GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #2 **September 23, 2008**

Time Allowed: 45 mins. **Name:**

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1. (5pts)

Wine is often placed in oak barrels during fermentation and aging. The barrels are made of wood strips loosely joined together and banded with strips of metal (as shown in picture). During fermentation, carbon dioxide is produced and the pressure inside the barrel may rise. It is therefore important that the metal bands are able to hold the barrel together.

A barrel in the shape of a cylinder, inside radius of 40cm, length of 1.1m and wall thickness of 1.2 cm, is filled with wine still undergoing fermentation at 1 atm. pressure. During the storage, the pressure inside the barrel increases to 5 atm. The bands are made of steel, 2cm wide and 0.9mm thick. As the pressure in the barrel increases, the inside radius of the barrel would tend to increase unless restrained. Hence there are normal stresses on the bands.

Given the data below, use ***dimensional analysis*** to estimate the minimum number of bands that should be wrapped around the curved wall of the barrel from the start to prevent disintegration. You may assume that the wall thickness of the barrel remains constant. The ends of the barrel are flat and secure.

Data: Properties of steel band: Young's modulus = 206.8 GPa, maximum elastic strain before snapping is 1.5%. The Poisson ratio for steel is 0.26. This is roughly the ratio of the normal stress (σ_N) on a surface to the tangential or lateral stress developed (σ_T). 1 atm = 101.325 kPa.

2. (5pts)

The heat transfer coefficient for film condensation can be derived from the following:

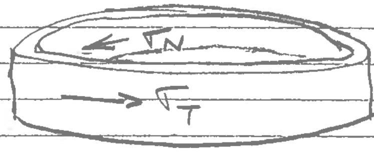
$$\frac{h_x x}{k} = \left(\frac{\lambda g \rho x^3}{4 \mu k \Delta T}\right)^{1/4}$$

Given the parameters below and assuming that the values were all determined at different laboratories, estimate a value for the heat transfer coefficient $\boldsymbol{h_x}$ and the error associated with the value.

Data: Thermal conductivity (k) = 0.6 ± 0.03 W/mK; density (ρ) = 997 ± 3 kg/m^3; viscosity (μ) = 1 ± 0.01 mPa s; length (x) = 0.75 ± 0.01m; latent heat of vaporization (λ) = 2260 ± 15 kJ/kg, temperature difference (ΔT) = 42 ± 3 °C

ENCH 501 Quiz #2 Solution Sept. 23, 2008

#1



Stresses on band

Consider a band. The band is stretched from inside by outwards expansion of the barrel. The force at the wall of the barrel is resisted at the inside surfaces of the bands.

The dimensionless quantity for this problem is

$$\frac{\sigma_N}{\sigma_T} = 0.26 \qquad (a)$$

σ_T is the elongation stress, with a maximum value of $E\varepsilon_{max}$. The normal stress σ_N is the force at the wall due to pressure change divided by the inside area of all the bands.

The differential pressure, $\Delta P = 4$ atm.

$\therefore$ Extra force on curved wall of barrel

$$F = \pi(D)L(\Delta P) = \pi(0.8)(1.1)(4)(101325)\ N$$

This force is assumed transmitted through the barrel wall.

Let the number of bands $= n$

The inside area of the bands $= n(\pi)dl$, where

$d = 0.824$ m (diam. of barrel + wall thickness)

$l = 2(10^{-2})$ m

$$\therefore \sigma_N = \frac{\pi(0.8)(1.1)(4)(1.01325)(10^5)}{n(\pi)(0.824)(2)10^{-2}} \qquad (b)$$

From eqs. ⓐ and ⓑ

$$\sigma_N = \frac{2.1642(10^7)}{n} = 0.26(206.8)(10^9)(0.015)$$

$$\therefore \quad n = 0.0268$$

That is, one band is enough. $\longrightarrow$

\# 2. The equation is re-written as

$$h_x = \left(\lambda \frac{g\rho}{4} x^{-1} k^3 \mu^{-1} \Delta T^{-1}\right)^{1/4} = f$$

where

$$\left.\begin{aligned} \lambda &= (2.26 \pm 0.015)10^6 \quad J/kg \\ \rho &= 997 \pm 3 \quad kg/m^3 \\ x &= 0.75 \pm 0.01 \quad m \\ k &= 0.6 \pm 0.03 \quad W/m\,K \\ \mu &= (1 \pm 0.01)10^{-3} \quad Pa.s \\ \Delta T &= 42 \pm 3 \quad K \end{aligned}\right\} \text{in SI units}$$

