ENCH 501 F18 **Quiz #1** Name/ID: _____

September 18, 2018 Time Allowed: 45 minutes Only Cheat Sheet permitted. aJ

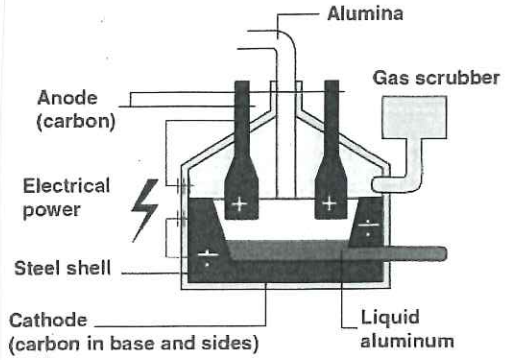
a) **10 points.**

Metallic aluminium is produced from mined bauxite in very energy-intensive processes. Bauxite ore contains 30 to 60% aluminium oxide (alumina, Al_2O_3) and it is refined by the Bayer Process. Hot sodium hydroxide (NaOH) is used to dissolve alumina and, on cooling, precipitates out aluminium hydroxide ($Al(OH)_3$). The hydroxide is heated in a rotary kiln to recover the alumina.

In batch processes, alumina is dissolved in molten cryolite (Na_3AlF_6) at around 960*C. A current is passed through the solution to both heat up the solution (and maintain its temperature) and to electrolytically reduce the alumina to aluminum. The Anode and the Cathode are typically both made of carbon, see sketch. CO_2 is produced in the process. It bubbles through the molten solution and induces liquid circulation. Assume that the liquid circulation is characterized by a velocity V and that this keeps the liquid temperature uniform. The gas released is typically captured and scrubbed. Heat loss from the top of the furnace is strictly dependent on the rate (kg/s) of CO_2 exhaust. An insulating layer of refractory brick is also typically layered between the cathode and the steel shell. The ambient temperature is T_a.

Identify all the <u>dimensional variables</u> and obtain the <u>dimensionless groups</u> for modeling such a furnace to maintain the temperature of the liquid mixtures at the desired value, as the furnace size or operating conditions are changed. Show your steps.

smelter



b) **Bonus: 2 points.**

Obtain values for x, y, u and v that satisfy the following equations:

$x + 7y + 3v + 5u = 16$

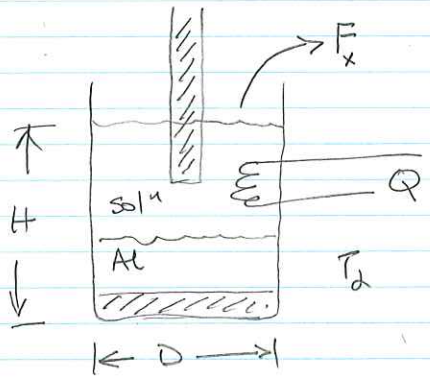
$8x + 4y + 6v + 2u = -16$

$2x + 6y + 4v + 8u = 16$

$5x + 3y + 7v + u = -16$

GTaji

ENCH 501 Quiz 1 F18 Solution



To maintain the bath temp, $\bar{T}$, the heat input must match the heat output.

Hence

$$\bar{T} \sim Q, V, F_x, \rho, \mu, C_{p_L}, k_i, H, D, \bar{T}_2, \delta$$

where F_x is CO_2 exhaust rate, Q is heat input rate, C_{p_L} is heat capacity of cryolite/alumina solution and (μ, ρ) its viscosity and density, k_i is thermal conductivity of insulation and δ its thickness. V is the characteristic velocity for the solution and $\bar{T}_2$ is the ambient temperature. H and D are the height and diameter of the tank.

Dimensions — 12 variables

$\bar{T}$	Q	V	F_x	ρ	μ	C_{p_L}	k_i	H	D	$\bar{T}_2$	δ
$[\bar{T}]$	$\frac{ML^2}{t^3}$	$\frac{L}{t}$	$\frac{M}{t}$	$\frac{M}{L^3}$	$\frac{M}{Lt}$	$\frac{L^2}{t^2\bar{T}}$	$\frac{ML}{t^3\bar{T}}$	L	L	$\bar{T}$	L

There are 4 dimensions — $M, L, t, \bar{T}$

Therefore there are 8 dimensionless groups.

