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ENCH 501: Mathematical Methods in Chemical Engineering
Quiz #1

Time Allowed: 30 mins. **September 19, 2000**

Student's Name: ____________________________

In her *Guide to Excruciatingly Correct Behavior*, Miss Manners states:

> There are three possible parts to a date of which at least two must be offered: entertainment, food and affection. It is customary to begin a series of dates with a great deal of entertainment, a moderate amount of food and the merest suggestion of affection. As the amount of affection increases, the entertainment can be reduced proportionately. When the affection has replaced the entertainment, we no longer call it dating. Under no circumstances can the food be omitted.

Entertainment, affection and food are assigned values using a point system as follows, given time t in months:

Entertainment,	$E = 100\, e^{-\alpha t}$;	$\alpha = 1/3$
Affection,	$A = 150\,(1 - \beta^{t})$;	$\beta = 0.6$
Food,	$F = 50$	

a) At what time is the total value accumulated a minimum or a maximum? Determine which of the latter.

b) When does the extremum occur relative to when A and E are equal, i.e. earlier or later?

You may make a rough sketch of the functions and the sum.

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13 DERIVATIVES

DEFINITION OF A DERIVATIVE

If $y = f(x)$, the derivative of y or $f(x)$ with respect to x is defined as

13.1 $$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

where $h = \Delta x$. The derivative is also denoted by y', df/dx or $f'(x)$. The process of taking a derivative is called *differentiation*.

GENERAL RULES OF DIFFERENTIATION

In the following, u, v, w are functions of x; a, b, c, n are constants [restricted if indicated]; $e = 2.71828\ldots$ is the natural base of logarithms; $\ln u$ is the natural logarithm of u [i.e. the logarithm to the base e] where it is assumed that $u > 0$ and all angles are in radians.

13.2 $$\frac{d}{dx}(c) = 0$$

13.3 $$\frac{d}{dx}(cx) = c$$

13.4 $$\frac{d}{dx}(cx^n) = ncx^{n-1}$$

13.5 $$\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$$

13.6 $$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

13.7 $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

13.8 $$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

13.9 $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

13.10 $$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

13.11 $$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{(Chain rule)}$$

13.12 $$\frac{du}{dx} = \frac{1}{dx/du}$$

13.13 $$\frac{dy}{dx} = \frac{dy/du}{dx/du}$$

DERIVATIVES OF TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS

13.14 $\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

13.15 $\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

13.16 $\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$

13.17 $\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$

13.18 $\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$

13.19 $\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$

13.20 $\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \left[-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right]$

13.21 $\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad [0 < \cos^{-1} u < \pi]$

13.22 $\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx} \qquad \left[-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right]$

13.23 $\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx} \qquad [0 < \cot^{-1} u < \pi]$

13.24 $\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} + \text{ if } 0 < \sec^{-1} u < \pi/2 \\ - \text{ if } \pi/2 < \sec^{-1} u < \pi \end{bmatrix}$

13.25 $\frac{d}{dx}\csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} - \text{ if } 0 < \csc^{-1} u < \pi/2 \\ + \text{ if } -\pi/2 < \csc^{-1} u < 0 \end{bmatrix}$

DERIVATIVES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

13.26 $\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx} \qquad a \neq 0, 1$

13.27 $\frac{d}{dx}\ln u = \frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$

13.28 $\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$

13.29 $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

13.30 $\frac{d}{dx}u^v = \frac{d}{dx}e^{v \ln u} = e^{v \ln u}\frac{d}{dx}[v \ln u] = vu^{v-1}\frac{du}{dx} + u^v \ln u \frac{dv}{dx}$

DERIVATIVES OF HYPERBOLIC AND INVERSE HYPERBOLIC FUNCTIONS

13.31 $\frac{d}{dx}\sinh u = \cosh u \frac{du}{dx}$

13.32 $\frac{d}{dx}\cosh u = \sinh u \frac{du}{dx}$

13.33 $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \frac{du}{dx}$

13.34 $\frac{d}{dx}\coth u = -\operatorname{csch}^2 u \frac{du}{dx}$

13.35 $\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \tanh u \frac{du}{dx}$

13.36 $\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \coth u \frac{du}{dx}$

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(a) The pertinent equations are:

$$E = 100\, e^{-\alpha t}$$
$$A = 150\,(1-\beta^t)$$
$$F = 50$$

Total $T = E + A + F = 50 + 100 e^{-\alpha t} + 150(1-\beta^t)$

At the extremum,

$$\frac{dT}{dt} = 0 = -100\,\alpha\, e^{-\alpha t} - 150\,\beta^t \ln\beta$$

Substitute $\alpha = \frac{1}{3}$ and $\beta = 0.6$

$$\frac{100}{3} e^{-t/3} = -150\,(0.6)^t \ln(0.6)$$

$$e^{-t/3} = 2.2987\,(0.6)^t$$

Use natural log (ln)

$$-t/3 = \ln(2.2987) + t\ln(0.6)$$

Solve

$$t = \frac{0.8324}{0.1775} = 4.6894 \text{ months.}$$

To find maximum or minimum, obtain $\frac{d^2T}{dt^2}$

$$\frac{d^2T}{dt^2} = 100\,\alpha^2 e^{-\alpha t} - 150\,(\ln\beta)^2\beta^t$$

Substitute for t, $\frac{d^2T}{dt^2} = -1.2391 < 0$

$\therefore$ At $t = 4.69$ months, the accumulated value is at the maximum.

(b) when $A = E$

$$100\, e^{-t/3} = 150(1 - 0.6^t)$$

$$\frac{2}{3} e^{-t/3} = 1 - 0.6^t$$

Solve – eg. chose t, evaluate left + right sides

$t = 1.174$ months.

This is less that when value is maximum.

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