The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Mathematical Methods in Chemical Engineering
Quiz #1

Time Allowed: 50 mins. **September 18, 2001**

Problem #1

Bitumen is extracted from tar sands in the hot water process by mixing pit-mined lumps of tar sands (which contains about 12% by wt. Of bitumen) with hot water at ~85°C and caustic, in a slightly inclined rotating drum. The bitumen is separated from sand grains by the shear stress which develops in the slurry formed, as the drum content tumbles. The efficiency of separation is assumed directly proportional to the shear stress (τ) and depends on the acceleration of gravity (g), a characteristic tar sand lump diameter (d), the density difference between sand and bitumen ($\rho_p - \rho_o$), the density and viscosity of water (ρ, μ), the interfacial tension (σ), the diameter of the drum (D), the angular rate of rotation of the drum (Ω) and the residence time (θ) of the mixture in the drum.

Obtain the dimensionless groups which describe the process.

Problem #2

In her *Guide to Excruciatingly Correct Behavior*, Miss Manners states:

> There are three possible parts to a date of which at least two must be offered: entertainment, food and affection. It is customary to begin a series of dates with a great deal of entertainment, a moderate amount of food and the merest suggestion of affection. As the amount of affection increases, the entertainment can be reduced proportionately. When the affection has replaced the entertainment, we no longer call it dating. Under no circumstances can the food be omitted.

Entertainment, affection and food are assigned values using a point system as follows, given time t in months:

Entertainment, $E = 100\, e^{-\alpha t}$; $\alpha = 1/3$

Affection, $A = 150\,(1 - \beta^{t})$; $\beta = 0.6$

Food, $F = 50$

a) At what time is the total value accumulated a minimum or a maximum? Determine which of the latter.

b) When does the extremum occur relative to when A and E are equal, i.e. earlier or later?

You may make a rough sketch of the functions and the sum.

DERIVATIVES OF TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS

13.14 $\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

13.15 $\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

13.16 $\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$

13.17 $\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$

13.18 $\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$

13.19 $\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$

13.20 $\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \left[-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right]$

13.21 $\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad [0 < \cos^{-1} u < \pi]$

13.22 $\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx} \qquad \left[-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right]$

13.23 $\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx} \qquad [0 < \cot^{-1} u < \pi]$

13.24 $\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} + \text{ if } 0 < \sec^{-1} u < \pi/2 \\ - \text{ if } \pi/2 < \sec^{-1} u < \pi \end{bmatrix}$

13.25 $\frac{d}{dx}\csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} - \text{ if } 0 < \csc^{-1} u < \pi/2 \\ + \text{ if } -\pi/2 < \csc^{-1} u < 0 \end{bmatrix}$

DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

13.26 $\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx} \qquad a \neq 0, 1$

13.27 $\frac{d}{dx}\ln u = \frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$

13.28 $\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$

13.29 $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

13.30 $\frac{d}{dx}u^v = \frac{d}{dx}e^{v \ln u} = e^{v \ln u}\frac{d}{dx}[v \ln u] = vu^{v-1}\frac{du}{dx} + u^v \ln u \frac{dv}{dx}$

DERIVATIVES OF HYPERBOLIC AND INVERSE HYPERBOLIC FUNCTIONS

13.31 $\frac{d}{dx}\sinh u = \cosh u \frac{du}{dx}$

13.32 $\frac{d}{dx}\cosh u = \sinh u \frac{du}{dx}$

13.33 $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \frac{du}{dx}$

13.34 $\frac{d}{dx}\coth u = -\operatorname{csch}^2 u \frac{du}{dx}$

13.35 $\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \tanh u \frac{du}{dx}$

13.36 $\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \coth u \frac{du}{dx}$

ENCH 501 Quiz #1 Solution Sept. 18, 2001

☐ Problem #1

$$\text{Efficiency} \sim \tau = f(g, d, \rho_p - \rho_o, \rho, \mu, \sigma, D, \Omega, \theta)$$

	τ	g	d	$\rho_p-\rho_o$	ρ	μ	σ	D	Ω	θ
Units	Pa, N/m^2, $\frac{kg}{m s^2}$	m/s^2	m	kg/m^3	$\frac{kg}{m^3}$	Pa·s, $\frac{N \cdot s}{m^2}$, $\frac{kg}{m s}$	Nm^{-1}, $\frac{kg}{s^2}$	m	s^{-1}	s
Dimensions	$\frac{M}{Lt^2}$	$\frac{L}{t^2}$	L	$\frac{M}{L^3}$	M/L^3	$\frac{M}{Lt}$	$\frac{M}{t^2}$	L	t^{-1}	t

Three dimensionless groups are immediately obvious

$$\frac{\rho_p - \rho_o}{\rho}, \quad \frac{d}{D} \quad \text{and} \quad \Omega\theta$$

Total number of dimensional variables, $n = 10$

Total number of dimensions, $j = 3$ (M, L, t)

$\therefore$ Total # of dimensionless variables, $k = n - j = 7$

of these, 3 have been identified by inspection.

