

CJ

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ENCH 501: Transport Phenomena Quiz #1**September 15, 2009****Time Allowed: 30 mins.****Name:****1. (6 points)**

Fresh tomatoes, *Lycopersicon esculentum*, is the world's second largest vegetable crop. It is the base for making tomato paste which is marketed directly to consumers and also used as ingredients for other products such as ketchup, soups and sauces. Fresh tomato contains 93 -95% (by wt) water content, the balance being solids. Pastes contain a minimum of 24% solids. A double concentrate has 30% solids and a triple concentrate has 38 - 40% solids. Ground tomatoes is concentrated by evaporation under a partial vacuum. The cost of removing the water by heating is a significant factor in the selling price of the paste.

A batch of fresh tomatoes containing 93% water is ground and it is to be concentrated by heating to form a paste with 40% solids and a density of 535 kg/m^3 . Heating is done through burning natural gas. Industrial and domestic burners are, however, inefficient and only 10% of the heat of combustion is utilized to evaporate the water. The rest of the energy is lost to the ambient. The heat of vaporization of water is $2,360 \text{ kJ/kg}$ at the operating temperature of 60°C .

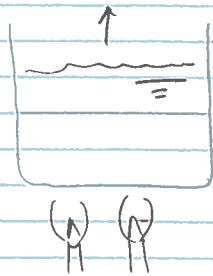
If the fresh tomatoes are purchased at 15¢/kg and the paste is sold wholesale at $\$2.20/\text{kg}$, what is the cost for the natural gas ($\$/\text{kJ}$) at which the operation will just break even - that is, at which the cost of production is equal to the selling price? Show all your steps. You may neglect all other costs such as personnel, maintaining a vacuum, repairs etc.

2. (4 points)

When air flows at high speeds across a wire, a humming sound can often be heard as the wire shed vortices in its wake. The frequency of the sound (f in $1/\text{s}$) depends on the air speed (U in m/s), the diameter of the wire (D in m), the density of air (ρ in kg/m^3) and its dynamic viscosity (μ in $\text{mPa} \cdot \text{s}$).

Obtain the dimensionless groups for this system. Show your steps.

(a)



Basis: 100 kg fresh tomatoes

This contains 93 kg water
7 kg solidsThe product - paste, contains
60% water and 40% solids.This 40% solids \equiv 7 kg \therefore The water present in paste =

$$7\left(\frac{3}{2}\right) = 10.5 \text{ kg}$$

 \therefore Total mass of paste = 17.5 kg \rightarrow

The mass of water evaporated =

$$93 - 10.5 = 82.5 \text{ kg}$$

and the energy required is

$$(82.5)(2360) = 194,700 \text{ kJ}$$

The actual energy used is 10 times this value.

Let the cost of natural gas = x \$/kJ

Then the break even point is given by

$$17.5(2.2) = (0.15)(100) + 1.947(10^6)x$$

$$\text{or } x = 1.207(10^{-5}) \text{ $/kJ} \rightarrow$$

(b) Given $f = \text{function}(u, D, \rho, \mu)$ units s^{-1} $\frac{m}{s}$ m $\frac{kg}{m^3}$ $mPa \cdot s$ Dimensions t^{-1} $\frac{L}{t}$ L $\frac{M}{L^3}$ $\frac{F \cdot t}{L^2} = \frac{M}{L \cdot t}$ # dimensions = 3 \rightarrow M, L, t

Use the π -theorem - 5 variables, 3 dimensions
 \therefore 2 dimensionless groups

$$\pi_1 = \underbrace{u^a D^b \rho^c f}_{\text{repeating variables}} \quad ; \quad \pi_2 = \underbrace{u^a D^b \rho^c} \mu$$

$$\pi_1 = \left(\frac{L}{t}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{1}{t}\right) = M^0 L^0 t^0$$

$$M: 0 = c$$

$$L: 0 = a + b - 3c$$

$$t: 0 = -a - 1$$

$$\left\{ \begin{array}{l} a = -1 \\ b = -a = 1 \\ c = 0 \end{array} \right.$$

$$\pi_1 = u^{-1} D f = \frac{f D}{u} \quad (\text{Strouhal number})$$

$$\pi_2 = \left(\frac{L}{t}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{L t}\right) = M^0 L^0 t^0 \quad \rightarrow$$

$$M: 0 = c + 1$$

$$L: 0 = a + b - 3c - 1$$

$$t: 0 = -a - 1$$

$$\left\{ \begin{array}{l} a = -1 \\ b = -1 \\ c = -1 \end{array} \right.$$

$$\pi_2 = \frac{\mu}{u D \rho} \quad (\text{inverse of Reynolds \#})$$

\therefore The dimensionless groups are:

$$\frac{f D}{u} \quad \text{and} \quad \frac{u D \rho}{\mu}$$

\rightarrow