

Quiz 2

Time Allowed: 40 minutes Only a "cheat sheet" can be consulted.

Sept 30, 2014 AJ

Q 1 (10 points) The data below were collected on sockeye salmon swimming in water *enroute* to spawning. Each fish was observed many times and only the average values for each fish is reported in the table by a single individual using a stopwatch to time a fish travelling between two markers.

- (a) Estimate the mean speed for the group of fishes and the error in the value. Show your steps.
- (b) If instead of all the data, you were given the data for the first 10 fishes, what error would you estimate for the mean value of speed for the 10 fishes? How does your result compare with the mean speed from part (a)?

Table - Estimates of swimming speed of sockeye salmon tracked between Vancouver Island and the mainland of British Columbia (Quinn, Thomas P., 1988, Can. J. Zool. v66, p2160-3)

Fish	No. of observations	Av. Swimming speed, cm/s			
1	12	68.5			
2	6	79.3			
3	4	55.6			
4	3	59.7			
5	4	73.8			
6	9	91			
7	4	38.2			
8	5	59.1			
9	3	36.5			
10	1	32.2			
11	7	95			
12	11	106			
13	9	51.2			
14	7	108			
15	5	32.8			
16	6	42.4			
17	4	33.2			
18	13	67.4			
19	5	102.6			
20	3	31.9			
21	9	69.8			
22	7	49.5			
23	10	39.5			
24	8	73.3			
25	4	51.9			
		$\bar{x} = 61.936$			

Inspect the data. There are no outliers because both the low and high speeds are from many observations. Thus all the values need to be used.

Q 2 (Bonus 2 points) On what variables would the settling time for sand grains in a bitumen extraction unit depend if the tank was not being continuously agitated? The suspension was initially thoroughly mixed. What dimensionless group(s) can you form? Assume the particles do not interact with or hinder one another.

#	obs	ave u	abs(u-uave)	abs(u-uave) ²
1	12	68.5	6.564	43.086096
2	6	79.3	17.364	301.508496
3	4	55.6	6.336	40.144896
4	3	59.7	2.236	4.999696
5	4	73.8	11.864	140.754496
6	9	91	29.064	844.716096
7	4	38.2	23.736	563.397696
8	5	59.1	2.836	8.042896
9	3	36.5	25.436	646.990096
10	1	32.2	29.736	884.229696
11	7	95	33.064	1093.228096
12	11	106	44.064	1941.636096
13	9	51.2	10.736	115.261696
14	7	108	46.064	2121.892096
15	5	32.8	29.136	848.906496
16	6	42.4	19.536	381.655296
17	4	33.2	28.736	825.757696
18	13	67.4	5.464	29.855296
19	5	102.6	40.664	1653.560896
20	3	31.9	30.036	902.161296
21	9	69.8	7.864	61.842496
22	7	49.5	12.436	154.654096
23	10	39.5	22.436	503.374096
24	8	73.3	11.364	129.140496
25	4	51.9	10.036	100.721296
sum	159	61.936	506.81	14341.52

$\bar{u} = 61.936$
 $\sum |u_i - \bar{u}| = 506.81$
 $\sum (u_i - \bar{u})^2 = 14341.52$

25 fishes	
u_ave	61.936
error	20.272
σ	24.445
u = 61.936 (+/-) 20.272	
u = 61.936 (+/-) 24.445	

10 fishes	
ave	59.390
error	15.517
σ	19.658
error in mean	
4.907	
u_ave_est = 59.390 (+/-) 4.907	
u_ave_est = 59.390 (+/-) 6.216	

1. (a) Mean speed for 25 fishes = 61.9 cm/s

Estimate errors by Ave. deviation or standard deviation method. Either will be accepted.

$$\text{Av. deviation } \Delta u = \pm \frac{\sum_{i=1}^{25} |u_i - \bar{u}|}{N} = \pm \frac{506.81}{25} = \pm 20.272 \text{ cm/s}$$

or by Standard deviation

$$\Delta u = \pm \sqrt{\frac{\sum_{i=1}^{25} (u_i - \bar{u})^2}{N-1}} = \pm \sqrt{\frac{14341.52}{24}} = \pm 24.445 \text{ cm/s}$$

Expected value =

$$61.9 \pm 20.3 \quad \text{or} \quad 61.9 \pm 24.4 \text{ cm/s}$$

Note the truncation of reported answer.

(b) For first 10 fishes

$$\text{Mean Speed, } \bar{u}_{\text{est}} = 59.39 \approx 59.4 \text{ cm/s}$$

Error in mean, by Av. deviation is

$$\Delta u = \pm \frac{\sum_{i=1}^{10} |\bar{u}_i - \bar{u}_{\text{est}}|}{N} = \frac{155.172}{10} \approx 15.5$$

$$\text{Error in mean, } \Delta \bar{u} = \frac{15.517}{\sqrt{10}} = 4.907$$

$$\bar{u}_{\text{expected}} = \bar{u}_{\text{est}} \pm \Delta \bar{u}$$

$$= 59.4 \pm 4.9 \longrightarrow$$

If the standard deviation is used, for 10 fishes

$$\Delta u = \pm \sqrt{\frac{3,477.93}{9}} = \pm 19.658$$

$$\text{Error in mean, } \pm \Delta \bar{u}_{\text{st}} = \frac{19.658}{\sqrt{10}} = 6.216$$

$$\bar{u}_{\text{expected}} = 59.4 \pm 6.2 \text{ cm/s} \longrightarrow$$

$\frac{2}{t}$	$= f(D, \Delta\rho, \mu, g)$
time	grain size Density difference Lig. viscosity gravity
Units	s m $\frac{\text{kg}}{\text{m}^3}$ Pa.s m/s^2

$$\text{Dimensions } [t] \quad [L] \quad \left[\frac{M}{L^3}\right] \quad \left[\frac{M}{Lt}\right] \quad \left[\frac{L}{t^2}\right]$$

There are 3 dimensions and 5 variables.

$$\text{Let } \pi_1 = D^a \Delta\rho^b \mu^c g^d \quad \text{and } \pi_2 = d'^a \rho'^b \mu'^c t'^d$$

where $D, \Delta\rho$ and μ were chosen as repeating variables.

$$\pi_1 = [L]^a \left[\frac{M}{L^3}\right]^b \left[\frac{M}{Lt}\right]^c \left[\frac{L}{t^2}\right]^d$$

$$\begin{array}{lcl} \text{mass} & 0 = & b + c \\ \text{length} & 0 = & a - 3b - c + d \\ \text{time} & 0 = & -c - 2d \end{array} \quad \left\{ \begin{array}{l} a = 3d \\ b = 2d \\ c = -2d \end{array} \right.$$

$$\text{or } \pi_1 = \left[\frac{D^3 \Delta\rho^2 g}{\mu^2} \right]^d \quad \text{Similarly } \pi_2 = \left[\frac{\Delta\rho D^2}{\mu t} \right]^d \longrightarrow$$