ENCH 501 F2017 **Quiz #1** Name:

September 19, 2017 Time allowed: 35 minutes aJ

1. Three numbers (x, y, z) form an arithmetic progression. That is, they differ by a constant in a series (e.g. y-x = z-y). Three other numbers (a, b, c) form a geometric series. That is, one is multiplied by a constant to get the other one in the series (e.g. b = βa, c = βb). You are given the following: x + a = 85, y + b = 76, z + c = 84 and x + y + z = 126.
   What are the 6 numbers?

2. Given: **u** = a + by + cy² + dy³ where a, b, c and d are constants,
   and the conditions: y = 0, u = 0 ; y = δ, u = U_α ; y = δ, $\frac{du}{dy} = 0$; y = 0 , $\frac{d^2u}{dy^2} = 0$,
   obtain an expression for **u** as a function of U_α, δ and y.

Solution

#1

$$x + a = 85$$
$$y + b = 76$$
$$z + c = 84$$
and $$x + y + z = 126$$
$$\therefore \quad a + b + c = 119$$

Consider the Aritmetic progression

Let $y - x = \varepsilon$
$z - y = \varepsilon$

$$-z + 2y - x = 0$$

or $$x - 2y + z = 0$$

From $\left.\begin{aligned} x + y + z &= 126 \\ x - 2y + z &= 0 \end{aligned}\right) \rightarrow 3y = 126 \quad \therefore y = 42$

Since $y + b = 76 \Rightarrow b = 34$

Consider the geometric progression.

$$a, b, c \equiv \frac{b}{\beta}, b, b\beta$$

$$\therefore \quad a + b + c = b\left(\frac{1}{\beta} + 1 + \beta\right) = 119 \quad \text{or}$$

$$1 + \beta + \beta^2 = \frac{119}{34}\beta \Rightarrow \beta^2 - 2.5\beta + 1 = 0$$

The roots (from $\alpha v^2 + \varepsilon v + \gamma = 0$ with roots

$v = \frac{-\varepsilon \pm \sqrt{\varepsilon^2 - 4\alpha\gamma}}{2\alpha}$) are $\beta = 2$ and $\frac{1}{2}$

Let $\beta = 2$, $a = 17$, $b = 34$ and $c = 68$

$x = 68$, $y = 42$ and $z = 16$

The second root work as well. Show.

#2. $u = a + by + cy^2 + dy^3$

$$\frac{du}{dy} = b + 2cy + 3dy^2$$

$$\frac{d^2u}{dy^2} = 2c + 6dy$$

Conditions

$y = 0 \quad u = 0$ (i)

$y = \delta \quad \frac{du}{dy} = 0$ (ii)

$y = \delta \quad u = U_\alpha$ (iii)

$y = 0 \quad \frac{d^2u}{dy^2} = 0$ (iv)

Use condition (i) $\Rightarrow$ $a = 0$

" " (iv) $\Rightarrow$ $c = 0$

Use " (ii) $0 = b + 3d\delta^2 \quad \times \delta$

" " (iii) $U_\alpha = b\delta + d\delta^3$ Subtract

$$-U_\alpha = 2d\delta^3 \Rightarrow d = -\frac{U_\alpha}{2\delta^3}$$

and $b = -3\left(-\frac{U_\alpha}{2\delta^3}\right)\delta^2 = \frac{3}{2}\frac{U_\alpha}{\delta}$

Substitute for a, b, c and d in original equation

$$u = \frac{3}{2}\frac{U_\alpha y}{\delta} - \frac{1}{2}\frac{U_\alpha y^3}{\delta^3} \quad \text{or}$$

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \longrightarrow$$