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ENCH 501: Transport Processes Quiz #1 **September 14, 2004**

Time Allowed: 50 mins. **Name:**

Problem #1 (10 points)

A centrifugal pump has an impeller of diameter D equal 0.329m. When pumping water at 20°C (ρ=998 kg/m^3; μ=1.003 mPa s), at an rpm Ω of 1160, the manufacturer provides the following data to relate the volume flow rate Q to the pressure rise ΔP from the inlet to the outlet.

Q, litres/min	756	1134	1512	1890	2268	2646
ΔP, atm.	2.467	2.399	2.330	2.198	1.988	1.576

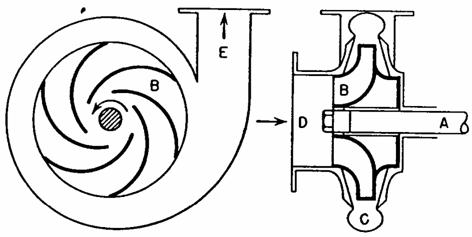

Diagram of a simple centrifugal pump.

(a) If ΔP = **f**(μ, ρ, D, Q, Ω), determine the dimensionless quantities which describe the pump.

(b) A similar pump with D equal 0.244m is to be used to pump gasoline (ρ=680 kg/m^3; μ=0.292 mPa s) at 20°C. Use the data above to prepare a table of ΔP versus Q for pumping the gasoline.

ENCH 501 QUIZ #1 Solution Sept. 14, 2004

a) Given: $\Delta P = f(\mu, \rho, D, Q, \Omega)$

	ΔP	μ	ρ	D	Q	Ω
units	Pa or N/m^2	Pa.s	$\frac{kg}{m^3}$	m	m^3/s	s^{-1}
Dimensions	M/Lt^2	$\frac{M}{Lt}$	$\frac{M}{L^3}$	L	L^3/t	$\frac{1}{t}$

\# of variables – n = 6
\# of dimensions – r = 3 ∴ # of dimensionless variables = 3

By inspection, $\frac{D^3\Omega}{Q}$ forms a dimensionless gp, Π_3

Hence reduce the number of variables by one, or

$$\Delta P = g(D, Q, \mu, \rho)$$

Choose 3 variables – D, Q, ρ

$$\therefore \Pi_1 = D^a Q^b \rho^c \mu^d$$

and $$\Pi_2 = D^a Q^b \rho^c (\Delta P)^d$$

With different choice other dimensionless variables will be derived.

- For Π_1

$$M^0L^0t^0 = L^a \left(\frac{L^3}{t}\right)^b \left(\frac{M}{L^3}\right)^c \left[\frac{M}{Lt}\right]^d$$

Group the coefficients

			Yeilds
mass	0 =	$c + d$	$a = d$
length	0 =	$a + 3b - 3c - d$	$b = -d$
time	0 =	$-b - d$	$c = -d$

$$\therefore \Pi_1 = \left[\frac{D\mu}{Q\rho}\right]^d$$

- For Π_2

$$M^0L^0t^0 = L^a \left[\frac{L^3}{t}\right]^b \left[\frac{M}{L^3}\right]^c \left[\frac{M}{Lt^2}\right]^d$$

mass	0 =	$c + d$	$a = 4d$
length	0 =	$a + 3b - 3c - d$	$b = -2d$
time	0 =	$-b - 2d$	$c = -d$

$$\Pi_2 = \left[\frac{D^4 \Delta P}{Q^2 \rho}\right]^d$$

Hence the dimensionless quantities that describe the pump are:

$$\frac{D^4 \Delta P}{Q^2 \rho} = F^+\left(\frac{D\mu}{Q\rho}, \frac{D^3 \Omega}{Q}\right)$$ This or alternative OK.

or

$$\frac{D^4 \Delta P}{Q^2 \rho} \cdot \frac{Q^2}{D^6 \Omega^2} = F^{++}\left(\frac{D\mu}{Q\rho} \cdot \frac{Q}{D^3\Omega}, \frac{D^3\Omega}{Q}\right)$$

or

$$\frac{\Delta P}{\rho D^2 \Omega^2} = F^\circ\left[\frac{\mu}{\rho D^2 \Omega}, \frac{D^3 \Omega}{Q}\right]$$

$$\frac{\Delta P}{\rho D^2 \Omega^2} = F\left[\frac{\rho D^2 \Omega}{\mu}, \frac{Q}{D^3 \Omega}\right]$$

dynamic kinematic efficiency

→

b) If the pumps are similar, each of the dimensionless group has identical values for both operations.

Let 1 denote the water pump + 2 the gasoline pump.

$$\frac{\rho_1 D_1^2 \Omega_1}{\mu_1} = \frac{\rho_2 D_2^2 \Omega_2}{\mu_2}$$

$$\frac{998(0.329)^2 1160}{1.003} = \frac{680(0.244)^2 \Omega_2}{0.292} \quad \text{or}$$

$$\Omega_2 = 901.1 \text{ rpm}, \text{ const. for } 2^{nd} \text{ pump.}$$

Also $$\frac{Q_1}{D_1^3 \Omega_1} = \frac{Q_2}{D_2^3 \Omega_2} \quad \text{or} \quad Q_2 = \frac{D_2^3 \Omega_2}{D_1^3 \Omega_1} Q_1$$

or $$Q_2 = \frac{(0.244)^3 (901.1)}{(0.329)^3 (1160)} Q_1$$

$$Q_2 = 0.31688 \; Q_1 \longrightarrow$$

And

$$\frac{\Delta P_1}{\rho_1 D_1^2 \Omega_1^2} = \frac{\Delta P_2}{\rho_2 D_2^2 \Omega_2^2}$$

$$\therefore \Delta P_2 = \frac{\rho_2 D_2^2 \Omega_2^2}{\rho_1 D_1^2 \Omega_1^2} \Delta P_1$$

$$= \frac{680 (0.244)^2 (981.1)^2}{998 (0.329)^2 (1160)^2} \Delta P_1$$

$$= 0.22615 \; \Delta P_1$$

Given Q_1 litres/min	ΔP_1 atm	Calculate Q_2 litres/min	ΔP_2 atm
756	2.467	239.56	0.558
1134	2.399	359.34	0.543
1512	2.330	479.12	0.527
1890	2.198	598.90	0.497
2268	1.988	718.68	0.450
2646	1.576	838.46	0.356

$\longrightarrow$