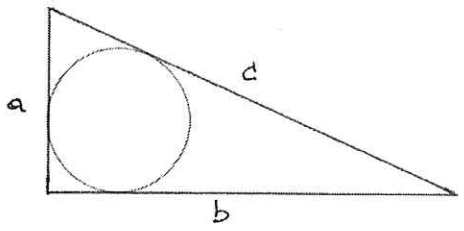
ENCH 501 Transport Phenomena University of Calgary Name_____

Quiz #1 / Time Allowed: 45 minutes Use of cheat sheet allowed. September 20, 2016 a)

a) **(2pts)** A certain brand of pencils normally sells for 60 cents each. During a "promotional sale", all the pencils were sold out within an hour and a total of $46.99 collected. What was the reduced price?

b) **(3pts)** A right-angled triangle is shown below. The length of the hypotenuse is c and the lengths of the two sides are a and b. The diameter of the inscribed circle is d. Prove that $a + b = c + d$.



c) **(5pts)** Certain fluids such as drilling mud, tooth paste and mayonnaise will not flow in a tube unless the pressure applied to overcome wall shear stress exceeds a minimum value called the yield stress. Once the fluid is moving, the friction factor f (dimensionless) depends on the diameter of the tube D, the viscosity of the tube μ, the average flow velocity V and the yield stress τ_o. What dimensionless group is f related to? Show your steps.

*(Buckingham, E. (1921). "On Plastic Flow Through Capillary Tubes". ASTM Proceedings. **21**: 1154–1156.)*

GJ

ENCH 501 Quiz #1 Solution Sept. 20, 2016

(a) The pencils sell normally for 60 ¢, ∴ the promotional sale price will be less.

The total amount collected is 4,699 cents. Assuming that each pencil's price is an integer S and n "whole" pencils were sold, then

$$S \times n = 4,699$$

Numbers other than "prime" would give multiple prices and fractions of a cent.

The prime #s 37 and 127 multiplied = 4699

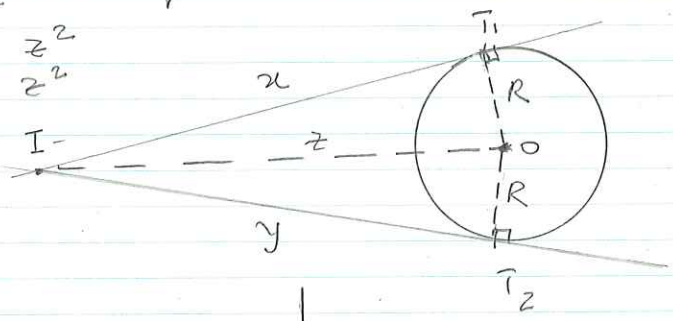
Hence the selling price / pencil = 37 ¢ and the number of S pencils sold = 127.

(b) Two key points are important for this problem.

(1) If 2 tangents to a circle intersect, the lengths of the tangents, from the intersect to the pts. of tangency, are equal.

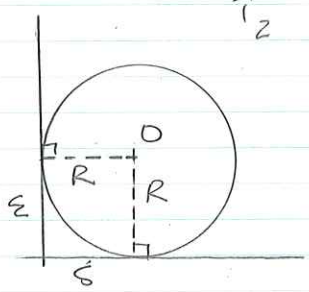
Pythagoras rule

$R^2 + x^2 = z^2$
$R^2 + y^2 = z^2$



or $x = y$, as obvious from the two triangles

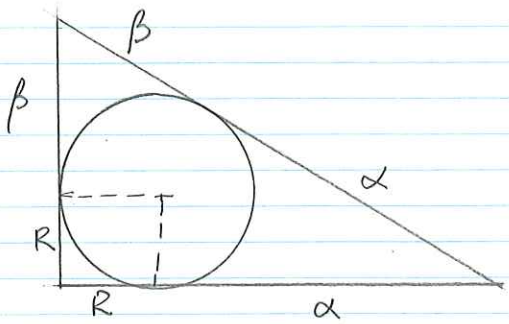
(2)



The normal through a tangent passes thro the centre of the circle, O.

That is $\varepsilon = S = R = \frac{D}{2}$

Now consider the diagram given,



$$a + b = \alpha + \beta + 2R = \alpha + \beta + d$$

$$c = \alpha + \beta$$

$$\therefore \quad a + b = c + d \longrightarrow$$

(c) $f \sim (D, \mu, V, T_0)$

Units	m	Pa.s	$\frac{m}{s}$	Pa
Dimensions	L	$\frac{M}{Lt}$	$\frac{L}{t}$	$\frac{M}{Lt^2}$

(i) By inspection, $\frac{T_0 D}{\mu V}$ (Archimedes #)

is dimensionless

$$\therefore \quad f = \text{function}\left(\frac{T_0 D}{\mu V}\right)$$

(ii) By Pi theorem

There are 3 dimensions — M, L, t and 4 dimensional variables. Therefore there is one dimensionless group

Let $\Pi = D^a \mu^b V^c T_0^d$

$$\Pi = L^{\circ} M^{\circ} t^{\circ} = L^{a} \left(\frac{M}{Lt}\right)^{b} \left(\frac{L}{t}\right)^{c} \left(\frac{M}{Lt^{2}}\right)^{d}$$

length	0	$=$	$a - b + c - d$			$a = d$
mass	0	$=$	b	$+ d$		$b = -d$
time	0	$=$	$-b$	$-c$	$-2d$	$c = -d$

$$\therefore \quad \Pi = \left(\frac{D\, T_0}{\mu V}\right)^{d}$$

Since d is a coefficient, it can take any value. The term within the bracket is the dimensionless group.