

**ENCH 501 Transport Phenomena****Mid-Term Examination, October 23, 2018****Time Allowed: 1.30 – 3.00 pm**

**Instructions:** Attempt all questions. Use of electronic calculators allowed but no other electronic device allowed. Open Notes, Open Book Examination.

**Problem 1 (15 points)**

One of the colligative properties of liquids is that their freezing points are depressed (or boiling points elevated) by addition of a solute to a solvent. This phenomenon has many useful applications. Glycols (ethylene or propylene) prevent the freezing of water in vehicular radiators in winter and reduce boil-off during car operations. Water-glycol solutions are rated often from -25°C to -45°C, based on the composition. Sodium or calcium chloride are spread on roads to remove ice by depressing the freezing point of water. Typical commercial road salt is effective down to ground surface temperatures of about -10°C, beyond which gravel is spread instead of salt. Another practical use is to quickly chill liquids such as water, soft drinks and beer in cans or soda-glass bottles below room temperature for functions and events. The problem is on the latter.

Ice is available from a freezer. A large insulated vessel is also available. This was filled with equal masses of the ice at -15°C and distilled water at a temperature of 0°C. Salt (NaCl) is then added to the water, while stirring continuously, until the salt concentration reached 16% by weight. The relationship for freezing point depression ( $\Delta T$ ) is given by

$\Delta T = K_f \cdot \phi \cdot i$  where  $K_f$  is the cryoscopic constant for the solvent,  $\phi$  is the molality of the solution (moles of solute per kilogram of solvent), and  $i$  is the van't Hoff factor (number of ions in solution for each dissolved solute molecule). For water  $K_f$  equals 1.853 kg.K/mol and NaCl has  $i$  equal 2. The molar mass of NaCl is 58.44 g/mol, and 18.016 g/mol for H<sub>2</sub>O.

A soda-glass bottle of beer (weighing 536.8g full and 202.3g empty) and an aluminum can of ginger ale (weighing 379.1g full and 15.8g empty) were both at 21°C when they were fully immersed in the salt solution that is being slowly stirred such that the convective heat transfer coefficient around the bottle and can is 4 W/m<sup>2</sup>K. Given the data as below,

a) Estimate the time required for each of the full beer bottle and can of ginger ale to reach 5°C from 21°C on immersion. Show and justify your steps.

b) If the vessel is metallic and uninsulated, and the rate of heat gain from the ambient is significant such that some ice melted to dilute the salt solution to 10% by weight of salt in 3 hours, how long would it take to cool the can of ginger ale from 21°C to 5°C? You may assume that the temperature of the salt solution changed linearly with time.

| <i>Material</i>   | <i>Density,<br/>kg/m<sup>3</sup></i> | <i>Heat capacity,<br/>J/kg K</i> | <i>Thermal conductivity,<br/>W/mK</i> | <i>Ext. Surface area,<br/>m<sup>2</sup></i> |
|-------------------|--------------------------------------|----------------------------------|---------------------------------------|---|
| Soda glass bottle | 2502                                 | 750                              | 1.4                                   | 0.0348                                      |
| Aluminum can      | 2700                                 | 903                              | 237                                   | 0.0292                                      |
| Beer              | 1008                                 | 4157                             | 0.635                                 | -   |
| Ginger Ale        | 1032.6                               | 3950                             | 0.566                                 | -   |

**Problem 2 (10 points)**

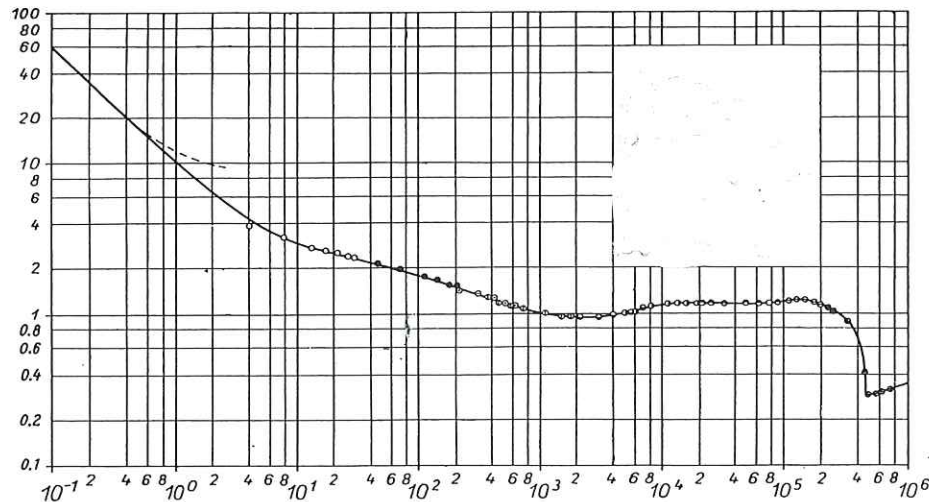
There are numerous reports of bridges washed away as a result of sudden rain downpours upstream of rivers, and of other weather events (like hurricanes) that cause water surges on beaches and in water ways. "Walls" of water from Tsunamis destroy buildings and other structures in the path. For bridges supported by piers (or vertical posts), if the force of water flowing across one of the pier is high enough, the pier and thus the entire bridge structure can collapse. To anticipate worst case scenarios, small-scale models are built and tested in flow tunnels (as was done for the 1988 Calgary Winter Olympic structures) to determine the maximum force a structure can withstand without collapsing under the force of a fluid flowing across it.

Each pier holding a bridge up is a vertical cylinder that is partially submerged in a river flowing underneath the bridge. During a flood, the river level rises and more of the pier is submerged.

a) Obtain the dimensionless groups relating relevant variables to the force on a pier. Show your steps.

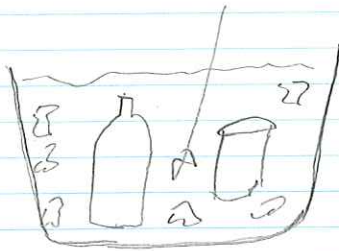
b) A model of a bridge is built with the diameter of a "pier" equals 16 cm. Kerosine at 15°C ( $\mu = 86.8(10^{-5})$  Pa.s and  $\rho = 776$  kg/m<sup>3</sup>) flows at a free-stream velocity of 0.7 m/s below the bridge and around the vertical cylinder. The height of the liquid level was varied while keeping the velocity of the stream constant. The model pier collapsed when the liquid level reached 1.3m. The real pier (prototype) has a diameter of 1.2m. The prototype and the model have dynamic and kinematic similarity. Water at 10°C ( $\mu = 1.308$  mPa.s,  $\rho = 1000$  kg/m<sup>3</sup>) flows around the prototype pier. At what stream velocity and water level will the pier collapse if, from other tests, it is established that a force of 48 N is sufficient to knock the pier off its base?

**Given** below is a plot for Drag Coefficient  $C_D = F/[\frac{1}{2}\rho U^2 LD]$  versus Reynolds number  $Re = DU\rho/\mu$  for flow across a cylinder where  $F$  is the force,  $U$  is freestream velocity,  $L$  is submerged length of cylinder,  $D$  is pier diameter,  $\rho$  is density and  $\mu$  is the dynamic viscosity of the fluid.



