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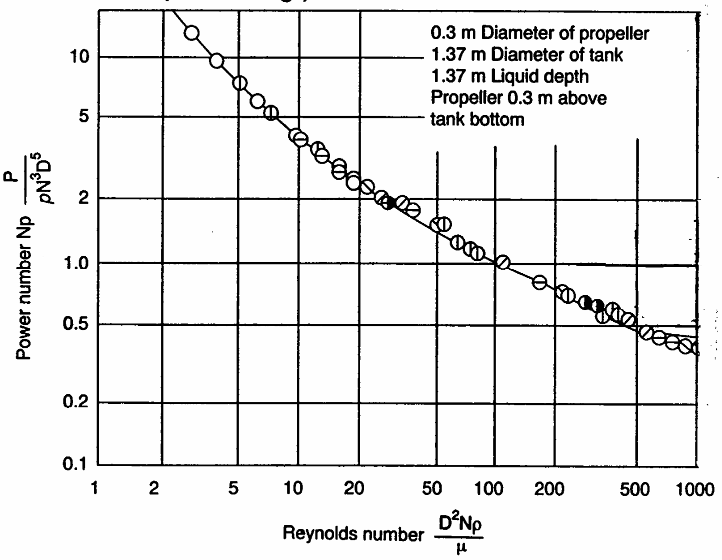
ENCH 501: Transport Processes Quiz #2 **September 21, 2004**

Time Allowed: 40 mins. **Name:**

Problem #1 (5 points)

In a polymerization operation, a liquid mixture (ρ= 1650 ± 60 kg/m^3; μ = 800±5 mPa s) is agitated in a tank by a propeller mixer with a diameter (**D**) of 0.5 ± 0.05 m. You are provided data for the power number (**Np**) versus Reynolds number (**Re**) for the propeller below. If the impeller rotation rate (**N**) is estimated as 15±1 rpm, what is the expected value of the power **P** of the motor required to drive the impeller? What is the error in the estimate of the power? Show all steps.

Data: (Note that the scales on the plot are log.)



Problem #2 (5 points)

The viscosity μ of a liquid can be determined using co-axial cylinders. The inner cylinder, radius ***a***, is suspended on a torsion string linked to a gauge to record torque G (force on the surface times the radius). The liquid is placed in the annular space between the cylinders and the outer cylinder, radius ***b***, is rotated at a steady rate Ω. The viscosity is calculated from -

$$\mu = \{G / (4\pi\Omega)\} \{ 1/a^2 - 1/b^2)$$

If the relative error for ***a*** is f_1 and the relative error for ***b*** is f_2 (and both G and Ω are exact), derive a relationship for the relative error in the viscosity. Show all steps.

If ***a***=4cm, ***b***=5cm and the maximum absolute error in the radii measurements is 0.01cm, what is the maximum relative error for the viscosity?

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#1 There is no explicit equation for $N_p = f(Re)$ given as a plot.

Expected value of Reynolds number (N in s^{-1})

$$Re = \frac{D^2 N \rho}{\mu} = (0.5)^2 \frac{15}{60} \frac{(1650)}{800 \times 10^{-3}}$$

$$= 128.9$$

Expected value of Power Number, from plot,

$$N_p = \frac{P}{\rho N^3 D^5} \simeq 0.9$$

$$\therefore \text{ Power, } P = 0.9(1650)(0.25)^3(0.5)^5$$

$$= 0.725 \text{ W}$$

To estimate error, determine lower & upper bound values for Re and Np

$$\text{Min } Re = (0.45)^2 \frac{(14)}{60} \frac{(1590)}{0.805} = 93.3$$

Corresponding $N_p \simeq 0.8$

$$\text{Max } Re = (0.55)^2 \frac{(16)}{60} \frac{(1710)}{0.795} = 173.5$$

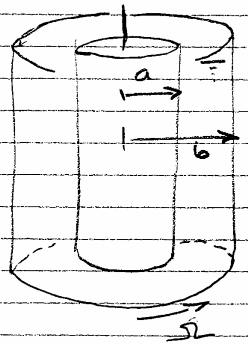
Corresponding $N_p \simeq 1.01$

$$\therefore N_p = \frac{P}{\rho N^3 D^5} \doteq 0.9 \pm 0.1$$

$$\therefore P = 0.725 \pm 0.081 \text{ W}$$

#2 The relationship is

$$\mu = \beta\left(\frac{1}{a^2}-\frac{1}{b^2}\right) \quad \text{where } \beta = \frac{G}{4\pi\Omega}, \text{ a const.}$$



$$d\mu = \frac{\partial \mu}{\partial a}\,da + \frac{\partial \mu}{\partial b}\,db$$

$$d\mu = -\beta\frac{2}{a^3}\,da + \beta\frac{2}{b^3}\,db$$

Given $f_1 = \pm\frac{da}{a}$ and $f_2 = \pm\frac{db}{b}$

$$\frac{d\mu}{\mu} = 2\left\{-\frac{f_1}{a^2} + \frac{f_2}{b^2}\right\} \Big/ \left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$

Relative error for viscosity, $\frac{d\mu}{\mu} = \frac{-2f_1 b^2 + 2f_2 a^2}{b^2 - a^2}$ $\longrightarrow$

Given $a = 4$ cm, $b = 5$ cm and $da = db = 0.01$ cm,

then $$\left.\frac{d\mu}{\mu}\right|_{max} = \frac{2}{9}\left(25 \cdot \frac{0.01}{4} + 16 \cdot \frac{0.01}{5}\right)$$

where $f_1 = -\frac{0.01}{4}$ and $f_2 = \frac{0.01}{5}$ to get largest error.

$$\left.\frac{d\mu}{\mu}\right|_{max} = 0.021$$

i.e. error in μ is $\simeq 2.1\%$

$\longrightarrow$

This is about 10 times the error of measuring the radii!