By inspection, $\pi_1 = \frac{H}{D}$, $\pi_2 = \frac{D}{\delta}$, $\pi_3 = \frac{\bar{T}}{\bar{T}_2}$

$\Pi_4 = \dfrac{V^2}{C_p T_\infty}$. Thus there are 4 more dimensionless groups to identify

Now let T = function $(Q, F_x, \rho, \mu, C_p, k_i, H)$

– dropping V, D, δ, T_∞

Let's choose H, ρ, μ, C_p as repeating variables

$$\Pi_5 = H^a \rho^b \mu^c C_p^d Q^e$$
$$\Pi_6 = H^a \rho^b \mu^c C_p^d F_x^e$$
$$\Pi_7 = H^a \rho^b \mu^c C_p^d T^e$$
$$\Pi_8 = H^a \rho^b \mu^c C_p^d k_i^e$$

where a, b, c, d and e are all different for each Π.

Determine the exponents

$$\Pi_5 \quad M^0 L^0 t^0 T^0 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{L^2}{t^2 T}\right)^d \left(\frac{ML^2}{t^3}\right)^e$$

				solve
M	0 =	$b + c + e$		$a = e$
L	0 =	$a - 3b - c + 2d + 2e$		$b = 2e$
t	0 =	$-c - 2d - 3e$		$c = -3e$
T	0 =	$-d$		$d = 0$

$$\Pi_5 = H^e \rho^{2e} \mu^{-3e} Q^e = \left(\frac{H \rho^2 Q}{\mu^3}\right)^e$$

$$\Pi_6 = M^0 L^0 t^0 T^0 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{L^2}{t^2 T}\right)^d \left(\frac{M}{t}\right)^e$$

M	0	=	$b + c + e$	$a = -e$
L	0	=	$a - 3b - c + 2d$	$b = 0$
t	0	=	$-c - 2d - e$	$c = -e$
T	0	=	d	$d = 0$

$$\Pi_6 = H^{-e} \mu^{-e} F_x^{\,e} = \left(\frac{F_x}{\mu H}\right)^e$$

$$\Pi_7 = M^0 L^0 t^0 T^0 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{L^2}{t^2 T}\right)^d \left(T\right)^e$$

M	0	=	$b + c$	$a = 2e$
L	0	=	$a - 3b - c + 2d$	$b = 2e$
t	0	=	$-c - 2d$	$c = -2e$
T	0	=	$-d + e$	$d = e$

$$\Pi_7 = H^{2e} \rho^{2e} \mu^{-2e} C_p^{\,e} T^e = \left(\frac{H^2 \rho^2 C_p T}{\mu^2}\right)^e$$

$$\Pi_8 = M^0 L^0 t^0 T^0 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{L^2}{t^2 T}\right)^d \left(\frac{ML}{t^3 T}\right)^e$$

M	0	=	$b + c + e$	$a = 0$
L	0	=	$a - 3b - c + 2d + e$	$b = 0$
t	0	=	$-c - 2d - 3e$	$c = -e$
T	0	=	$-d - e$	$d = -e$

$$\Pi_7 = \mu^{-e} C_p^{-e} k^e = \left(\frac{k}{C_p \mu}\right)^e$$

Hence

$$\frac{T}{T_\infty} = \text{func}\left(\frac{H}{D}, \frac{D}{\delta}, \frac{H\rho^2 Q}{\mu^3}, \frac{V^2}{C_p T_\infty}, \frac{F_x}{\mu H}, \frac{H^2 \rho^2 C_p T}{\mu^2}, \frac{k}{C_p \mu}\right)$$

Dependents on the variables chosen as repeating's, other valid dimensionless groups can be obtained.

Problem 2 – Bonus

1. $x + 7y + 3v + 5u = 16$
2. $8x + 4y + 6v + 2u = -16$
3. $2x + 6y + 4v + 8u = 16$
4. $5x + 3y + 7v + u = -16$

Add equations 2 and 3

$$10(x + y + v + u) = 0 \quad (5)$$

Add equations 1 and 4

$$6x + 10y + 10v + 6u = 0 \quad (6)$$

Add all the equations 1 → 4

$$16x + 20y + 20v + 16u = 0$$

or $4(x+u) + 5(y+v) = 0$

or $4(x + y + v + u) + y + v = 0 \quad (7)$

Use eq. (5) + (7) $y + v = 0 \Rightarrow y = -v$ and $x = -u \quad (8)$

Substitute (8) into (1)

$$4u - 4v = 16 \quad \text{or} \quad u - v = 4 \quad (9)$$

Substitute (8) into (3)

$$6u - 2v = 16 \quad \text{or} \quad 3u - v = 8 \quad (10)$$

Subtract (9) from (10) $2u = 4 \therefore u = 2$

From (9) $v = -2$

From (8) $x = -2, \; y = 2$

$\therefore (x, y, u, v) \rightarrow (-2, 2, 2, -2)$ →