The freezing point depression is given by

$$\Delta T = K_f \cdot \phi \cdot i$$



The solution is made up to 16% by wt. salt (NaCl) or

0.16 kg salt / 0.84 kg water

This is  $\frac{0.16}{0.84} = 0.19047$  / kg water  
kg salt

or  $\frac{190.47}{58.44} = 3.2593$  moles NaCl / kg water

This is the molality of the solution.

The freezing point depression is hence

$$\Delta T = 1.853 \times 3.2593 \times 2 = 12.0791^\circ\text{C}$$

That is the temperature external to the beer bottle and the can =  $-12.0791^\circ\text{C}$ .

- Each of the containers with the liquid inside may be treated as a composite body and the lumped capacity method may be used if

$$\frac{h \left( \frac{V}{A} \right)}{k} < 0.1$$

For bottle of beer,  $V = \frac{0.2023}{2502} + \frac{0.3345}{1008} = 4.127(10^{-4})$  m<sup>3</sup>  
glass beer





where

$$\beta = \frac{hA}{m_1 c_{p1} + m_2 c_{p2}} = \frac{4(0.0348)}{(0.2023)(750) + (0.3345)(4157)}$$

$$= 9.0258 (10^{-5}) \text{ s}^{-1}$$

$$\ln \left( \frac{17.0791}{33.0791} \right) = -\beta t$$

$$t = 7323.94 \text{ s} \quad \text{or} \quad \approx 2.034 \text{ hrs} \rightarrow$$

For the can of ginger ale, the calculation is similar

$$\beta = \frac{4(0.0292)}{(0.0158)(903) + 0.3633(3950)}$$

$$= 8.059 (10^{-5}) \text{ s}^{-1}$$

$$\ln \left( \frac{17.0791}{33.0791} \right) = -\beta t$$

$$t = 8202.53 \text{ s} \quad \text{or} \quad 2.278 \text{ hrs} \rightarrow$$

The can <sup>of ale</sup> cools slower than the bottle of beer.

These times can be reduced considerably if the solution is stirred more strongly.

(b) For this part,  $T_x$  around the containers is time dependent. The solution is diluted from 16% to 10% by weight through the melting of ice.

The freezing point depression for the 10% solution is determined as follows:

$$\frac{0.1 \text{ salt}}{0.9 \text{ water}} = 0.1111 \text{ kg salt / kg water.}$$

$$\text{The molality is } \frac{111.1}{58.44} = 1.9013 \frac{\text{moles salt}}{\text{kg water}}$$

The freezing point depression

$$\Delta = 1.853(1.9013)(2) = 7.0461^\circ\text{C}$$

$$\text{Hence } T_d(t) = -12.0791 + \frac{5.033}{3(3600)} t$$

with  $t$  in seconds

$$\text{Let } T_d(t) = a + bt$$

$$\text{where } a = -12.0791^\circ\text{C}$$

$$b = 4.6602(10^{-4}) \frac{^\circ\text{C}}{\text{s}}$$

The energy balance on the can of ale is

$$\begin{array}{c} \text{Input} \\ \downarrow \\ 0 \end{array} + \begin{array}{c} \text{Loss} \\ \downarrow \\ 0 \end{array} = \text{Output} + \text{Accum}$$

$$hA(T - T_d)$$

$$\frac{d}{dt} \left[ (m_1 C_{p1} + m_2 C_{p2})(T - T_{ref}) \right]$$

where  $1 \equiv \text{aluminum}$

$2 \equiv \text{ale}$

Hence

$$\frac{dT}{dt} = - \frac{hA}{m_1 C_{P1} + m_2 C_{P2}} (T - T_\alpha)$$

If  $\beta = \frac{hA}{m_1 C_{P1} + m_2 C_{P2}}$ , the equation becomes

$$\frac{dT}{dt} + \beta T = \beta T_\alpha \quad \text{where } T_\alpha = T_\alpha(t)$$

subject to the condition  $t=0$ ,  $T = T_0 = 21^\circ\text{C}$

Using the method of integrating factor

$$e^{\int \beta dt} \frac{dT}{dt} + e^{\int \beta dt} \beta T = e^{\int \beta dt} \beta (a + bt)$$

$$e^{\beta t} T = \beta \int e^{\beta t} (a + bt) dt + C$$

$$= \beta \frac{a}{\beta} e^{\beta t} + \beta b \frac{e^{\beta t}}{\beta^2} (\beta t - 1) + C_0$$

$$\text{or } T = a + \frac{b}{\beta} (\beta t - 1) + C_0 e^{-\beta t}$$

when  $t=0$ ,  $T = T_0$

$$\therefore T_0 = a - \frac{b}{\beta} + C_0$$

$$\text{or } C_0 = T_0 - a + \frac{b}{\beta}$$



$$T = a + \frac{b}{\beta} (\beta t - 1) + \left( T_0 - a + \frac{b}{\beta} \right) e^{-\beta t}$$

$$a = -12.0791 \text{ } ^\circ\text{C}$$

$$b = 4.6602 (10^{-4}) \text{ } ^\circ\text{C/s}$$

$$\beta = 8.059 (10^{-5}) \text{ s}^{-1}$$

$$T_0 = 21^\circ\text{C}$$

To find the time when  $T = 5^\circ\text{C}$ , choose  $t$  and calculate function. Compare to  $5^\circ\text{C}$ .

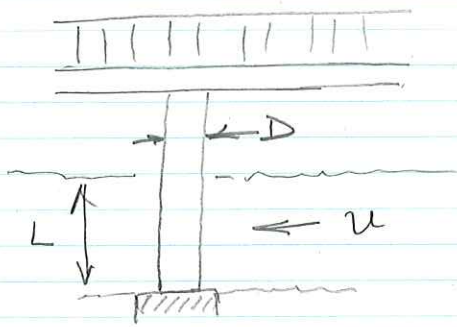
The iteration gives

$$t = 9142.38 \text{ s} \quad \text{or} \quad \approx 2.54 \text{ hrs.}$$





# 2



The force  $F$  on the pier would depend on

$u$  - stream velocity

$D$  - pier diam.

$L$  - length submerged

$\mu$  - fluid viscosity

$\rho$  - fluid density

(a)

$$\text{Hence } F = f(u, D, L, \mu, \rho)$$

| Variables  | $F$              | $u$           | $D$ | $L$ | $\mu$          | $\rho$            |
|------------|------------------|---------------|-----|-----|----------------|-------------------|
| units      | N                | m/s           | m   | m   | Pa.s           | kg/m <sup>3</sup> |
| Dimensions | $\frac{ML}{t^2}$ | $\frac{L}{t}$ | $L$ | $L$ | $\frac{M}{Lt}$ | $\frac{M}{L^3}$   |

There are 3 dimensions &  $\therefore$  3 dimensionless  $\pi$ 's.

