The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #2 September 22, 2009

Time Allowed: 35 mins. Name:

A condenser is an important component of many heat transfer devices such as distillation columns, air conditioners and refrigerators. Consider a vertical pipe through which saturated vapor is passed. The vapor condenses on the inner wall as heat is lost to the ambient and the condensate runs down the wall in a laminar stream which grows in thickness (δ) as it descends. An important design issue is estimating the rate of condensation as controlled by the rate of heat transfer from the vapor-liquid interface to the pipe wall.

The parameter of interest is the average heat transfer coefficient (***h***) per unit area of the pipe wall and unit temperature difference between the vapor and the wall. A functional relationship between ***h*** and other variables is given as:

$$h = f(\Delta\theta, \rho\lambda, k, \rho g, \mu, L, D)$$; h in W/m²K

where $\Delta\theta$ is temperature difference (°C) between vapor and the pipe wall, L and D are the length and the diameter (m) of the tube, λ is the heat of condensation per unit condensate mass (kJ/kg), ρ is the density of the condensate (kg/m^3), k is the thermal conductivity (W/mK) of the liquid, g is the acceleration of gravity (m/s^2) and μ is the dynamic viscosity (mPa s) of the condensate.

a) Obtain the dimensionless groups for the variables.

b) Relate your answer in part a) to the following result derived theoretically by Nusselt in 1916,

$$h = 0.943\left(\frac{g\rho^2\lambda k^3}{L\mu\Delta\theta}\right)^{1/4}$$

i.e. re-write Nussselt's equation in terms of your dimensionless groups.

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Given

$$h = f(\Delta\theta, \rho\lambda, k, \rho g, \mu, L, D) \quad —(1)$$

Units $\frac{W}{m^2K}$ K $\frac{kg}{m^3}\frac{kJ}{kg}$ $\frac{W}{mK}$ $\frac{kg}{m^3}\frac{m}{s^2}$ $Pa\cdot s$ m m

These can be simplified into

$\frac{kg}{s^3K}$ K $\frac{kg}{ms^2}$ $\frac{kg\text{-}m}{s^3K}$ $\frac{kg}{m^2s^2}$ $\frac{kg}{ms}$ m m

There are 4 dimensions — M, L, t and T
mass, length, time and temperature

Dimensions

$\frac{M}{t^3T}$ T $\frac{M}{Lt^2}$ $\frac{ML}{t^3T}$ $\frac{M}{L^2t^2}$ $\frac{M}{Lt}$ L L

(a)

There are 8 variables and 4 dimensions, hence there are 4 dimensionless groups.

By inspection, 2 dimensionless groups can be identified:

$$\pi_1 = \frac{L}{D} \quad \text{geometric similarity}$$

$$\pi_2 = \frac{hL}{k} \quad \longrightarrow$$

Equation (1) can be reduced to determine the other 2 dimensionless gps. We drop D and h — any 2 variables

$$0 = g(\Delta\theta, \rho\lambda, k, \rho g, \mu, L)$$

and choose 4 repeating variables (same as number of dimensions) —

$\Delta\theta,\ \rho\lambda,\ k,\ \rho g$

$$\Pi_3 = (\Delta\theta)^a (\rho\lambda)^b k^c (\rho g)^d \mu$$

and

$$\Pi_4 = (\Delta\theta)^{a'} (\rho\lambda)^{b'} k^{c'} (\rho g)^{d'} L$$

solve

$$\Pi_3 = (T)^a \left(\frac{M}{Lt^2}\right)^b \left(\frac{ML}{t^3 T}\right)^c \left(\frac{M}{L^2t^2}\right)^d \left(\frac{M}{Lt}\right)$$

$$\left.\begin{aligned} T &: 0 = a - c \\ M &: 0 = b + c + d + 1 \\ L &: 0 = -b + c - 2d - 1 \\ t &: 0 = -2b - 3c - 2d - 1 \end{aligned}\right\} \quad \begin{aligned} a &= 1 \\ b &= -4 \\ c &= 1 \\ d &= 2 \end{aligned}$$

$$\therefore \Pi_3 = (\Delta\theta)(\rho\lambda)^{-4}(k)(\rho g)^2 \mu = \frac{\Delta\theta\, k \rho^2 g^2 \mu}{\rho^4 \lambda^4}$$

$$= \frac{\Delta\theta\, k g^2 \mu}{\rho^2 \lambda^4}$$

$\longrightarrow$

$$\Pi_4 = (T)^{a'} \left(\frac{M}{Lt^2}\right)^{b'} \left(\frac{ML}{t^3 T}\right)^{c'} \left(\frac{M}{L^2t^2}\right)^{d'} L$$

$$\left.\begin{aligned} T &: 0 = a' - c' \\ M &: 0 = b' + c' + d' \\ L &: 0 = -b' + c' - 2d' + 1 \\ t &: 0 = -2b' - 3c' - 2d' \end{aligned}\right\} \quad \begin{aligned} a' &= 0 \\ b' &= -1 \\ c' &= 0 \\ d' &= 1 \end{aligned}$$

$$\therefore \Pi_4 = (\rho\lambda)^{-1}(\rho g) L = \frac{gL}{\lambda}$$

$\longrightarrow$

Hence

$$\frac{hL}{k} = f\left(\frac{\Delta\theta k \mu g^2}{\rho^2 \lambda^4}, \frac{gL}{\lambda}, \frac{L}{D} \right)$$

$$\pi_2 \qquad \Pi_3 \qquad \Pi_4 \qquad \Pi_1 \longrightarrow$$

(b) Nusselt's equation is

$$h = 0.943 \left(\frac{g\rho^2\lambda k^3}{L\mu\Delta\theta} \right)^{1/4}$$

multiply both sides by L/k

$$\frac{hL}{k} = (0.943)\left(\frac{g\rho^2\lambda \cancel{k^3} L^{\cancel{4}\,3}}{\cancel{L}\mu\Delta\theta k^{\cancel{4}}} \right)^{1/4}$$

$$= (0.943)\left(\frac{\rho^2\lambda^4}{\Delta\theta k\mu g^2} \cdot \frac{g^3 L^3}{\lambda^3} \right)^{1/4}$$

i.e.

$$\Pi_2 = 0.943\left(\frac{\Pi_4^3}{\Pi_3} \right)^{1/4}$$

and we assume L/D is held constant

and therefore part of 0.943 in Nusselt's expression. $\longrightarrow$

Please note that other dimensionless quantities may be formed that are equally valid.