ENCH 501 Transport Phenomena University of Calgary Name________________________________

Quiz #2 / **Time Allowed**: 35 minutes Only a "cheat sheet" is allowed. September 22, 2015 AJ

Q. 1 (5 points)

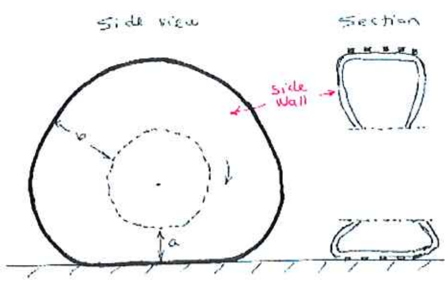
Crystals of salt are to be dissolved in water in a large cylindrical tank. The tank's diameter is $\mathbf{D_t}$ (m) and the solution within is agitated by an impeller of diameter $\mathbf{D_i}$ (m). The impeller is rotated at a constant rate $\boldsymbol{\omega}$ (1/s). At the surface of a particle with diameter **d**, salt dissolves in water and diffuses away from the wall at a rate dependent on $\mathbf{D_{AB}}$ (the diffusivity, m^2/s) and the convective motion in the tank moves the dissolved salt into the bulk liquid at a rate proportional to $\mathbf{k_L}$ (the mass transfer coefficient, m/s). The viscosity $\boldsymbol{\mu}$ and the density of the fluid $\boldsymbol{\rho}$ are also important. Determine the dimensionless groups.

Q. 2 (5 pts)

Vehicles are rated for the maximum extra load they can carry. This maximum load is based primarily on the total load **F** that each tire can carry without failure. A car's owner's manual specifies the load and the recommended inflation pressure for the tires of the vehicle. The cold pressure **P** in the tires is often ignored. Proper inflation keeps the shape of the tire as round as possible to minimize flexing of the side walls as the tire rotates. Flexing generates heat and this weakens the tire wall structure. Under-inflated tires or tires with heavy loads are deformed more than normal and they wear out faster or may disintegrate. High average driving speeds **V** and weaker wall construction (equate to wall thickness, $\boldsymbol{\delta}$) decrease the service life $\boldsymbol{\tau}$ of tires.

a) Obtain the dimensionless groups from the dependence of service life on other variables.

b) Draw a sketch of dimensionless service time versus dimensionless load for a tire, as you would expect.



ENCH 501 Solution Quiz 2 Sept. 22, 2015

1 The variables are:

$$d \sim f(D_{AB}, k_L, \omega, D_t, D_i, \mu, \rho)$$

8 variables in 3 dimensions - M, L, t

Hence 5 dimensionless groups are to be formed.

By inspection, $\Pi_1 = \frac{D_i}{D_t}$ and $\Pi_2 = d/D_i$ are dimensionless.

The other 3 can also be obtained by inspection - e.g.

$$\frac{D_{AB}}{k_L d}, \quad \frac{\omega D_i}{k_L} \quad \text{and} \quad \frac{D_i^2 \omega \rho}{\mu} \quad \text{but}$$

let's get them by the Pi theorem,

Since 2 dimensionless groups for geometric similarity have been identified, 2 of the variables should be dropped - say D_i + D_t

$$\therefore \quad d = \text{function}(D_{AB}, k_L, \omega, \mu, \rho)$$

Units	m	m^2/s	m/s	$\frac{1}{s}$	Pa.s	kg/m^3
Dimensions	L	L^2/t	$\frac{L}{t}$	$\frac{1}{t}$	$\frac{M}{Lt}$	$\frac{M}{L^3}$

Choose 3 variables that do not form a dimensionless group - (same as # of dimensions)

Note: D_{AB}, k_L and ω cannot be chosen,

as $\frac{D_{AB}\,\omega}{k_L^2}$ is dimensionless.

Let's choose D_{AB}, k_L and ρ

$$\Pi_3 = D_{AB}^{a} k_L^{b} \rho^{c} d^{d}$$

$$\Pi_4 = D_{AB}^{a'} k_L^{b'} \rho^{c'} w^{d'}$$

$$\Pi_5 = D_{AB}^{a''} k_L^{b''} \rho^{c''} \mu^{d''}$$

For Π_3 $$M^0 L^0 t^0 = \left[\frac{L^2}{t}\right]^{a} \left[\frac{L}{t}\right]^{b} \left[\frac{M}{L^3}\right]^{c} \left[L\right]^{d}$$

$$\left.\begin{array}{lll} L & 2a + b - 3c + d & = 0 \\ M & c & = 0 \\ t & -a - b & = 0 \end{array}\right\} \begin{array}{l} a = -b = -d \\ b = d \\ c = 0 \end{array}$$

$$\Pi_3 = \left[D_{AB}^{-1} k_L d\right]^{d} \quad \text{or} \quad \frac{k_L d}{D_{AB}} \quad \text{if } d = 1$$

For Π_4 $$M^0 L^0 t^0 = \left[\frac{L^2}{t}\right]^{a'} \left[\frac{L}{t}\right]^{b'} \left[\frac{M}{L^3}\right]^{c'} \left[\frac{1}{t}\right]^{d'}$$

$$\left.\begin{array}{lll} L & 2a' + b' - 3c' & = 0 \\ M & c' & = 0 \\ t & -a' - b' - d' & = 0 \end{array}\right\} \begin{array}{l} a' = d' \\ b' = -2d' \\ c' = 0 \end{array}$$

$$\Pi_4 = \left[D_{AB} k_L^{-2} w\right]^{d'} \quad \text{or} \quad \frac{D_{AB} w}{k_L^2} \quad \text{if } d' = 1$$