By inspection,  $\frac{L}{D} = \pi_1$

Using the Buckingham Pi theorem (Notes)

$$\pi_2 = \frac{D \rho u}{\mu}$$

$$\text{and } \pi_3 = \frac{F}{\rho u^2 D^2}$$

$$\frac{\pi_3}{\pi_1} = \frac{F}{\rho u^2 L D}$$

(b) for the model

$$Re_m = \frac{D u_p}{\mu} = \frac{(0.16)(0.7)(774)}{86.8(10^{-5})}$$

$$= 1.0013(10^5)$$

from plot  $C_{D_m} \approx 1.1 = \frac{F}{\frac{1}{2} \rho u^2 L D}$

If the model and prototype have dynamic and kinematic similarity, then

$$Re_m = Re_p = 1.0013(10^5)$$

$$\text{and } C_{D_m} = C_{D_p} = 1.1$$

$$Re_p = \frac{(1.2) u (1000)}{1.308(10^{-3})} = 1.0013(10^5)$$

$$\therefore u = 0.1091 \text{ m/s} \rightarrow$$

$$C_{D_p} = \frac{48}{\frac{1}{2}(1000)(0.1091)^2(L)(1.2)} = 1.1$$

$$L = 6.106 \text{ m} \rightarrow$$

$$R = 8.314$$

$$\text{J/mol K}$$



A mass bottle  $m_1$  mass of beer  $m_2$  mixture of beer and ginger ale  
 mass of can  $w_1$  mass of ginger ale  $w_2$  - shandy.

Freezer - herile section - at  $-15^\circ\text{C}$  | Room temp -  $20^\circ\text{C}$   
 beer + ginger ale

Molar NaCl -  $58.44 \text{ g/mol}$

One of the colligative properties of liquids is that freezing pts can be depressed (or boiling point elevated) by addition of a solute to a solvent. This phenomenon has many useful applications, Glycols (ethylene or propylene) are used to prevent the freezing of radiator

liquids in winter and they may be rated from  $-25$  to  $-45^\circ\text{C}$ . <sup>sprinkled on roads</sup> Salt depresses the freezing point of water up to certain temperature and prevent roads from being slippery at temperatures above  $-10^\circ\text{C}$ .

(At colder temperatures, gravel is <sup>spread</sup> ~~sprayed~~ on the roads.)  
 It is suggested that <sup>practical use</sup> ~~another~~ is to quickly chill ~~beer~~ <sup>liquids</sup> such as water, beer and soft drinks for parties than simply submerging the containers (glass, cans or plastic) in a mixture of ice and water. You have been commissioned by a local brewery to study this matter.

You are provided with ~~the~~ <sup>in bottles</sup> cases of beer and <sup>in aluminum cans</sup> a case of ginger ale. Your freezer at home has an ice-maker at a temperature of  $-12^\circ\text{C}$ . The dimensions of the container is given below. You have a big bucket and you filled it with ice <sup>from the freezer</sup> and <sup>added</sup> distilled water at  $0^\circ\text{C}$ . You then added salt (sodium chloride) until the solution is 10% by wt. salt.

(1)

soda-lime glass

mass bottle - empty - 202.3 g

$$\rho = \cancel{1040} 2520 \text{ kg/m}^3$$

$$C_p = 49 \text{ J/m}^3\text{K}$$

$$k = \cancel{0.7-1.3} \text{ W/mK}$$

0.7-1.3 cal.

- nearly full

- 536.8 g

$$V_1 = 338 \text{ ml.}$$

can - empty

15.8 g

$$C_p = 284 \text{ J/m}^3\text{K}$$

nearly full

- 379.1 g

$$V_1 = 305 \text{ ml.}$$

$$k = 237 \text{ W/mK}$$

2.2a

$$\text{ml for mass} = 26.981 \text{ g/ml.}$$

Can

bottle

Diam - 6.2 cm.

5.7 cm

H - 12.8

18

bottle



$$\pi D H + \frac{\pi D^2}{4}$$

$$\frac{\text{can}}{\pi D H + \frac{\pi D^2}{4}}$$

$$3.14158 (0.062) (0.1109)$$

$$+ \frac{\pi (0.062)^2}{4}$$

$$\pi (0.057) (0.18)$$

$$+ \frac{\pi (0.057)^2}{4} =$$

$$\frac{\pi_3}{\pi_1} = \frac{F}{\rho u^2 D^2} \frac{D}{L}$$

$$\frac{N}{m^2} \cdot s$$

$$\frac{\text{kg}}{m^2} \frac{m}{s^2} \cdot s$$

Algebra — / concepts — heat exchanger

Dimensional Analysis

$\Delta T_{lm}$

Heat

Mass

Momentum

Error Analysis

- / Lumped capacity
- x Shape factor

Beer

$$\left( \begin{array}{l} P \sim 1008 \text{ kg/m}^3 \\ C_p \\ k = 0.635 \text{ W/mK} \end{array} \right)$$

Glycerol

$$P = \frac{1}{\frac{0.912}{978 \text{ kg/m}^3} + \frac{0.087}{1599.1 \text{ kg/m}^3}}$$

1599.1 -  
0.31046  
kg/m<sup>3</sup> (22)  
5/1592.54

water

$$k = 0.57 \text{ W/mK}$$

$$C_p = 4187 \text{ W/mK}$$

$$P = 1000$$

beer

$$P \sim \text{Mark 25}^{\circ}\text{P} \cdot 3.73$$

| Temp | k       |
|------|---------|
| 20°P | 3.8     |
| 25°P | 3.46    |
| 30°P | 4.0-4.1 |

Glycerol (91.2%)

$$k = 3950 \text{ J/kg}^{\circ}\text{C}$$

$$k = 0.576 \text{ W/mK}$$

glycerol - 91.2% H<sub>2</sub>O

Carbohydrate 8.7%



15% by wt

— 0.15 kg salt / 0.85 kg solvent

0.17647 kg salt / kg solvent

$$\frac{176.47}{58.44} = 3.019688$$

$$\Delta T = 1.853 \times 3.019688 \times 2$$

$$\frac{1 \text{ kg K}}{\text{mol}} \quad \text{mS}$$

$$= 11.190965^\circ\text{C} \longrightarrow$$

16% by wt

0.16 salt / 0.84 kg water

0.190476 kg salt / kg water

$$\frac{190.476}{58.44} = 3.259346$$

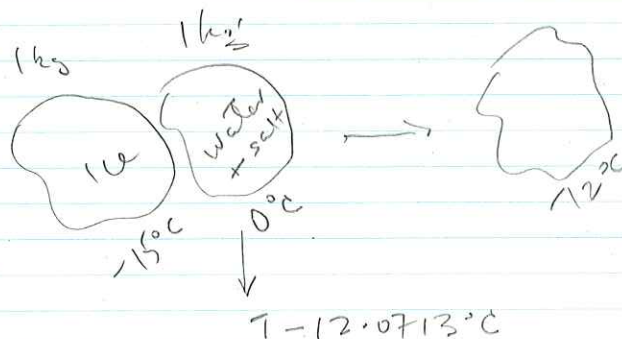
$$\Delta T = 1.853 \times 3.259346 \times 2$$

$$= 12.079137^\circ\text{C}$$

10% by wt

$$\frac{111.11}{58.44} \times 1.853 \times 2$$

$\Delta T = 7.7$



$$m_{ice} (\Delta H_f) = m_w c_p (0 - T_f)$$

$m c_p (\Delta T)$   
 is with drawn from the water

## Colligative properties

Here's an equation for it:  $\Delta T = K_f \cdot b \cdot i$

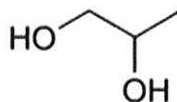
here  $\Delta T$  is the change in freezing point in Kelvins (Celsius will work too since they have the same scale).  $K_f$  is the cryoscopic constant of the solvent, different for every material. For water  $K_f = 1.853 \text{ K} \cdot \text{kg/mol}$ .  $b$  is the molality of the solution, which is the amount of solute expressed in mols per kilogram of solvent.  $i$  is called the van't Hoff factor. This is the number of ions in solution for each dissolved molecule. For salt, NaCl this number is 2