Estimate for h_x

$$= \left[\frac{(2.26)(10^6)}{4}\frac{(9.81)}{0.75}\frac{(0.6)^3}{10^{-3}}\frac{997}{42}\right]^{1/4}$$

$$= 441.2 \quad W/m^2K$$

Since the data sets are independent,

$$\Delta h_x = \left[\left(\frac{\partial f}{\partial \lambda}\right)^2 (\Delta\lambda)^2 + \left(\frac{\partial f}{\partial x}\right)^2 (\Delta x)^2 + \right.$$

$$\left(\frac{\partial f}{\partial \rho}\right)^2 (\Delta\rho)^2 + \left(\frac{\partial f}{\partial k}\right)^2 (\Delta k)^2 + \left(\frac{\partial f}{\partial \mu}\right)^2 (\Delta\mu)^2$$

$$\left. + \left(\frac{\partial f}{\partial (\Delta T)}\right)^2 (\Delta[\Delta T])^2 \right]^{\frac{1}{2}}$$

and

$$\frac{\partial f}{\partial \lambda} = \frac{1}{4} \lambda^{-3/4} \left(\frac{g \rho x^{-1} k^3 \mu^{-1} \Delta T^{-1}}{4} \right)^{1/4}$$

$$= \frac{1}{4\left[(2.26)(10^6)\right]^{3/4}} \left[\frac{9.81\,(997)}{4} \frac{(0.6)^3}{0.75} \frac{1}{10^{-3}} \frac{1}{42} \right]^{1/4}$$

$$= 4.88806\,(10^{-5})$$

$$\frac{\partial f}{\partial x} = -\frac{1}{4} x^{-5/4} \left[\frac{\lambda g \rho k^3}{4 \mu \Delta T} \right]^{1/4}$$

$$= -\frac{1}{4} \frac{1}{(0.75)^{5/4}} \left[\frac{2.26\,(10^6)(9.81)\,997\,(0.6)^3}{4\,(10^{-3})(42)} \right]^{1/4}$$

$$= -147.0678$$

$$\frac{\partial f}{\partial \rho} = \frac{1}{4} \rho^{-3/4} \left[\frac{\lambda g k^3}{4 x \mu \Delta T} \right]^{1/4}$$

$$= \frac{1}{4} \frac{1}{(997)^{3/4}} \left[\frac{2.26\,(10^6)(9.81)(0.6)^3}{4\,(0.75)(10^{-3})\,42} \right]^{1/4}$$

$$= \quad 0.1106$$

$$\frac{\partial f}{\partial k} = \frac{3}{4} k^{-\frac{1}{4}} \left[\frac{\lambda g \rho}{4 x \mu \Delta T} \right]^{\frac{1}{4}}$$

$$= \frac{3}{4} \frac{1}{(0.6)^{1/4}} \left[\frac{2.26(10^6)(9.81)(997)}{4(0.75)(10^{-3})(42)} \right]^{\frac{1}{4}}$$

$$= 551.5044$$

$$\frac{\partial f}{\partial \mu} = -\frac{1}{4} \mu^{-5/4} \left[\frac{\lambda g \rho k^3}{4 x \Delta T} \right]^{\frac{1}{4}}$$

$$= -\frac{1}{4} \frac{1}{(10^{-3})^{5/4}} \left[\frac{2.26(10^6)(9.81)(997)(0.6)^3}{4(0.75)(42)} \right]^{\frac{1}{4}}$$

$$= -1.103(10^5)$$

$$\frac{\partial f}{\partial (\Delta T)} = -\frac{1}{4} (\Delta T)^{-5/4} \left[\frac{\lambda g \rho k^3}{4 x \mu} \right]^{\frac{1}{4}}$$

$$= -\frac{1}{4} \frac{1}{(42)^{5/4}} \left[\frac{2.26(10^6)(9.81)\,997\,(0.6)^3}{4(0.75)(10^{-3})} \right]^{\frac{1}{4}}$$

$$= -2.6262$$

$$\Delta h_x = \Big[\{4.8806(10^{-5})\}^2 \{0.015(10^6)\}^2 +$$

$$(147.0678)^2 (0.01)^2 + (0.1106)^2 (3)^2$$

$$+ (551.504)^2 (0.03)^2 + \{1.103(10^5)\}^2 \{(10^{-5})^2\}$$

$$+ (2.6262)^2 (3)^2 \Big]^{\frac{1}{2}}$$

$$= (339.8393)^{\frac{1}{2}} = 18.435$$

$$\therefore \quad h_x = 441.2 \pm 18.4 \quad W/m^2 K$$

$\longrightarrow$