GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #1 September 20, 2005

Time Allowed: 50 mins. Name:

The drag coefficient (C_D) for a body moving in a fluid is defined as [$F_D/(\frac{1}{2}\rho V^2 A)$] where F_D is the drag force, V is the relative velocity between the body and the fluid, ρ is the fluid density and A is the projected (or frontal) area of the body.

Data collected in a water-flow tunnel on a 1:160 scale model of a submersible to be used in off-shore drilling for oil is provided below.

a) Identify the dimensional variables which affect the magnitude of the drag force on a body submerged in a flowing liquid, and determine the dimensionless variables for the system. Show all your steps.

b) Estimate the drag force of the full-scale submersible (prototype) which has a length of 64m and a diameter of 12m, if the submersible is moving at a speed of 0.524 km/hr in sea water which has a current of 0.5 km/hr along the same direction.

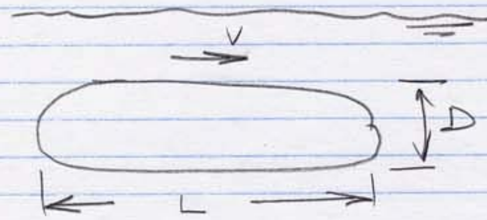
c) If the submersible reversed direction and maintained its speed as before directly against the current, estimate the force on the submersible. (It may be useful to examine the trends for or make a plot of the dimensionless groups you determined above for the model data.)

Data: Properties of fresh : sea water at 5°C μ = 1.54 : 1.61 mPa s ; ρ = 998 : 1202 kg/m^3

Experimental data for model stationary in flowing fresh water:

Water Speed, m/s	0.6	0.749	0.823	1.034	1.202	1.323	1.412	1.515	1.604
Drag Force, N	3.1	4.41	5.19	7.97	10.7	12.9	14.7	16.9	18.9

ENCH 501 Quiz #1 Solution Sept. 20, 2005



a/ The variables which affect the drag force on the submersible are V, D, L, μ, ρ

where μ = viscosity, ρ = density of medium.

i.e. $F_D = f(V, D, L, \mu, \rho)$

Determine dimensionless quantities by the Pi-theorem.

Quantity	F_D	V	D	L	μ	ρ
Dimension	$\frac{ML}{t^2}$	$\frac{L}{t}$	L	L	$\frac{M}{Lt}$	M/L^3

Total of 6 dimensional variables and 3 fundamental dimensions.

$\therefore$ # of dimensionless quantities = 6 - 3 = 3

By Inspection, $\pi_3 = L/D$ is dimensionless. Therefore there are 2 more dimensionless quantities to determine

Choose 3 convenient variables L, V, ρ and let

$$\pi_1 = L^a V^b \rho^c F^d, \quad \pi_2 = L^{a'} V^{b'} \rho^{c'} \mu^{d'}$$

Substitute dimensions

$$\pi_1 = M^0 L^0 t^0 = L^a \left(\frac{L}{t}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{ML}{t^2}\right)^d$$

Equate exponents $\begin{cases} \text{length} & 0 = a + b - 3c + d \\ \text{mass} & 0 = c + d \\ \text{time} & 0 = -b - 2d \end{cases}$

Solve algebraic equations $\Rightarrow a = -2d,\ b = -2d,\ c = -d$

$$\therefore \pi_1 = L^{-2d} V^{-2d} \rho^{-d} F^d = \left(F/\rho L^2 V^2\right)^d$$

$$\therefore \pi_1 = F/\rho L^2 V^2$$

Similarly, solve for $\Pi_2 = \left(\frac{\rho V L}{\mu}\right)$

Hence, $\frac{F}{\rho L^2 V^2} = g\left(\frac{\rho V L}{\mu}, \frac{L}{D}\right)$; $Re = \frac{\rho V L}{\mu} \cdot \frac{D}{L} = \frac{\rho V D}{\mu}$

By multiplying $\frac{F}{\rho L^2 V^2}$ by $\left(\frac{L}{D}\right)^2$ one obtains $\frac{F}{\rho D^2 V^2} = C_F$

force coefficient

The drag coefficient $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{F_D}{\frac{1}{2}\rho V^2 \frac{\pi D^2}{4}}$

Hence $C_D = C_F \cdot \frac{8}{\pi}$

$\longrightarrow$

b) For the problem, geometric similarity is indicated, i.e. L/D is the same for both model and prototype.

When the current and the prototype are moving in the same direction, the prototype is moving faster by 0.024 km/hr

or $(24/3600)$ m/s.

For prototype $Re_p = \frac{12\,(24/3600)(1202)}{1.61\,(10^{-3})}$

For model $Re_m = \frac{(12/160)(V)(998)}{1.54\,(10^{-3})}$

For kinematic similarity, $Re_p = Re_m$

$\therefore \quad V = 1.2288$ m/s

Interpolate F_D for model from data provided:

Linear interpolation, $F_D = 11.19$ N

For dynamic similarity, $C_F\big|_{model} = C_F\big|_{prototype}$

Hence

$$\frac{11.188}{(998)(1.2288)^2\left(\frac{12}{160}\right)^2} = \frac{F_p}{(1202)\left(\frac{24}{3600}\right)^2(12)^2}$$

$\therefore$ Drag force on prototype is

$$F_p = 10.15 \text{ N} \longrightarrow$$

c) When the submersible's direction is reversed, the relative velocity becomes 1.024 km/hr.

$$\therefore \quad Re_p = \frac{12\left(\frac{1024}{3600}\right)(1202)}{1.61(10^{-3})} = Re_m$$

$$\doteq 2.55(10^6)$$

The corresponding model speed $V = 52.43$ m/s

This far exceeds the values in the Data!

For the model calculate Re and C_F values.

V	$Re = DV\rho/\mu$	$C_F = F/\rho V^2 D^2$
0.6	29,162	1.534
0.749	36,404	1.4
0.823	40,001	1.365
1.034	50,256	1.328
1.202	58,422	1.319
1.323	64,303	1.313
1.412	68,288	1.313
1.515	73,635	1.312
1.604	77,961	1.309

As Re increases past about 60,000, C_F approaches a constant, $\simeq 1.31$

$\therefore$ For the problem

$$C_D = \frac{8}{\pi} \cdot 1.31 = \frac{F_D}{\frac{1}{2}\rho V^2\left(\frac{D^2\pi}{4}\right)} = \frac{F_D}{\frac{1}{2}(1202)\left(\frac{1024}{3600}\right)^2(12)^2\frac{\pi}{4}}$$

$$\vec{F_D} = 18,345.7 \text{ N}$$

The drag force has increased substantially, and thus the power output of the engine will be significant.