So if we dissolve 1.5 mol (87.75g) of table salt in 1 Kg (1L) of water our equation is:

$$\Delta T = 1.853 \cdot 1.5 \cdot 2 = 5.5 \text{ K}$$

The phenomenon of freezing-point depression has many practical uses. The radiator fluid in an automobile is a mixture of water and [ethylene glycol](#). As a result of freezing-point depression, radiators do not freeze in winter (unless it is extremely cold, e.g.  $-30$  to  $-40^\circ\text{C}$  ( $-22$  to  $-40^\circ\text{F}$ )). Road salting takes advantage of this effect to lower the freezing point of the ice it is placed on. Lowering the freezing point allows the street ice to melt at lower temperatures, preventing the accumulation of dangerous, slippery ice. Commonly used [sodium chloride](#) can depress the freezing point of water to about  $-21^\circ\text{C}$  ( $-6^\circ\text{F}$ ). If the road surface temperature is lower, NaCl becomes ineffective and other salts are used, such as [calcium chloride](#), [magnesium chloride](#) or a mixture of many. These salts are somewhat aggressive to metals, especially iron, so in airports safer media such as [sodium formate](#), [potassium formate](#), [sodium acetate](#), [potassium acetate](#) are used instead.

## Propylene glycol



Propylene glycol

[Propylene glycol](#) is considerably less toxic than ethylene glycol and may be labeled as "non-toxic antifreeze". It is used as antifreeze where ethylene glycol would be inappropriate, such as in food-processing systems or in water pipes in homes where incidental ingestion may be possible. For example, the [FDA](#) allows [propylene glycol to be added](#) to a large number of [processed foods](#), including [ice cream](#), [frozen custard](#), salad dressings, and [baked goods](#), and it is commonly used as the main ingredient in the "[e-liquid](#)" used in [electronic cigarettes](#).

Propylene glycol [oxidizes](#) when exposed to air and heat, forming [lactic acid](#).<sup>[9][10]</sup> If not properly [inhibited](#), this fluid can be very corrosive,<sup>[citation needed]</sup> so [pH buffering](#) agents such as [dipotassium phosphate](#) and [potassium bicarbonate](#) are often added to propylene glycol, to prevent acidic corrosion of metal components. Pre-inhibited propylene glycol solutions can also be used instead of pure propylene glycol to prevent corrosion.

Besides cooling system corrosion, [biological fouling](#) also occurs. Once bacterial slime starts to grow, the corrosion rate of the system increases. Maintenance of systems using glycol solution includes regular monitoring of freeze protection, [pH](#), [specific gravity](#), inhibitor level, color, and biological contamination.

Propylene glycol should be replaced when it turns a reddish color. When an aqueous solution of propylene glycol in a cooling or heating system develops a reddish or black color, this indicates that iron in the system is corroding significantly. In the absence of inhibitors, propylene glycol can react with oxygen and metal ions, generating various compounds including organic acids (e.g., formic, oxalic, acetic). These acids accelerate the corrosion of metals in the system. [\[11\]\[12\]\[13\]\[14\]](#)

## Glycerol

Ge, Xinlei; Wang, Xidong (2009). "Estimation of Freezing Point Depression, Boiling Point Elevation, and Vaporization Enthalpies of Electrolyte Solutions". *Industrial & Engineering Chemistry Research*. **48** (10): 5123–5123. [doi:10.1021/ie900434h](#). [ISSN 0888-5885](#).

The following figures were found in a published report, but have not been checked out in detail. A 10% salt solution was said to lower the melting point to -6°C (20°F) and a 20% salt solution was said to lower it to -16°C (2°F).



# The fastest way to chill your beer

For an icy drink in under 10 minutes

By [Sara Chodosh](#) May 29, 2017



These people already have perfectly chilled beer. Why don't you?

Unsplash

You always forget to chill the booze. Somehow the burgers get prepped and the decorations get hung, but fifteen minutes before the cookout begins, you're stuck with lukewarm [beer](#).

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Article Continues Below:



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Well, not anymore. Because if there's one thing science can help us with, it's thermodynamics. And that's really all this problem is—how do you lower the temperature of a liquid inside a metal can (or glass bottle) as quickly as possible?

## The salt water method

This is the most basic and practical way to cool drinks, because virtually everyone has the materials to do it. The process is simple:

1. Fill a container (something insulated like a cooler is best) with ice, water, and lots of salt. You really can't overdo it on the salt here, but you can overdo it on ice. Make sure to add enough water to surround the drinks, since that gives you the maximal surface area to cool the beer inside.
2. Submerge your cans and/or bottles as much as possible. Make sure that they aren't packed too tightly, since you want the icy water to flow around each container. If you've put the drinks in a cooler with a top, close the top.
3. Wait. This should only take about five minutes.
4. That's it! Enjoy your chilled beer.

## Why salt water works so well

Plain water freezes at 32°F. Salt water can get much colder than that and still remain a liquid. You might remember this from your high school chemistry class—it's called "freezing point depression," and it happens because dissolving salt in water lowers the liquid's freezing point. Table salt breaks down into sodium and chloride ions, which physically interfere with water molecules' ability to form the crystalline structure we call ice. This means the solution has to be colder than normal in order to freeze.

Incidentally, salt in water will also raise the boiling point, leading to the old wives' tale that you shouldn't add salt to water if you're trying to boil it. While it's technically true, the amount of salt you add when cooking won't raise the boiling point by any substantial amount, so salt away.

Now, back to beer chilling.

If you put your beer in an ice-water bath, it will cool the beer fairly effectively because liquid conducts heat well: The icy water pulls heat from the beer; the beer gets cooler. But no matter how much ice you add to a standard water bath, it will only ever reach temperatures just above 32°F. If it got any colder than that, you'd just end up with more ice. Even if you keep your ice at 0°F, the standard temperature for freezers, once it melts into ice water, the liquid won't get any colder than that 32°F sticking point.

So what happens when you add salt to the bath? Because salt lowers the melting point of water, if you add salt to ice, the ice will melt. You might assume that, because the ice is melting faster, the salt has somehow heated up the ice faster than normal. But that's not what's happening—the

salt isn't raising the temperature of the ice; it's converting ice into salt water of the same temperature.

Salty ice water can get much colder than regular water, though. While salty 0°F ice will still melt, its temperature won't increase to 32°F like it would in regular water. Instead, the salt will turn it into 0°F water. Combined with the rest of the water in the bath, you suddenly have a watery, salty slurry that's well below 32°F. And because the water bath is colder, your beer will chill faster.

## **But what if you spin the bottles?**

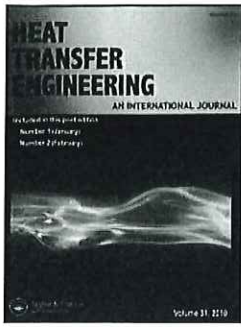
In case five minutes is too long to wait, there is one more thing you can do to speed the cooling process along: Spin it! (Pull it! Bop it!)

