GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #1 **September 18, 2012**

Time Allowed: 35 mins. **Name:**

Question #1 (3 points)

a) (1 pt) Given the conditionally convergent series:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ...., \qquad -1 < x \leq 1$$

for x=1, how different is the sum of the first 20 terms from the true value for ln(2)?

b) (1pt) Demonstrate that, for x=1, ln(1+x) can be shown equal to zero.

c) (1 pt) Given a > b, under what condition is $\frac{a}{b} = \frac{b}{a}$? Show an example.

Question #2 (7 points)

When shallow pools of liquids are heated from below, complex patterns of convection currents are observed in the liquid. The heat transfer coefficient **h** (W/m^2 K) between the liquid and the heated surface is considered related to the following dimensional variables - **δ**, depth of pool (m); **k**, thermal conductivity of liquid (W/mK); **g**, acceleration of gravity (m/s^2); **β**, coefficient of volume expansion (1/V dV/dT, K^{-1}); **dT/dz**, temperature gradient (K/m); **μ**, viscosity (Pa.s); **ρ**, density (kg/m^3); and **C_p**, heat capacity (kJ/kg K).

Determine the dimensionless groups. Show important steps.

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\# 1

a) True value (calculator)

$$\ln 2 = 0.693147$$

first 20 terms –

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \cdots - \frac{1}{20} = 0.668438$$

$$\frac{0.668438 - 0.693147}{0.693147} = -0.035647$$

i.e. summation low about 3.6%

$\longrightarrow$

b) $$\ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \cdots$$

$$= \left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \cdots\right) - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} \cdots\right)$$

$$= \left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \cdots\right) + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} \cdots\right) - 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} \cdots\right)$$

$$= 0$$

$\longrightarrow$

c) Given $a > b$, let $b = -a$

then $\frac{a}{b} = \frac{a}{-a}$ and $\frac{b}{a} = \frac{-a}{a}$

or $\frac{a}{b} = \frac{b}{a} = -1$

$\longrightarrow$

#2 $h = f(\delta, k, g, \beta, \frac{dT}{dz}, \mu, \rho, C_p)$

Dimensional variables = 9

Dimensions - $M, L, t, T = 4$

$\therefore$ 5 dimensional groups are to be found.

By inspection

$$\Pi_5 = \frac{h\delta}{k} \quad (Nu)$$

$$\Pi_4 = \frac{k}{\rho C_p} = \frac{\mu/\rho}{C_p \mu/k} = \frac{\nu}{\alpha} \quad (Pr)$$

$$\Pi_3 = \delta \beta \frac{dT}{dz}$$

$\therefore$ 2 dimensional gps remain to be found.

Drop 3 of the dimensional variables - e.g. h, k and dT/dz

The remaining variables are - $\delta, g, \beta, \mu, \rho, C_p$

Dimensions

$$\delta - [L]$$

$$g = [L]/[t]$$

$$\beta = 1/[T]$$

$$\mu = [M]/[L][t]$$

$$\rho = [M]/[L]^3$$

$$C_p = [L]^2/[t]^2[T]$$

$$\left(C_p - \frac{kJ}{kg\,K} = \frac{N.m}{kg\,K} = \frac{kg\,m^2}{s^2\,K} = \frac{m^2}{s^2 K} \right)$$

Let $\Pi_1 = \delta^a g^b \beta^c \rho^d \mu^e$

and $\Pi_2 = \delta^{a'} g^{b'} \beta^{c'} \rho^{d'} C_p^{e'}$

δ, g, β, ρ are rotating variables

$$\therefore \quad \Pi_1 = L^a \left(\frac{L}{t^2}\right)^b \left(\frac{1}{T}\right)^c \left(\frac{M}{L^3}\right)^d \left(\frac{M}{Lt}\right)^e$$

Exponents

$$\begin{array}{llll} L & 0 & = & a + b - 3d - e \\ M & 0 & = & d + e \\ t & 0 & = & -2b - e \\ T & 0 & = & -c \end{array}$$

$a = -\frac{3}{2}e$
$b = -\frac{1}{2}e$
$c = 0$
$d = -e$

$$\Pi_1 = \left(\delta^{-\frac{3}{2}} g^{-\frac{1}{2}} \rho^{-1} \mu\right)^e$$

$$= \left(\frac{\mu}{\rho g^{\frac{1}{2}} \delta^{3/2}}\right)^e$$

and

$$\Pi_2 = L^{a'} \left(\frac{L}{t^2}\right)^{b'} \left(\frac{1}{T}\right)^{c'} \left(\frac{M}{L^3}\right)^{d'} \left(\frac{L^2}{t^2 T}\right)^{e'}$$

$$\begin{array}{llll} L & 0 & = & a' + b' - 3d' + 2e' \\ M & 0 & = & d' \\ t & 0 & = & -2b' - 2e' \\ T & 0 & = & -c' - e' \end{array}$$

$a = -e'$
$b = -e'$
$c = -e'$
$d = 0$

$$\Pi_2 = \left(\delta^{-1} g^{-1} \beta^{-1} c_p\right)^{e'}$$

$$= \left(\frac{c_p}{\delta g \beta}\right)^{e'}$$

Other dimensionless groups are valid.

Overall:

$$\frac{h\delta}{k} = F\left(\frac{k}{\rho c_p},\ \delta\beta\frac{dT}{dz},\ \frac{\mu}{\rho g^{\frac{1}{2}} \delta^{\frac{3}{2}}},\ \frac{c_p}{\delta g \beta}\right)$$