a.J

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ENCH 501: Transport Processes Quiz #2 September 26, 2006

Time Allowed: 50 mins. Name:

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1) (4 points) Potable (drinking) water is packaged and sold by different companies in plastic bottles of various sizes. Some of the smaller bottles are labeled to contain 16 US fl oz or 473.2 ml water. It is suspected that there are variations in the quantity of water sold. 20 bottles labeled as above are collected from a store and the amount of water in each carefully measured. The data is as given below. Estimate the probable quantity of water in a bottle and the errors if a) the bottles are from one company ~~any~~ and in one package, and b) all the bottles are from different companies.

Data: Volumes of water in plastic bottles in US fluid ounces

16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

2) (6 points) The standard equation used to calculate oil production rate (q) from a well in terms of reservoir and fluid properties is given as follows:

$$q = \frac{kh(P_e - P_{wf})}{C_o \mu B(\ln\frac{r_e}{r_w} + s)}$$

In the equation, k is reservoir permeability, h is the net pay thickness, P_e is external boundary pressure, P_{wf} is flowing bottomhole pressure, μ is fluid viscosity, B is oil formation volume factor, r_e is external boundary radius, r_w is wellbore radius and s is the skin factor.

Given the following values, estimate the pressure drop (in Pa) from the boundary to the well bore and the associated error. C_o is a constant and equals 1867.4.

Data: B = 1.1 ±0.1 res m^3/STm^3; μ= 2±0.02 mPa s, k = $(1\pm0.08)10^{-14}$ m^2, h=10±0.3m, r_e=575±5m, r_w = 0.1±0.01m; q = $(1\pm0.1)\ 10^{-3}$ m^3/s and s = 0.

* More realistic numbers for k and q are:

$k = (1 \pm 0.08) 10^{-12}\ m^2$

$q = (1 \pm 0.1) 10^{-5}\ m^3/s$

ENCH 501 Quiz #2 Solution September 26, 2006

1 Given Data:

Volume, V_i	$V_i - \bar{V}_{est}$
16.03	0.02
16.04	0.03
16.05	0.04
16.05	0.04
16.02	0.01
16.01	0
15.96	- 0.05
15.98	- 0.03
16.02	0.01
15.99	- 0.02
16.02	0.01
15.97	- 0.04
15.96	- 0.05
16.01	0
15.99	- 0.02
16.03	0.02
16.04	0.03
16.02	0.01
16.01	0
16.00	- 0.01

$$\bar{V}_{est} = \frac{320.2}{20} = 16.01$$

(a) If all bottles are from the same company, the data would be correlated

$$\therefore \quad \Delta V = \frac{\sum_{i=1}^{20} |(V_i - \bar{V})|}{20} = \frac{0.44}{20} = 0.022$$

$$\therefore \quad V = 16.01 \pm 0.022 \text{ US fl oz}$$

(b) If the bottles are from different companies, the data would probably be independent. Hence

$$\Delta \bar{V}_{est} = \sigma = \sqrt{\frac{\sum_{i=1}^{N} (V_i - \bar{V}_{est})^2}{N-1}} = \sqrt{\frac{0.0146}{19}} = 0.0277$$

where $N = 20$

The error $\Delta V = \frac{\sigma}{\sqrt{N}} = \frac{0.0277}{\sqrt{20}} = 0.0062$

$\therefore \quad V = 16.01 \pm 0.0062$ US fl. oz. $\longrightarrow$

2. Re-write eq.

$$\Delta P = P_e - P_{wf} = \frac{C_o \, q B \mu \ln\left(r_e/r_w\right)}{kh}$$

Use method of propagation of errors.

The value of ΔP with given data is:

$$\Delta P = \frac{1867.4\,(10^{-3})(1.1)\,2(10^{-3}) \ln\left(575/0.1\right)}{(10^{-14})(10)}$$

$$= 3.5565\,(10^{11}) \quad Pa$$

For the error

$$\Delta(\Delta P) \simeq \frac{\partial(\Delta P)}{\partial q}\Delta q + \frac{\partial(\Delta P)}{\partial B}\Delta B +$$

$$\frac{\partial(\Delta P)}{\partial \mu}\Delta \mu + \frac{\partial(\Delta P)}{\partial \eta}\Delta \eta + \frac{\partial(\Delta P)}{\partial k}\Delta k$$

$$+ \frac{\partial(\Delta P)}{\partial h}\Delta h.$$

where $\eta = \ln \frac{r_e}{r_w} = \ln r_e - \ln r_w$

$$\Delta(\ln r_e) = \frac{\Delta r_e}{r_e}$$

(from Pythagorean theorem: $\Delta Q = \sqrt{\left(\frac{\partial Q}{\partial q_1}\right)^2 (\Delta q_1)^2 + \cdots}$)

and $\Delta(\ln r_w) = \frac{\Delta r_w}{r_w}$

$$\therefore \eta \pm \Delta\eta = \ln\left(\frac{575}{0.1}\right) \pm \left(\left|\frac{\Delta r_e}{r_e}\right| + \frac{\Delta r_w}{r_w}\right)$$

$$= 8.657 \pm (0.0087 + 0.1)$$

$$= 8.657 \pm 0.1087$$

Solve for each of the error components.

$$\frac{\partial(\Delta P)}{\partial q}\Delta q = \frac{1867.4\,(1.1)(2)(10^{-3})(8.657)}{10^{-14}(10)} \cdot 10^{-4}$$

$$= 3.5566\,(10^{10})$$

$$\frac{\partial(\Delta P)}{\partial B}\Delta B = \frac{(1867.4)(10^{-3})2(10^{-3})(8.657)}{10^{-14}(10)}(0.1)$$

$$= 3.2332\,(10^{10})$$

$$\frac{\partial(\Delta P)}{\partial \mu}\Delta\mu = \frac{(1867.4)(10^{-3})(1.1)\,8.657)}{10^{-14}(10)}(0.02)(10^{-3})$$

$$= 3.5566\,(10^{9})$$

$$\frac{\partial(\Delta P)}{\partial \eta}\Delta\eta = \frac{(1867.4)(10^{-3})(1.1)\,2(10^{-3})}{(10^{-14})(10)}(0.1087)$$

$$= 4.4657\,(10^{9})$$

$$\frac{\partial(\Delta P)}{\partial k}\Delta k = -\frac{C_o q B \mu \eta}{k^2 h}\Delta k$$

$$= \frac{1867.4(10^{-3})(1.1)(2)(10^{-3})(8.657)}{(10^{-14})^2(10)}(0.08)(10^{-14})$$

$$= -2.8453(10^{10})$$

$$\frac{\partial(\Delta P)}{\partial h}\Delta h = -\frac{C_o q B \mu \eta}{k h^2}\Delta h$$

$$= \frac{1867.4(10^{-3})(1.1)(2)(10^{-3})8.657}{10^{-14}(10)^2}(0.3)$$

$$= 1.067(10^{10})$$

$\therefore$ $\Delta(\Delta P)$ error in pressure drop $= (3.5566 + 3.2332 + 0.35566$

$+ 0.44657 + 2.8453 + 1.067)10^{10}$

$= 1.150(10^{11})$

$\therefore$ $\Delta P = (3.5565 \pm 1.1500)10^{11}$ Pa

The error is 32.4 %

Alternate methods of estimating the error for ΔP may be valid — e.g. estimating max. & min. ΔP.