Water is already a great heat conductor, but it still takes time for heat to transfer from the beverage inside the bottle or can to the bath outside it. Inside the container, the beer along the edges will be slightly colder than the fluid at the center. Likewise, on the outside of the container, the water closest to the beverage will be slightly warmer. You can alleviate some of that unevenness by spinning the bottles (gently, so as not to shake them up). This will speed up the heat redistribution from beer to water.

One company seems to sell a device explicitly for this purpose, though if you haven't thought to chill your beer yet, it's unlikely that you thought to buy a special gadget to do so. In a pinch, you could rig up a spinning device from a can of compressed air. Or hook up a bottle to a power drill—just remember to remove the drill bit so you don't put a hole through the can.

Alternatively, just wait the five minutes for the salt bath to work. It's not worth interrupting your celebration for a trip to the emergency room.





## Thermal Performance of Aluminum and Glass Beer Bottles

Robert T. Bailey & Wayne L. Eban

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# Thermal Performance of Aluminum and Glass Beer Bottles

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Aluminum beer bottles recently have been introduced as an alternative to traditional glass bottles or aluminum cans. Advertisements claim that the new aluminum bottles keep beer “colder longer” than glass bottles. Because the thermal conductivity of aluminum is over 150 times that of typical soda-lime glass, and because the bottle wall is thinner, this claim appears counterintuitive. In this investigation, the thermal performance of commercially available aluminum and glass beer bottles was examined using experimental, analytical, and computational methods. It was found that when exposed to ambient air, glass and aluminum bottles perform in a nearly identical manner with respect to keeping their contents cold. Each bottle showed an approximately 15°C temperature rise over a 2.7-hour period. Heat transfer is controlled by natural convection and thermal radiation at the outer bottle surface; hence, the difference in thermal conductivity between the bottles has no significant impact on the temperature transient. Computational simulations also predict that when an aluminum bottle is immersed in an ice-water bath, the liquid cools more quickly than in glass due to the lower thermal resistance of the aluminum versus the glass; when held in the hand, the glass bottle allows the liquid to warm more slowly than the aluminum bottle.

## INTRODUCTION

Beer has traditionally been packaged in two types of containers—glass bottles and aluminum cans. Recently, a relatively new type of container, the aluminum beer bottle, has been introduced as an alternative [1, 2]. The wall of this bottle is thicker than a typical aluminum can but thinner than its glass counterpart. Various statements have been made about the advantages of aluminum bottles over other containers including, “the beer stays colder longer—up to 50 minutes—versus glass containers” [3]. Because the thermal conductivity of aluminum is over 150 times that of typical soda-lime glass [4], and because the bottle wall is thinner, this claim appears counterintuitive. Some experimental results and analyses have been reported informally [5–7], but no engineering studies examining this claim in detail have been published in the open literature. Therefore, the purpose of this investigation is to compare the thermal performance of commercially available aluminum and glass beer bottles using experimental, analytical, and computational methods and to draw conclusions about their relative abilities to keep liquids cold.

## EXPERIMENTAL INVESTIGATION OF WARMING IN AMBIENT AIR

The experimental portion of the study was designed and performed by five undergraduate engineering students at Loyola College in Maryland as part of the capstone laboratory requirement for their junior experimental methods course. The students worked with commercial beer bottles available from the same vendor and having approximately equal internal volumes: (1) an impact-extruded [8] aluminum bottle manufactured by CCL Container (Hermitage, PA, USA) and (2) an amber soda-lime glass bottle (Figure 1).

Both bottles were thoroughly washed/rinsed, allowed to dry, and then filled with 355 ml (12 oz.) of de-ionized water (chosen to simulate the properties of beer, which is mostly water), capped with aluminum foil to minimize evaporation, and cooled overnight to just below 6°C in a refrigerator. The bottles with their contents were then removed from the refrigerator, placed on a wooden tabletop, exposed to ambient air at 24°C, and allowed to warm up for approximately 2.7 hours with temperature readings taken at nine intervals.

Two identical iron-constantan (type J) thermocouples (model JMTSS-062U-12; Omega Engineering, Inc., Stamford, CT, USA) attached to digital thermometers (model DSS-115JC; Omega) were used to measure the temperature change of the water in the beer bottles over time. The data were recorded every

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Figure 1 Aluminum and glass bottles.

Table 1 Experimental results for warming in ambient air

| Elapsed time (s) | Water temperature (°C) |                 |
|------------------|------------------------|-----------------|
|                  | Glass bottle           | Aluminum bottle |
| 0                | 5.8                    | 5.4             |
| 1070             | 8.5                    | 8.2             |
| 2075             | 11.6                   | 11.0            |
| 3165             | 14.2                   | 13.3            |
| 4000             | 15.7                   | 14.6            |
| 5060             | 17.2                   | 16.0            |
| 6100             | 18.2                   | 17.6            |
| 7050             | 19.0                   | 18.4            |
| 8055             | 20.2                   | 19.2            |
| 9680             | 20.8                   | 20.2            |

HEAT TRANSFER ANALYSIS OF WARMING IN AMBIENT AIR

To facilitate an understanding of the physical reasons for the experimental results, an engineering analysis was performed.

Insights from Steady State

Heat transfer between the ambient air and the water inside a bottle takes place via three mechanisms: thermal radiation between the outer bottle surface and the external environment, conduction through the bottle wall, and natural convection at the inner and outer surfaces of the bottle. The bottle geometry is such that the radial direction is the dominant heat transfer pathway. Each mechanism presents a resistance to heat flow, so radial heat transfer in the steady-state case can be described by

$$Q = UA(T_{amb} - T_L) \tag{1}$$

where U is the overall heat transfer coefficient, expressed as

$$U = \frac{1}{\frac{1}{h_o + h_r} + \frac{L}{k_b} + \frac{1}{h_i}} \tag{2}$$

2 seconds using a digital storage oscilloscope (Nicolet model 310; LDS Test and Measurement, Middleton, WI, USA). To determine that the temperature measuring system (thermocouples plus instrumentation) was functioning properly, a calibration was first performed using a single-temperature reference (an ice-water bath). The bias errors at 0°C were determined to be -0.5 and -0.7°C for the thermocouples used with the aluminum and glass bottles, respectively. The total measurement uncertainty is conservatively estimated to be ±2.7°C, obtained by adding an uncertainty of ±2.2°C for a standard-grade type J thermocouple [9] to the oscilloscope uncertainty of ±0.5°C as stipulated by the manufacturer. The thermocouples were subsequently suspended at the same depth (approximately 38 mm from the bottom) in the radial centers of the filled bottles that had just been simultaneously retrieved from the refrigerator.

The data obtained from the experiment are presented in Table 1 and Figure 2. It can be observed that the water in the aluminum bottle heated up at a slightly lower overall rate than the water in the glass bottle. However, the differences between the temperatures are well within the level of uncertainty of the thermocouples. The experiment was repeated four times by other Loyola College researchers with essentially the same result in each instance.