For Π_5 $\quad M^0 L^0 t^0 = \left[\frac{L^2}{t}\right]^{a''}\left[\frac{L}{t}\right]^{b''}\left[\frac{M}{L^3}\right]^{c''}\left[\frac{M}{Lt}\right]^{d''}$

$$\left.\begin{array}{lrr} L & 2a'' + b'' - 3c'' - d'' & = 0 \\ M & c'' + d'' & = 0 \\ t & -a'' - b'' \quad - d'' & = 0 \end{array}\right\} \quad \begin{array}{l} a'' = -d'' \\ b'' = 0 \\ c'' = -d'' \end{array}$$

$$\Pi_5 = \left[D_{AB}^{-1}\rho^{-1}\mu\right]^{d''} \quad \text{or} \quad \frac{\mu}{D_{AB}\rho} \quad \text{for } d'' = 1$$

Hence we obtain from the Buckingham Pi theorem that

$$\frac{d}{D_i} = f\left(\frac{D_i}{D_t}, \frac{k_L d}{D_{AB}}, \frac{D_{AB}\omega}{k_L^2}, \frac{\mu}{D_{AB}\rho}\right)$$

Π_3, Π_4 and Π_5 appear different from the ones obtained by inspection. They are related;

$$\frac{D_{AB}}{k_L d} = \Pi_3^{-1}$$

$$\frac{D_i\omega}{k_L} = \frac{D_{AB}\omega}{k_L^2} \cdot \frac{k_L d}{D_{AB}} \cdot \frac{D_i}{d} = \Pi_4 \cdot \Pi_3 \cdot \Pi_2^{-1}$$

$$\frac{D_i^2\omega\rho}{\mu} = \frac{D_{AB}\rho}{\mu} \cdot \frac{D_{AB}\omega}{k_L^2} \cdot \left(\frac{k_L d}{D_{AB}}\right)^2 \cdot \left(\frac{D_i}{d}\right)^2 =$$

$$\Pi_5^{-1}\,\Pi_4\,\Pi_3^2\,\Pi_2^{-2}$$

2 service life $\tau = f(\text{Load, Pressure, Speed, wall thickness})$

		F	P	V	δ
units	s	N	Pa (N/m^2)	m/s	m
dimensions	$[t]$	$\left[\frac{ML}{t^2}\right]$	$\left[\frac{M}{Lt^2}\right]$	$\left[\frac{L}{t}\right]$	$[L]$

(a) There are 3 fundamental dimensions and 5 variables. ∴ There are 2 dimensionless gps.

These can be found by inspection -

$$\Pi_1 = \frac{V\tau}{\delta} \quad \text{and} \quad \Pi_2 = \frac{F}{\delta^2 P}$$

Each can be obtained using the Pi theorem. E.g.

$$\Pi_2 = \delta^a V^b P^c F^d \quad \text{— with } \delta, V + P \text{ 'repeating'}$$

$$M^0 L^0 t^0 = [L]^a \left[\frac{L}{t}\right]^b \left[\frac{M}{Lt^2}\right]^c \left[\frac{ML}{t^2}\right]^d \Rightarrow \begin{aligned} a &= -2d \\ b &= 0 \\ c &= -d \end{aligned}$$

$$\text{or} \quad \Pi_2 = \left[\delta^{-2} P^{-1} F\right]^d \quad \text{or} \quad F/\delta^2 P$$

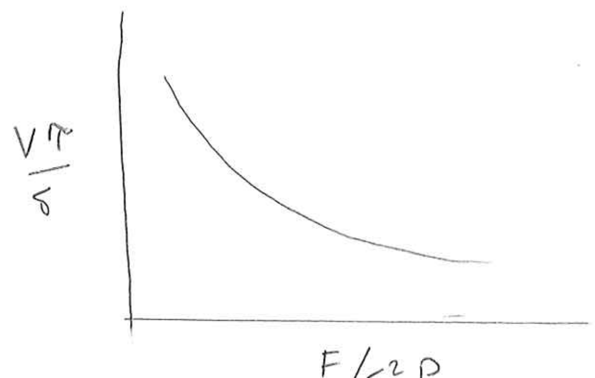
(b) From the statement of problem

$$\tau \propto 1/F,\ 1/V,\ \delta,\ P$$

$$\frac{V\tau}{\delta} = f\left(\frac{F}{\delta^2 P}\right)$$

$$\text{OR} \quad \tau \sim \frac{\delta}{V}\left(\frac{F}{\delta^2 P}\right)^{\phi}$$

where $\phi < 0$



Additional Information – Q 2.

1. A tire is inflated to 35 psig off a vehicle. It is now mounted and the vehicle lowered to the ground so that the tire now carries part of the weight of the vehicle. What happens to the tire pressure – increases, stays the same or decreases?

A. The pressure stays the same. The tire is not a balloon that applied external pressure tends to contract. The tire deforms in shape however.

2. When hot, tire pressures are 5–6 psia ($> \frac{1}{3}$ atm) above the cold tire pressure.