GJ Sept 25

University of Calgary - Department of Chemical & Petroleum Engineering

Fall 2012

ENCH 501 Transport Phenomena **Quiz #2** **Name** ____________________

Time Allowed: 35 minutes

Question #1 (4 points)

An object is to be photographed. The object, as estimated by the photographer, is at a distance x = 5±0.1 m from the lens. The image in the camera, as specified by the manufacturer, is at a distance y = 3±0.05 cm from the lens.
Estimate the focal length (***f***) of the lens and the error if ***f*** is given by the expression ***f*** = xy/(x+y).

Question #2 (6 points)

It desired to estimate the vapor pressure of propane. The data available from a laboratory are at -42.05±1°C (vapor pressure equal 1±0.06 atm.) and at 2.09±1°C (vapor pressure equal 5±0.06 atm.)
Estimate the ***latent heat of vaporization*** (kJ/kmol), ***the vapor pressure*** and their ***errors*** at -25.09°C.
The Clasius-Clapeyron equation is: $\boldsymbol{ln}(P_2/P_1) = (\Delta H_v/R)(1/T_1 - 1/T_2)$; where R is the universal gas constant equal 8.314 kJ/kmol K.

ENCH 501 Quiz #2 Solution Sept. 25, 2012

#1 $f = \dfrac{xy}{x+y}$; x and y are independent

$$\therefore \Delta f \doteq \pm \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\Delta y)^2}$$

$$\doteq \pm \frac{\sqrt{x^4 (\Delta y)^2 + y^4 (\Delta x)^2}}{(x+y)^2}$$

Subst. Values

$$\Delta f = \pm \frac{\sqrt{5^4 (0.0005)^2 + (0.03)^4 (0.1)^2}}{(5.03)^2}$$

$$= \pm \frac{0.0125}{(5.03)^2} = \pm 0.000494 \text{ m}$$

The focal length

$$f = \frac{5(0.03)}{5.03} = 0.02982 \text{ m}$$

$$\therefore f = 2.982 \pm 0.0494 \text{ cm}$$

$\longrightarrow$

#2 The Clasius - Clapeyron eq. is used to interpolate

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_v}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

with T is absolute scale

P_1 = 1atm , T_1 = −42.05 ± 1 °C or 231.1 K

P_2 = 5 atm , T_2 = 275.24 K

The data is treated as correlated as they are from the same lab.

Estimate ΔHv first at the 2 conditions

$$\ln\left(\frac{5}{1}\right) = \frac{\Delta H_v}{8.314}\left[\frac{1}{231.1} - \frac{1}{275.24}\right]$$

$$\Delta H_v = 19{,}282.5 \text{ kJ/kmol}$$

$$= 438.239 \text{ kJ/kg}$$

(In tables, ΔH_v @ 231.1 K is 426.2 kJ/kg, and @ 275.24 K, is 373.1 kJ/kg. Estimated ΔH_v is outside of this range.)

With calculated ΔH_v, estimate P_{vp} at −25.09 °C or 248.06 K

$$\ln\left(\frac{P_{vp}}{1}\right) = \frac{19282.5}{8.314}\left[\frac{1}{231.1} - \frac{1}{248.06}\right]$$

$$P_{vp} = 1.986 \text{ atm.}$$

Now to estimate the errors.

* for ΔH_v, apply the Clausius-Clapeyron eq. from $P = 1.986$ atm to 5 atm

i.e. $$\ln\left(\frac{5}{1.986}\right) = \frac{\Delta H_{v_1}}{R}\left[\frac{1}{248.06} - \frac{1}{275.24}\right]$$

$$\Delta H_{v_1} = 19{,}283.19 \text{ kJ/kmol}$$

This is very close to the value for P from 1 → 5 atm.

Therefore, estimate the max. & min for P = 1 → 5 atm.

max $$\ln\left(\frac{5.06}{0.94}\right) = \frac{\Delta H_{v_{max}}}{R}\left[\frac{1}{232.1} - \frac{1}{274.24}\right]$$

$$\Delta H_{v_{max}} = 21{,}138.2 \text{ kJ/kmol}$$

min $$\ln\left(\frac{4.94}{1.06}\right) = \frac{\Delta H_{v_{min}}}{R}\left[\frac{1}{230.1} - \frac{1}{276.24}\right]$$

$$\Delta H_{v_{min}} = 17{,}627.94 \text{ kJ/kmol}$$

ΔH_v kJ/kmol	17,627.94 min	19 282.5 estimate	21,138.2 max
	$\underbrace{\qquad}_{1654.6}$		$\underbrace{\qquad}_{1855.7}$

$$\therefore \quad \Delta H_v = 19{,}282.5 \pm 1855.7 \text{ kJ/kmol}$$

using the higher deviation. →

* for P_{vp} — by max / min method

max $\ln \frac{P_v}{1.06} = \frac{21,138.2}{8.314}\left[\frac{1}{230.1} - \frac{1}{249.06}\right]$

$P_{v_{max}} = 2.458$ atm

min $\ln \frac{P_v}{0.94} = \frac{17426.8}{8.314}\left[\frac{1}{232.1} - \frac{1}{247.06}\right]$

$= 1.624$ atm

P_{vp} atm $\underbrace{\underset{min}{1.624} \quad \underset{estimate}{1.986}}_{0.362}$ $\underbrace{\underset{estimate}{1.986} \quad \underset{max}{2.458}}_{0.472}$

use the higher deviation

$P_{vp} = 1.986 \pm 0.472$ atm

⟶

#2

Alternate method

$$\ln \frac{P}{P_1} = \frac{\Delta H_v}{R}\left[\frac{1}{T_1} - \frac{1}{T}\right] = f$$

By method of propagation of errors, correlated data

$$d \ln P \doteq \frac{\partial f}{\partial \Delta H_v}\Delta(\Delta H_v) + \frac{\partial f}{\partial T}\Delta T \doteq \frac{\Delta P}{P}$$

or

$$\frac{\Delta P}{P} \simeq \frac{1}{R}\left[\frac{1}{T_1} - \frac{1}{T}\right]\Delta(\Delta H_v) - \frac{\Delta H_v}{RT^2}\Delta T$$

$$\frac{\Delta P}{P} \simeq \ln\left(\frac{P}{P_1}\right)\frac{\Delta(\Delta H_v)}{\Delta H_v} - \frac{\Delta H_v}{RT}\left(\frac{\Delta T}{T}\right)$$

from data provided, assume $\Delta P = \pm 0.06$ atm

Estimate ΔH_v from

$$\ln\left(\frac{5}{1}\right) = \frac{\Delta H_v}{8.314}\left(\frac{1}{231.1} - \frac{1}{275.24}\right)$$

$$\Delta H_v = 19282.6 \text{ kJ/kg}$$

Estimate P (at $T = 248.06$ K) from

$$\ln\left(\frac{P}{1}\right) = \frac{19282.6}{8.314}\left(\frac{1}{231.1} - \frac{1}{248.06}\right)$$

$$P = 1.986 \text{ atm}$$

Estimate $\Delta(\Delta H_v)$

$$\frac{0.06}{1.986} \simeq \ln\left(\frac{1.986}{1}\right)\frac{\Delta(\Delta H_v)}{19282.6} - \frac{19282.6}{8.314(248.06)^2}$$

$$\pm\Delta(\Delta H_v) = 1908.32 \text{ kJ/kmol}$$

(close to other method)