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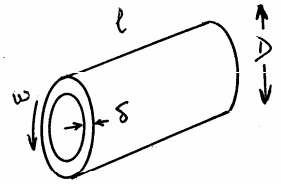
ENCH 501: Transport Processes Quiz #1 **September 16, 2003**

Time Allowed: 45 mins. **Name:**

Problem #1 (4 points)

Power loss in a journal bearing (P) varies with the length of the bearing (ℓ), the diameter (D), the clearance or width of gap filled with lubricant (δ), the rate of rotation (ω), the fluid viscosity (μ) and the pressure (p).

Determine the dimensionless quantities which describe the system.



Problem #2 (6 points)

The pressure rise Δp, of a liquid flowing at a steady rate through a centrifugal pump, depends on the pump diameter D, the angular rotation rate of the motor ω, the volumetric flow rate Q, and the density of the liquid ρ.

The following data are given for a prototype and a test model which is geometrically similar. Determine the two values missing from the table. Show your steps.

Variable	*Prototype*	*Model*
Δp, kPa	?	29.3
Q, m^3/min	1.25	?
ρ, kg/m^3	800	999
ω, rad/s	183	367
D, mm	150	50

aJ

ENCH 501 Quiz #1 SOLUTION Sept. 16, 2003

Problem #1

$$\underset{\text{power loss}}{P} = f(\ell, D, \delta, \omega, \mu, \rho)$$

Units	W or $\frac{N\cdot m}{s} = \frac{kg\,m^2}{s^3}$	m	m	m	s^{-1}	Pa·s $\frac{N}{m^2}\cdot s = \frac{kg}{m\cdot s}$	$Pa = N/m^2 = \frac{kg}{m\,s^2}$
Dimensions	$\frac{ML^2}{t^3}$	L	L	L	t^{-1}	$\frac{M}{Lt}$	$\frac{M}{Lt^2}$

There are 7 dimensional variables and 3 fundamental dimensions (M, L, t), ∴ There are 4 dimensionless gps.

By inspection, there are 2 dimensionless gps for length — ℓ/D and δ/D

$\therefore \pi_3 = \ell/D$ and $\pi_4 = \delta/D$

Define $P = g(D, \omega, \mu, \rho)$

Choose D, ω and ρ as three repeating variables.

$$\pi_1 = D^a \omega^b \rho^c P^d \quad \text{and} \quad \pi_2 = D^{a'} \omega^{b'} \rho^{c'} \mu^{d'}$$

Solve. $M^0 L^0 t^0 = \frac{L^a}{t^b}\,\frac{M^c}{L^c t^{2c}}\,\frac{M^d L^{2d}}{t^{3d}}$ for π_1

mass	$0 =$	$c + d$
length	$0 =$	$a - c + 2d$
time	$0 =$	$-b - 2c - 3d$

$a = -3d$, $b = -d$, $c = -d \Rightarrow \pi_1 = \left[\frac{P}{D^3 \omega \rho}\right]^d$

$$M^0 L^0 t^0 = L^{a'} \frac{M^{c'}}{t^{b'}} \frac{M^{d'}}{L^{c'} t^{2c'}} L^{d'} t^{x'} \quad \text{for} \quad \Pi_2$$

mass	$0 =$	$c' + d'$
length	$0 =$	$a' - c' - d'$
time	$0 =$	$-b' - 2c' - x'$

$a' = 0$, $b' = d'$, $c' = -d'$ $\Rightarrow$ $\Pi_2 = \left[\frac{\mu w}{\rho}\right]^{d'}$

Hence the dimensionless quantities are:

$$\left[\frac{P}{D^3 w \rho}\right] = F\left[\frac{\mu w}{\rho}, \frac{\lambda}{D}, \frac{\delta}{D}\right]$$

Pls. note that other dimensionless variables can be created by combinations of the above.

Problem #2

Identify the dimensionless quantities first and then invoke similarity.

As above

$$\Delta P = f(D, w, Q, \rho)$$

Units	Pa or $N/m^2 = \frac{kg}{m s^2}$	m	s^{-1}	m^3/s	kg/m^3
Dimensions	$\frac{M}{Lt^2}$	L	t^{-1}	L^3/t	M/L^3

dimensional variables = 5
of fundamental dimensions = 3
$\therefore$ 2 dimensionless groups.

Choose D, w and ρ as repeating variables.

$\Pi_1 = D^a w^b \rho^c Q^d$ and $\Pi_2 = D^{a'} w^{b'} \rho^{c'} (\Delta P)^{d'}$

For Π_1 $\quad M^0 L^0 t^0 = L^a \frac{1}{t^b} \frac{M^c}{L^{3c}} \frac{L^{3d}}{t^d}$

mass $\quad 0 = c$

length $\quad 0 = a - 3c + 3d$

time $\quad 0 = -b - d$

$a = -3d, \quad b = -d, \quad c = 0 \Rightarrow \Pi_1 = \left[\frac{Q}{D^3 w}\right]^d$

For Π_2 $\quad M^0 L^0 t^0 = L^{a'} \frac{1}{t^{b'}} \frac{M^{c'}}{L^{3c'}} \frac{M^{d'}}{L^{d'} t^{2d'}}$

mass $\quad 0 = c' + d'$

length $\quad 0 = a' - 3c' - d'$

time $\quad 0 = -b' - 2d'$

$a' = -2d', \quad b' = -2d', \quad c' = -d' \Rightarrow \Pi_2 = \left[\frac{\Delta P}{w^2 \rho D^2}\right]^{d'}$

For Dynamic similarity, Π_2 must be the same for both systems.

For Kinematic similarity, Π_1 should be the same.

$$\Pi_1 = \frac{1.25}{150^3 (183)} \Bigg|_{\text{prototype}} = \frac{?}{50^3 (367)}$$

Note: units from table is not converted since they cancel.

∴ Volume rate for model = 0.0928 m^3/min →

$$\Pi_2 = \frac{?}{(150)^2 183^2 800} = \frac{29.3}{(50)^2 (367)^2 999}$$

∴ Pressure drop for prototype = 52.5 kPa →

Note: It is important to choose 3 repeating variables which do not combine to form a dimensionless quantity. Try D, w and Q for example.