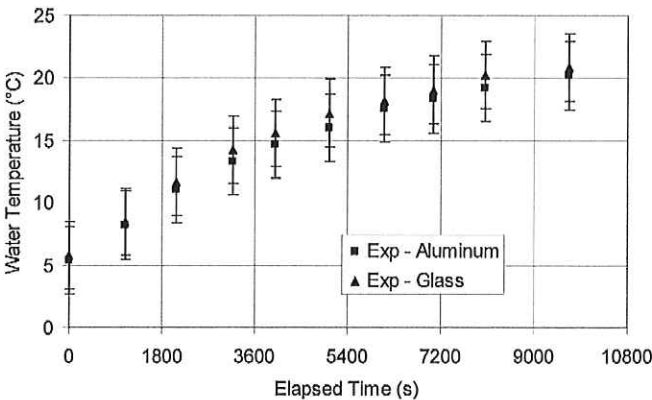


Figure 2 Experimental results for warming in ambient air.

For this analysis, it is assumed that the curved bottle can be modeled as a plane wall because the radius of the bottle is much greater than the thickness of the bottle wall.

The steady-state equations are not applicable to the time-dependent case; however, some insight into the heat transfer behavior of the system can be gleaned by examining the overall thermal resistance. If one of the resistance terms in the denominator of Eq. (2) is substantially greater than the other two, then the overall resistance to heat transfer will be controlled by this mechanism. For the case of the liquid-filled bottle heating up under ambient conditions, the sum of the external convection coefficient,  $h_o$  (solid-gas interface), and the radiation coefficient,  $h_r$ , will be much smaller than the internal convection coefficient,  $h_i$  (solid-liquid interface). This means that the resistance posed by natural convection at the inner boundary will be much smaller than that posed by convection and low-temperature radiation at the outer boundary. Further, because the wall thickness is relatively small for either the glass or aluminum bottles, the conductive resistance through the wall also is expected to be much smaller than the combined convective/radiative resistance at the outer wall surface. Therefore, differences in the thermal conductivity of the wall material may not have a significant effect on the insulating characteristics of the bottle under ambient warming conditions because the rate of heat transfer to the liquid will be controlled by natural convection and thermal radiation at the outer surface.

### Transient Formulation

In the transient case, the temperature profile in the bottle wall is governed by

$$\frac{\partial T_b}{\partial t} = \alpha_b \frac{\partial^2 T_b}{\partial x^2} \quad (3)$$

Under the assumption that the bottle can be modeled as a small convex object in a large cavity, and treating the surfaces as diffuse and gray, the boundary condition at the outer bottle surface is a combination of thermal radiation and natural convection:

$$-k_b \frac{\partial T_b}{\partial x} \bigg|_{x=0} = (h_o + h_r)[T_{amb} - T_b(0, t)] \quad (4)$$

where

$$h_r = \varepsilon_o \sigma [T_{amb} + T_b(0, t)]^2 T_{amb} + [T_b(0, t)]^2 \quad (5)$$

At the inner bottle surface, a natural convection boundary condition is applied:

$$-k_b \frac{\partial T_b}{\partial x} \bigg|_{x=L} = h_i [T_b(L, t) - T_L(t)] \quad (6)$$

The initial condition for the wall is given by

$$T_b(x, 0) = T_{init} \quad (7)$$

Table 2 Material and surface properties

|                                 | Glass | Aluminum          | Water |
|---------------------------------|-------|-------------------|-------|
| $\rho$ (kg/m <sup>3</sup> ) [4] | 2500  | 2702              | 1000  |
| $k$ (W/m·K) [4]                 | 1.4   | 237               | 0.590 |
| $C$ (J/kg·K) [4]                | 750   | 903               | 4189  |
| $\varepsilon^a$                 | 0.93  | 0.85 <sup>b</sup> | n/a   |

<sup>a</sup>The emissivities of the bottle surfaces were determined experimentally.

<sup>b</sup>The outer surface of the aluminum bottle is covered with a thin polymer layer, which causes the emissivity to be considerably higher than that normally attributed to clean aluminum.

Inside the bottle, both conduction and natural convection will act to minimize temperature gradients within the liquid, so that the average temperature of the liquid can be found from

$$\frac{dT_L}{dt} = \frac{h_i A_i}{\rho_L C_L V_L} [T_b(L, t) - T_L] \quad (8)$$

subject to the initial condition

$$T_L(0) = T_{init} \quad (9)$$

The values for the material properties, surface properties, and geometric parameters shown in Eqs. (3) through (9) are listed in Tables 2 and 3.

### Lumped Capacitance Approximation

The temperature gradient within the bottle wall will not be significant if the resistance to conduction through the wall is small compared to the resistance to external heat transfer at either surface. This is expressed mathematically using the Biot number, defined in this case as

$$Bi = \frac{(h_o + h_r)L}{k_b} \quad (10)$$

If  $Bi$  is less than 0.1, it will be acceptable to lump the wall as if it were at a single temperature [4]. Under these conditions, Eqs. (3) through (6) can be simplified and combined as

$$\rho_b C_b V_b \frac{dT_b}{dt} = (h_o + h_r) A_o [T_{amb} - T_b] + h_i A_i [T_L - T_b] \quad (11)$$

and the initial condition becomes

$$T_b(0) = T_{init} \quad (12)$$

Table 3 Bottle geometric parameters

|                         | Glass bottle          | Aluminum bottle       |
|-------------------------|-----------------------|-----------------------|
| $d_o$ (m)               | 0.0619                | 0.0587                |
| $L$ (m)                 | $3.56 \times 10^{-3}$ | $7.11 \times 10^{-4}$ |
| $H$ (m)                 | 0.150                 | 0.138                 |
| $A_i$ (m <sup>2</sup> ) | 0.0306                | 0.0299                |
| $V_L$ (m <sup>3</sup> ) | $3.55 \times 10^{-4}$ | $3.55 \times 10^{-4}$ |



## Natural Convection Coefficients

The natural convection coefficient at the inner bottle surface can be approximated using the following expression for internal flow within a vertical cylinder [10]:

$$\frac{h_i H}{k} = 0.55 \text{ Ra}^{0.25} \quad (13)$$

while the natural convection coefficient at the outer surface can be approximated using an expression for external flow involving a vertical cylinder [11]:

$$\frac{h_o H}{k} = \frac{4}{3} \frac{\text{Ra}^2}{5(20 + 21 \text{ Pr})} + \frac{4(272 + 315 \text{ Pr})H}{35(64 + 63 \text{ Pr})d_o} \quad (14)$$

where

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)H^3}{\nu\alpha} \quad (15)$$

$$\text{Pr} = \frac{\nu}{\alpha} \quad (16)$$

The fluid properties and temperatures are those of water (inner surface) or air (outer surface).

Using Eqs. (13) and (14), the average convection coefficients (Table 4) were calculated. Values for the average radiation heat transfer coefficient at the outer surface also are listed in Table 4 and were calculated using Eq. (5). Because the Biot numbers shown in this table justify the use of Eq. (11), it was decided to lump the bottle wall at a single temperature. Eqs. (8) and (11) were solved simultaneously for the bottle wall and water temperatures over time using a fourth-order Runge-Kutta method [12]. Heat transfer through the top and bottom of the bottle was approximated by increasing the surface area for heat transfer to include these areas. The results of the lumped computational simulations are presented in Table 5 and Figure 3.

