AJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #2 **September 27, 2005**

Time Allowed: 40 mins. **Name:**

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1) "Radiant heating" is a relatively new technique for heating up homes. It involves embedding coils of tubes in the floor and passing warm aqueous solutions of ethylene glycol or hot water through the tubes. This heats up the floor which, in turn, heats up the air close to the ground. Since warm air is lighter than cooler air, the air in the enclosed space becomes hydrodynamically unstable and a circulation is set up. The principles for this application are well understood. If a fluid is confined between two flat plates and the bottom plate is heated while the top plate is cooled, no convection is observed until the temperature difference ΔT between the plates is large enough. The motion is observed to start when the Rayleigh number (Ra) exceeds a critical value.

(a) (6 pts) Unused engine oil is confined between two large horizontal plates separated by a gap L of 15 ± 0.5mm. The bottom plate is maintained at a temperature of 30.9 ± 0.3°C and the top is maintained at 23.2 ± 0.2°C. Estimate the critical Rayleigh number and the error in the value if convective motion was observed to start at these conditions.

Data: Rayleigh number, $Ra = \dfrac{g\beta\Delta T L^3}{\nu\alpha}$ where $\alpha = \dfrac{k}{\rho C_p}$ and $\nu = \dfrac{\mu}{\rho}$

The acceleration of gravity g is a constant which equals 9.81 m/s^2.
Engine oil properties: The coefficient of volumetric expansion β is (0.7 ± 0.05) 10^{-3} K^{-1}. The density ρ is 888.2 ± 2 kg/m^3. The viscosity μ is 1.06 ± 0.03 Pa.s, the thermal conductivity k is 0.145 ± 0.005 W/m K and the heat capacity C_p is 1.88 ± 0.04 kJ/kg K.

(b) (1 pt) Which three of the variables would you suggest should be measured with better accuracy to reduce the error?

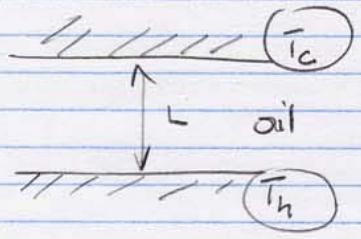
2) (3 pts) A detector is in use for scanning a location for radioactivity. The rates of signals per minute at different sites are reported as follows:

81, 95, 101, 108, 90, 115, 85, 124, 105, 88, 117, 98, 109, 132, 97

Estimate the mean, the average deviation and the standard deviation.

ENCH 501 Quiz #2 Solution

#1



$$Ra = \frac{g\beta \Delta T L^3}{\nu \alpha} = \frac{g\beta \Delta \bar{T} L^3}{(\mu/\rho)(k/\rho C_p)} = \frac{g\beta \rho^2 C_p \Delta T L^3}{\mu k}$$

The critical Rayleigh number

$$Ra_c = \frac{(9.81)(0.7)(10^{-3})(888.2)^2(1880)(7.7)(15)^3(10^{-9})}{(1.06)(0.145)}$$

$$= 1722.02 \longrightarrow$$

Estimate the error using propagation of errors or relative error method.

$$\text{e.g. } \frac{\Delta Ra}{Ra} = \frac{\Delta\beta}{\beta} + 2\frac{\Delta\rho}{\rho} + \frac{\Delta C_p}{C_p} + \frac{\Delta(\Delta\bar{T})}{\Delta\bar{T}} + 3\frac{\Delta L}{L} + \frac{\Delta\mu}{\mu} + \frac{\Delta k}{k}$$

Substituting values

Note: error for $\Delta\bar{T} = 0.5°C$

$$\frac{\Delta Ra}{Ra} = \frac{0.05}{0.7} + \frac{2(2)}{888.2} + \frac{0.04}{1.88} + \frac{0.5}{7.7} + \frac{3(0.5)}{15} + \frac{0.03}{1.06} + \frac{0.005}{0.145}$$

$$= 0.0714 + 0.0045 + 0.0213 + 0.0649 + 0.1 + 0.0283 + 0.0345$$

$$= 0.3249$$

$$\Delta Ra_c = 1722.02(0.3249)$$

$$= 559.48$$

$$\therefore \quad Ra_c = 1722 \pm 559.5$$

The three variables that contributed most to the error are β, ΔT and L. These should be measured to higher degrees of accuracy.

#2.

Table of detector signals

i	x_i	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
1	81	22	484
2	95	8	64
3	101	2	4
4	108	5	25
5	90	13	169
6	115	12	144
7	85	18	324
8	124	21	441
9	105	2	4
10	88	15	225
11	117	14	196
12	98	5	25
13	109	6	36
14	132	29	841
N = 15	97	6	36
Sum	1545	178	3018

For a Gaussian distribution,

≃ 58% of data will be within 103 ± 11.87

and

≃ 68% of data will be within 103 ± 14.68

$\bar{x} = 103$ average

mean = 103

Average Deviation, $\Delta x = \dfrac{\Sigma \lvert x_i - \bar{x} \rvert}{N} = \pm 11.87$

Standard Deviation, $\sigma_x = \sqrt{\dfrac{\Sigma (x_i - \bar{x})^2}{N-1}} = \pm 14.68$