Hence

$$\tau = h(g, D, \rho, \mu, \sigma, \Omega)$$

Choose 3 variables which do not combine to form a dimensionless group — D, ρ, μ

$$\Pi_1 = D^a \rho^b \mu^c \tau$$

$$\Pi_2 = D^a \rho^b \mu^c g$$

$$\Pi_3 = D^a \rho^b \mu^c \sigma$$

$$\Pi_4 = D^a \rho^b \mu^c \Omega$$

Using the Buckingham Pi theorem method

$$\Pi_1 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{m}{Lt}\right)^c \frac{m}{Lt^2}$$

mass $0 = b + c + 1$
length $0 = a - 3b - c - 1$
time $0 = -c - 2$

$c = -2$, $b = 1$, $a = 2$

$$\Pi_1 = \frac{D^2 \rho \tau}{\mu^2}$$

$$\Pi_2 = L^a \left(\frac{m}{L^3}\right)^b \left(\frac{m}{Lt}\right)^c \frac{L}{t^2}$$

mass $0 = b + c$
length $0 = a - 3b - c + 1$
time $0 = -c - 2$

$c = -2$, $b = 2$, $a = 3$

$$\Pi_2 = \frac{D^3 \rho^2 g}{\mu^2}$$

$$\Pi_3 = L^a \left(\frac{m}{L^3}\right)^b \left(\frac{m}{Lt}\right)^c \frac{m}{t^2}$$

mass $0 = b + c + 1$
length $0 = a - 3b - c$
time $0 = -c - 2$

$c = -2$, $b = 1$, $a = 1$

$$\Pi_3 = \frac{D \rho \sigma}{\mu^2}$$

$$\Pi_4 = L^a \left(\frac{m}{L^3}\right)^b \left(\frac{m}{Lt}\right)^c \frac{1}{t}$$

mass $0 = b + c$
length $0 = a - 3b - c$
time $0 = -c - 1$

$c = -1$, $b = -c = 1$, $a = 2$

$$\Pi_4 = \frac{D^2 \rho \Omega}{\mu}$$, Reynolds number.

Hence
$$\left[\frac{D^2 \rho \tau}{\mu^2}\right] = \phi\left[\frac{D^3 \rho^2 g}{\mu^2}, \frac{D \rho \sigma}{\mu^2}, \frac{D^2 \rho \Omega}{\mu}, \frac{\rho_p - \rho_0}{\rho}, \frac{d}{D}, \Omega\theta\right]$$

☐ Problem # 2

(a) The pertinent equations are:

$$E = 100\, e^{-\alpha t}$$

$$A = 150\,(1-\beta^{t})$$

$$F = 50$$

13.29

Total $T = E + A + F = 50 + 100 e^{-\alpha t} + 150\,(1-\beta^{t})$

At the extremum,

$$\frac{dT}{dt} = 0 = -100\,\alpha\, e^{-\alpha t} - 150\,\beta^{t} \ln\beta$$

Substitute $\alpha = \frac{1}{3}$ and $\beta = 0.6$

$$\frac{100}{3} e^{-t/3} = -150\,(0.6)^{t} \ln(0.6)$$

$$e^{-t/3} = 2.2987\,(0.6)^{t}$$

Use natural logs (ln)

$$-t/3 = \ln(2.2987) + t \ln(0.6)$$

Solve

$$t = \frac{0.8324}{0.1775} = 4.6894 \text{ months.}$$

To find maximum or minimum, obtain $\frac{d^2T}{dt^2}$

$$\frac{d^2T}{dt^2} = 100\,\alpha^2 e^{-\alpha t} - 150\,(\ln\beta)^2 \beta^{t}$$

Substitute for t, $\frac{d^2T}{dt^2} = -1.2391 < 0$

$\therefore$ At $t = 4.69$ months, the accumulated value is at the maximum.

(6) When $A = E$

$$100\, e^{-t/3} = 150(1 - 0.6^t)$$

$$\frac{2}{3} e^{-t/3} = 1. - 0.6^t$$

Solve — eg. chose t, evaluate left + right sides

$t = 1.174$ months.

This is less that when value is maximum. ⟶