As a check on the lumped approximation, an implicit finite-difference solution to Eqs. (3) through (7) was also generated. The finite-difference equations are of the form

$$a_i(T_b)_{j,m+1} - b_i(T_b)_{j+1,m+1} - c_i(T_b)_{j-1,m+1} = d_j \quad (17)$$

$$1 \leq j \leq N$$

where

$$a_1 = 1 + 2 \text{ Fo} + \frac{2(h_o + h_r)\frac{E_0}{19}}{\rho_b C_b \frac{E_0}{19}} \quad (18)$$

Table 4 Average heat transfer parameters used in numerical simulations of warming in ambient air

|  | Glass bottle         | Aluminum bottle      |
|--|----------------------|----------------------|
| $h_o$ ( $\text{W}/\text{m}^2\cdot\text{K}$ ) | 4.3                  | 4.5                  |
| $h_i$ ( $\text{W}/\text{m}^2\cdot\text{K}$ ) | 125                  | 128                  |
| $h_r$ ( $\text{W}/\text{m}^2\cdot\text{K}$ ) | 5.3                  | 4.8                  |
| Bi   | $2.4 \times 10^{-2}$ | $2.8 \times 10^{-5}$ |

Table 5 Computational results for warming in ambient air

| Elapsed time (s) | Water temperature ( $^{\circ}\text{C}$ ) |                 |
|------------------|--|-----------------|
|                  | Glass bottle                             | Aluminum bottle |
| 0                | 5.8                                      | 5.4             |
| 1070             | 9.0                                      | 8.6             |
| 2075             | 11.6                                     | 11.1            |
| 3165             | 13.9                                     | 13.4            |
| 4000             | 15.4                                     | 14.8            |
| 5060             | 17.0                                     | 16.4            |
| 6100             | 18.2                                     | 17.7            |
| 7050             | 19.2                                     | 18.7            |
| 8055             | 20.0                                     | 19.6            |
| 9680             | 21.1                                     | 20.7            |

$$b_1 = 2 \text{ Fo} \quad (19)$$

$$c_1 = 0 \quad (20)$$

$$d_1 = (T_b)_{1,m} + \frac{2\frac{E_0}{19}(h_o + h_r)T_{\text{amb}}}{\rho_b C_b \frac{E_0}{19}} \quad (21)$$

$$a_j = 1 + 2 \text{ Fo} \quad 1 < j < N \quad (22)$$

$$b_j = \text{Fo} \quad 1 < j < N \quad (23)$$

$$c_j = \text{Fo} \quad 1 < j < N \quad (24)$$

$$d_j = (T_b)_{i,m} \quad 1 < j < N \quad (25)$$

$$a_N = 1 + 2 \text{ Fo} + \frac{2h_i\frac{E_0}{19}}{\rho_b C_b \frac{E_0}{19}} \quad (26)$$

$$b_N = 0 \quad (27)$$

$$c_N = 2 \text{ Fo} \quad (28)$$

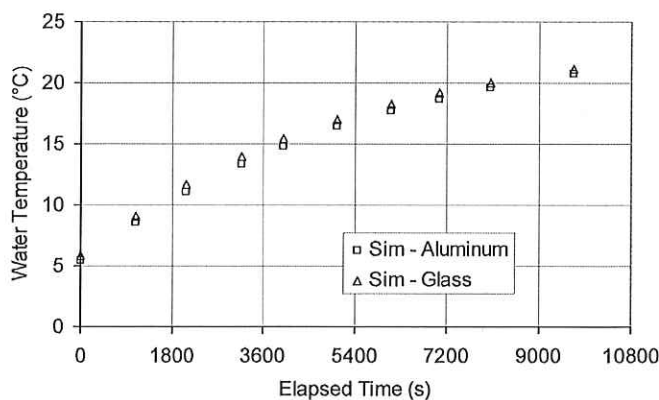


Figure 3 Computational results for warming in ambient air (lumped capacitance computational model).

$$d_N = (T_b)_{N,m} + \frac{2 h_i (T_L)_m}{\rho_b C_b} \quad (29)$$

$$Fo = \frac{\alpha_b}{L^2} \quad (30)$$

and

$$(T_L)_{m+1} = (T_L)_m + \frac{h_i A_i}{\rho_L C_L V_L} \frac{[(T_b)_{N,m+1} + (T_b)_{N,m}] - (T_L)_m}{2} \quad (31)$$

At each time  $t = (m+1)\Delta t$ , the tridiagonal system of  $N$  simultaneous linear equations [represented collectively by Eqs. (17) through (30)] was solved for the  $N$  unknown temperatures within the bottle wall using the Thomas algorithm [13]. The liquid temperature was then updated via Eq. (31). This was repeated until final time,  $t = M\Delta t$ , was reached. Using this approach, it was verified that the temperature gradient in the aluminum or glass wall was minimal during heating in ambient air, and the lumped capacitance approximation was therefore justified.

It should be noted that the inner surface of the aluminum bottle is coated with a thin polymer layer (epoxy-phenolic food-grade liner) to keep the liquid from directly contacting the metal. The outer surface of the aluminum bottle also is coated with a thin, protective polymeric layer (modified polyester). These coatings were not included in the conduction analysis because they are extremely thin and would not significantly change the overall thermal resistance posed by conduction, convection, and radiation. The emissivities of the outer surfaces of the coated aluminum and glass bottles at 23°C were used in the thermal radiation analysis and were determined using a digital thermometer (USA Taylor TruTemp model 3519, Oak Brook, IL, USA) and an infrared thermometer (model OSXL653 Omega) with adjustable emissivity capability. The measured emissivity values compared well with those from the literature for similar materials [4].

## DISCUSSION

The experimental and computational results for warming in ambient air are presented together in Figure 4. Using the convection and radiation heat transfer coefficients shown in Table 4, the agreement between the experimental and computational approaches is very good, falling well inside the bounds of experimental uncertainty.

A recent experimental study [7] examined a similar configuration using glass and aluminum beer bottles and reported a temperature increase of 8.3°C for 1 hour. This compares well with the plots in Figure 4, which show a temperature increase between 8°C and 9°C for the same time period.

The experimental and computational results suggest that the water within the glass bottle warmed at a slightly faster rate than that in the aluminum bottle, but the two rates are effectively

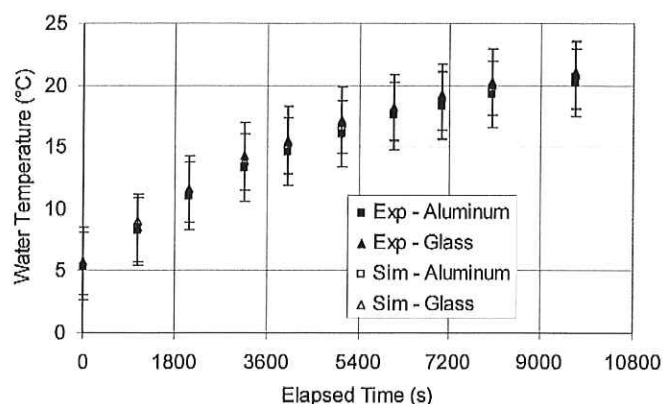


Figure 4 Computational and experimental results for warming in ambient air (lumped capacitance computational model).

equivalent when measurement uncertainty is considered. This trend was duplicated in the computational runs, demonstrating that the difference in thermal conductivities of the glass and aluminum was not an important factor in this physical situation. Two factors that do favor more rapid warming in glass are that the diameter of the glass bottle is slightly larger, providing more area for convection and radiation heat transfer at the outer surface, and that the emissivity of the glass is slightly higher than the plastic-coated aluminum. However, working against these factors was the smaller thermal mass of the aluminum bottle.

