

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #2**September 25, 2007****Time Allowed: 45 mins.****Name:**

Particles of sand, fungal spores, micro-organisms and salts suspended in streams of gases and liquids often deposit on the surfaces of conveying ducts. If the particles consolidate and agglomerate while sticking to the surface, fouling occurs. Fouling reduces the rates of heat transfer across the wall. There are three regimes for particle deposition - the diffusion, inertia and impaction regimes. The regime controlling a deposition process is determined by calculating a dimensionless particle relaxation time, t^+ , given as:

$$t^+ = \frac{\rho_p d_p^2 U^2}{18 \mu \nu}$$

where $t^+ < 0.1$ for the diffusion regime; $0.1 \leq t^+ \leq 10.2$ for the inertia regime; and $t^+ > 10.2$ for the compaction regime.

Given the data below, estimate the relaxation time and the error (a) if the data is assumed correlated, and (b) if the data is assumed independent. Show your steps.

For each of the above, what are the regimes of particle deposition? Explain.

Data: Density of particle, $\rho_p = 2700 \pm 50 \text{ kg/m}^3$; diameter of particle, $d_p = 20 \pm 0.3 \text{ } \mu\text{m}$; average velocity of stream, $U = 0.485 \pm 0.005 \text{ m/s}$; viscosity of suspending fluid, $\mu = 1.2 \pm 0.01 \text{ mPa s}$; and density of suspending fluid, $\rho = 992 \pm 8 \text{ kg/m}^3$. Kinematic viscosity of fluid, $\nu = \mu/\rho$.

Given
$$t^+ = \frac{\rho_p d_p^2 U^2}{18 \mu^2} = \frac{\rho \rho_p d_p^2 U^2}{18 \mu^2}$$

Estimate for $t^+ = \frac{(992)(2700)(20)^2(10^{-12})(0.485)^2}{18(1.2)^2(10^{-6})}$

$= 9.72$

To estimate errors, use method of propagation of errors, i.e.

$$\Delta t^+ = \frac{\partial t^+}{\partial \rho} \Delta \rho + \frac{\partial t^+}{\partial \rho_p} \Delta \rho_p + \frac{\partial t^+}{\partial d_p} \Delta d_p + \frac{\partial t^+}{\partial U} \Delta U$$

$+ \frac{\partial t^+}{\partial \mu} \Delta \mu$ for correlated data (1)

$$\frac{\partial t^+}{\partial \rho} = \frac{\rho_p d_p^2 U^2}{18 \mu^2} = 9.801(10^{-3})$$

$$\frac{\partial t^+}{\partial \rho_p} = \frac{\rho d_p^2 U^2}{18 \mu^2} = 3.601(10^{-3})$$

$$\frac{\partial t^+}{\partial d_p} = \frac{2 \rho \rho_p d_p U^2}{18 \mu^2} = 972,263.3$$

$$\frac{\partial t^+}{\partial U} = \frac{\rho \rho_p d_p^2 (2U)}{18 \mu^2} = 40.093$$

$$\frac{\partial t^+}{\partial \mu} = \frac{-2 \rho \rho_p d_p^2 u^2}{18 \mu^3} = -16,204.39$$

Substitute the above with $\rho = 8 \text{ kg/m}^3$,

$$\rho_p = 50 \text{ kg/m}^3, \quad \Delta d_p = 0.3(10^{-6}) \text{ m},$$

$$\Delta u = 0.005 \text{ m/s} \quad \text{and} \quad \Delta \mu = -10^{-5} \text{ Pa}\cdot\text{s}$$

into equation ①

① For correlated data, $\Delta t^+ = 0.9126$

$$\therefore t^+ = 9.72 \pm 0.9126 \quad \text{②}$$

For independent data set,

$$\Delta t^+ = \sqrt{\left(\frac{\partial t^+}{\partial \rho}\right)^2 (\Delta \rho)^2 + \left(\frac{\partial t^+}{\partial \rho_p}\right)^2 (\Delta \rho_p)^2 + \left(\frac{\partial t^+}{\partial d_p}\right)^2 (\Delta d_p)^2 + \left(\frac{\partial t^+}{\partial u}\right)^2 (\Delta u)^2 + \left(\frac{\partial t^+}{\partial \mu}\right)^2 (\Delta \mu)^2}$$

Use the numbers above to get

$$\Delta t^+ = \sqrt{0.1900869} = 0.436$$

② $\therefore$ for independent data set,

$$t^+ = 9.72 \pm 0.436 \quad \text{③}$$

The result. For the correlated data suggests that since t^+ is in the range of 8.807 to 10.6326, some particles will deposit by inertia and some by compaction.

For independent data, all particles will deposit in the inertia regime.

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