \left(C_p \mu / k\right) =$ Prandtl #, P_r

apply the rule for Π_3

$$\Pi_3 = \left[\frac{L^2}{Tt^2}\right]^{a'} \left[\frac{M}{Lt}\right]^{b'} \left[\frac{M}{L^3}\right]^{c'} \left[L\right]^{e'} \left[\frac{L}{t}\right]^{f'}$$

$$\Pi_3 = \left(\frac{du\rho}{\mu}\right)^{f'}$$

Let $f' = 1$, $\quad \Pi_3 = \dfrac{du\rho}{\mu} = Re$

$\therefore$ The three dimensionless groups are

$$\underset{Nu}{\frac{hd}{k}} = f\left(\underset{Re}{\frac{du\rho}{\mu}}, \underset{Pr.}{\frac{C_p\mu}{k}}\right)$$

$\longrightarrow$