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ENCH 501: Transport Phenomena Quiz #1 September 20, 2011

Time Allowed: 35 mins. Name:

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1. (4 points)

An elaborate glass chandelier has a mass of 1,281 kg. It is to be suspended from the ceiling of a great hall. A solid steel rod is to connect the chandelier to the ceiling.
Estimate the minimum diameter that you would recommend for the rod.

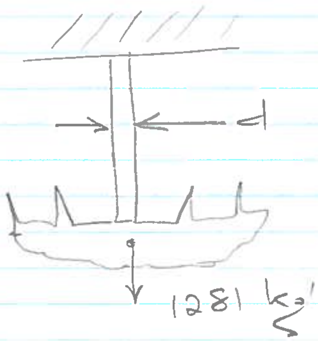
Data: The Young's modulus for structural steel is 200 GPa and the maximum allowable strain (at the yield point) is 1.25 (10^{-3}). The acceleration of gravity is 9.81 m/s^2.

2. (6 points)

A vertical pipe is exposed to ambient air on the outside while saturated steam is passed through the inside. Steam is condensed as it flows. Data on this simple arrangement is to be used to design refrigerator and air-conditioning coils. The dimensional variables and parameters that control the heat transfer process are given as the heat transfer coefficient, h (W/m^2 K), the temperature difference between the steam and ambient, $\Delta\theta$ (°C), the length of pipe, L (m), the latent heat of condensation, λ (kJ/kg), the density of the condensate, ρ (kg/m^3), the thermal conductivity of the condensate, k (W/mK), the viscosity of the condensate, μ (Pa.s) and the acceleration of gravity, g (m/s^2).
Determine the dimensionless groups for the system. Show all important steps.

ENCH 501 Quiz 1 Solution. Sept 20, 2011

1



To ensure that the rod will not break,

$$\frac{mg}{A} \le E\,\varepsilon_{max} \quad \text{Hooke's Law}$$

(Tension) $\frac{\text{force of gravity}}{\text{area}}$ — Young's modulus times max. strain

$\therefore$ A (cross-sectional area) $\geq \frac{mg}{E\,\epsilon_{max}} = \frac{1281(9.81)}{200(10^9)(1.25)(10^{-3})}$

$= 5.0264\,(10^{-5})\ m^2$

Since $A = \frac{\pi d^2}{4}$, $d = 8\,(10^{-3})\ m$

The minimum diameter of the rod should be 8 mm.

$\longrightarrow$

2

The dimensional variables are:

$$h = f(\Delta\theta, L, \lambda, \rho, k, \mu, g)$$

	h	$\Delta\theta$	L	λ	ρ	k	μ	g
units	W/m^2K	K	m	J/kg	kg/m^3	W/mK	Pa.s	m/s^2
Dimensions	$\frac{M}{t^3T}$	T	L	L^2/t^2	$\frac{M}{L^3}$	$\frac{ML}{t^3T}$	$\frac{M}{Lt}$	$\frac{L}{t^2}$

where $W = J/s = \frac{N.m}{s} = \frac{M.a.m}{s} = \frac{ML^2}{t^3}$

$Pa = N/m^2 = \frac{M.a}{m^2} = \frac{M}{Lt^2}$

first step, find dimensionless quantities by inspection – there are 8 dimensional quantities.

$$\frac{hL}{k}, \quad \frac{\lambda}{gL}$$

There are 4 fundamental dimensions – M, L, t, T

∴ There should be 8 − 4 = 4 dimensionless groups, of which 2 have been identified.

Re-write first equation, dropping 2 dimensional groups – I choose h and λ.

$$\therefore \quad k = f(\Delta\theta, L, \rho, \mu, g)$$

Choose 4 repeating variables, $\Delta\theta$, L, ρ and μ that contain all fundamental dimensions

$$\Pi_1 = \Delta\theta^a L^b \rho^c \mu^d g$$

$$\Pi_2 = \Delta\theta^a L^b \rho^c \mu^d k$$

Use the Buckingham Pi Theorem for $\Pi_1 + \Pi_2$

$$\Pi_1 = T^a L^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{Lt}\right)^d \left(\frac{L}{t^2}\right) = 0$$

mass	$0 = c + d$	$a = 0$
length	$0 = b - 3c - d + 1$	$b = 3$
time	$0 = -d - 2$	$c = 2$
temp.	$0 = a$	$d = -2$

$$\therefore \quad \Pi_1 = (\Delta\theta)^0 L^3 \rho^2 \mu^{-2} g = \frac{L^3 \rho^2 g}{\mu^2}$$

→

For Π_2

$$0 = T^a L^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{Lt}\right)^d \left(\frac{ML}{t^3 T}\right)$$

mass	$0 = c + d + 1$		$a = 1$
length	$0 = b - 3c - d + 1$		$b = 2$
time	$0 = -d - 3$		$c = 2$
temp	$0 = a - 1$		$d = -3$

$$\therefore \quad \Pi_2 = (\Delta\theta)^1 L^2 \rho^2 \mu^{-3} k = \frac{\Delta\theta L^2 \rho^2 k}{\mu^3}$$

For the problem: the groups are

$$\frac{hL}{k} = \text{Func}\left(\frac{\lambda}{gL}, \frac{L^3\rho^2 g}{\mu^2}, \frac{(\Delta\theta) L^2 \rho^2 k}{\mu^3}\right)$$

→

Note that the final answer depends on the choice of the dimensional quantities used to determine the Πs, other answers are correct, as long as the groups formed are dimensionless.

The ratio

$$\frac{(\Delta\theta)L^2\rho^2 k}{\mu^3} \cdot \frac{\mu^2}{L^3\rho^2 g} = \frac{\Delta\theta k}{L\mu g} = \frac{\text{conduction}}{\text{viscous}}$$

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