GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #2 September 28, 2010

Time Allowed: 35 mins. Name:

The director of athletics has asked you for an estimate of the pressure inside a leather soccer ball. The dry mass of an uninflated ball was measured as 425g. After filling the ball with air at 25±2°C, the mass became 476g. Both mass determinations were made with a balance accurate to ±5g. The inside diameter of the spherical inflated ball is 22±0.3 cm. Assume the data are correlated.

Use the ***van der Waals equation of state*** to estimate the pressure in the ball and the error for your estimate.

Data: van der Waals equation, $\left[P+\frac{a}{V_m^2}\right]\left[V_m - b\right] = RT$, where a = 1.33atm$\left\{\frac{m^3}{kmol}\right\}^2$;

b = 0.0366 $\frac{m^3}{kmol}$; R = 0.08205 $\frac{m^3 atm}{kmol K}$; V_m is molar volume and the molar mass of air is 28.96 kg/kmol.

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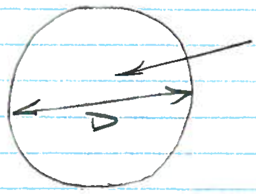
(a)



mass of air = $51 \pm 5g$

$T = 25 \pm 2\,°C$

$D = 22 \pm 0.3\,cm$

Given data correlated.

Re-arrange the van der Waals equation

$$P = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

where $T = 25°C = 298.15K$

$\forall$, volume $= \frac{4}{3}\pi R^3 = \frac{\pi}{6} D^3$

$D = 0.22\,m$

Volume of air, $\forall = 5.575\,(10^{-3})\ m^3$

Mass of air $= 51\,g$ or 1.761 moles

$$\therefore\ V_m = \frac{5.575\,(10^{-3})}{1.761\,(10^{-3})} = 3.1657\ m^3/kmol$$

Substitute values

$$P = \frac{0.08205\,(298.15)}{3.1657 - 0.0366} - \frac{1.33}{(3.1657)^2}$$

$$= 7.818 - 0.1327 = 7.6853\ atm$$

$\longrightarrow$

The variables are T, D and m (mass)

molar volume $V_m = \frac{\forall}{m}(M)$ where M = molar mass air.

volume of air $\forall = \frac{\pi}{6} D^3$

$\therefore\ V_m = \beta \frac{D^3}{m}$, where $\beta = \frac{M\pi}{6} = 15.1634\ kg/kmol$

Applying method of propagation of errors to the van der Waals equation

$$\Delta P = \frac{\partial P}{\partial V_m}\Delta V_m + \frac{\partial P}{\partial T}\Delta T$$

where

$$\Delta V_m = \frac{\partial V_m}{\partial D}\Delta D + \frac{\partial V_m}{\partial m}\Delta m$$

and $\Delta T = \pm 2\,^\circ C$

from previous equation for V_m

$$\Delta V_m = \frac{3D^2\beta}{m}\Delta D - \frac{\beta D^3}{m^2}\Delta m$$

with $\Delta D = \pm 0.003\ m$; $\Delta m = \pm 5g$

$$\Delta V_m = \frac{3(0.22)^2(15.1634)}{51(10^{-3})}(3)(10^{-3}) + \frac{(15.1634)(0.22)^3\,5(10^{-3})}{(51)^2(10^{-6})}\ m^3/\mathrm{kmol}$$

$$= 0.1295 + 0.3104 = \pm 0.4399\ m^3/\mathrm{kmol}$$

Hence

Error in pressure
$$\Delta P = \left[-\frac{RT}{(V_m-b)^2} + \frac{2a}{V_m^3}\right]\Delta V_m + \frac{R}{(V_m-b)}\Delta T$$

$$= \left|\left[\frac{-(0.08205)(298.15)}{(3.1657-0.0366)^2} + \frac{2(1.33)}{3.1657^3}\right]\pm 0.4399\right| + \left|\left[\frac{0.08205}{(3.1657-0.0366)}\right]\pm 2\right| = \pm 1.1146\ \mathrm{atm}$$