## FOLLOW-ON SIMULATIONS

Exposure to ambient air is one physical situation that a beer bottle is expected to encounter. There are others, including

- Additional Case I: immersion in an ice-water bath to facilitate cooling the beer, and
- Additional Case II: being hand-held as the beer is consumed.

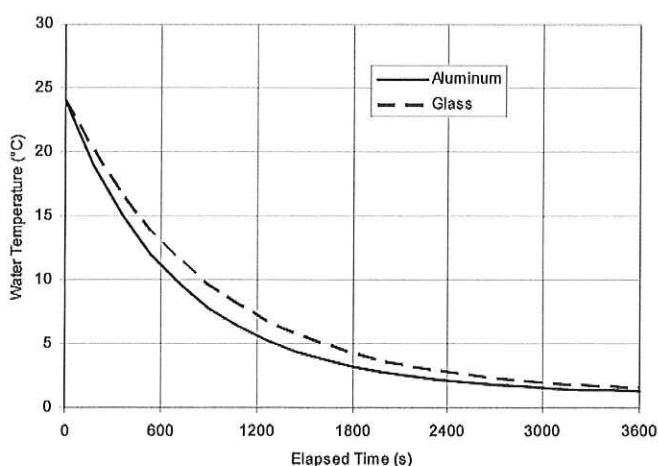


Figure 5 Computational results for Additional Case I: immersion cooling in an ice-water bath (finite-difference computational model).



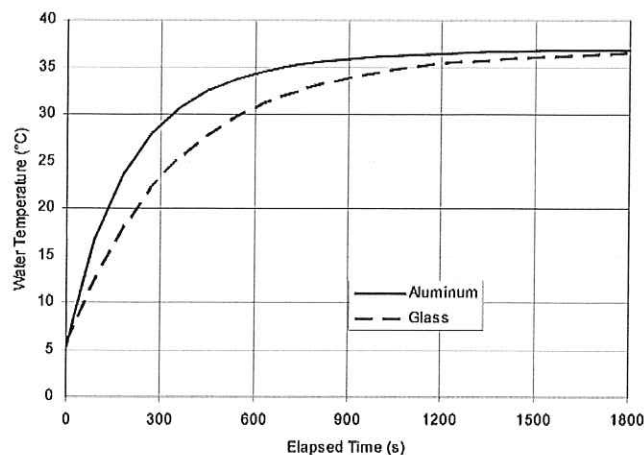


Figure 6 Computational results for Additional Case II: warming while hand-held (finite-difference computational model).

Because these situations present a different environment for heat transfer, the finite-difference model mentioned earlier was used to examine the performance of aluminum and glass in these two cases. This model accounted for the potential temperature gradient within the bottle wall. This gradient was significant only for the glass bottle, and seven grid points ( $N = 7$ ) within the wall were found to be adequate to resolve the temperature profile there.

In Additional Case I, the convection coefficient at the outer surface was changed to  $100 \text{ W/m}^2\cdot\text{K}$  to simulate liquid natural convection, and the external fluid temperature (now water) was decreased to  $1^\circ\text{C}$ . The liquid temperature as a function of time for this case is shown in Figure 5. Here, it can be seen that the liquid in the aluminum bottle cools more rapidly than in glass. Because the resistance to heat transfer at the external surface of the bottle has been reduced, the internal resistance to conduction across the wall plays a role, and the thinner, more conductive aluminum transmits heat more readily. Thus, the liquid chills more rapidly in the aluminum bottle.

In Additional Case II, the external boundary condition was changed to a constant temperature at  $37^\circ\text{C}$  (human body temperature). The hand would not be able to maintain this temperature in the real case, and some of the bottle would still be exposed to air, but for the purpose of this analysis, the idealized constant-temperature condition serves to illustrate the differences in heat transfer behavior from the ambient-air warming scenario. The liquid temperature for Additional Case II is shown in Figure 6. The same characteristics that caused the liquid in the aluminum

bottle to cool faster in Additional Case I now result in a faster warming for aluminum. Hence, the liquid warms more slowly in the glass bottle.

The impact of conduction resistance on the six transients investigated can also be understood by examining Table 6, which presents the ratio of conduction resistance to total thermal resistance for each scenario. In all cases, the conduction resistance for the aluminum bottle comprises less than 0.05% of the total resistance. The conduction resistance fraction for the glass bottle is also relatively low in the ambient warming scenario (2.2%). Hence, the aluminum and glass bottles perform similarly in this scenario. In contrast, the conduction resistance fraction for the glass bottle is 13% for Additional Case I (ice-water bath cooling) and 28% for Additional Case II (warming in the hand). Conduction through the glass bottle wall plays a more significant role in these cases, and the glass is more effective at limiting heat transfer to and from the liquid.

## CONCLUSIONS

When simply exposed to ambient air, glass and aluminum bottles perform in a nearly identical manner with respect to keeping their contents cold. Experimental and computational results are in agreement, and each bottle showed an approximately  $15^\circ\text{C}$  temperature rise over a 2.7-hour period. The thermal conduction resistance of the glass or aluminum bottle is small compared to the total thermal resistance, which includes convection and radiation at the outer surface and convection at the inner surface; hence, the thermal conductivity of the bottle material plays no significant role in the physics. Instead, heat transfer is controlled by natural convection and thermal radiation at the outer surface of the bottle.

Computational simulations predict that when an aluminum bottle is immersed in an ice-water bath, the beer cools more quickly than in glass due to the lower thermal resistance of the aluminum versus the glass. When held in the hand, the glass bottle allows the beer to warm more slowly than the aluminum bottle.

## ACKNOWLEDGMENTS

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Table 6 Ratio of conduction resistance to total thermal resistance for all cases

| Scenario                               | $R_{\text{cond}}/R_{\text{total}}$ |                       |
|--|------------------------------------|-----------------------|
|  | Glass bottle                       | Aluminum bottle       |
| Warming in ambient air                 | $2.21 \times 10^{-2}$              | $2.61 \times 10^{-5}$ |
| Immersion cooling in an ice-water bath | $1.33 \times 10^{-1}$              | $1.80 \times 10^{-4}$ |
| Warming while hand-held                | $2.79 \times 10^{-1}$              | $4.21 \times 10^{-4}$ |



product in this paper does not imply endorsement or criticism by the authors or Loyola College.

## NOMENCLATURE

|    |   |
|----|---|
| A  | bottle surface area ( $\text{m}^2$ )  |
| Bi | Biot number (dimensionless)   |
| C  | specific heat ( $\text{J/kg}\cdot\text{K}$ )                                      |
| d  | diameter (m)  |
| Fo | Fourier number (dimensionless)  |
| g  | acceleration due to gravity ( $\text{m/s}^2$ )                                    |
| h  | convection or radiation heat transfer coefficient ( $\text{W/m}^2\cdot\text{K}$ ) |
| H  | height of liquid in bottle (m)  |
| k  | thermal conductivity ( $\text{W/m}\cdot\text{K}$ )                                |
| L  | bottle wall thickness (m)   |
| M  | total number of time steps  |
| N  | number of grid points in the radial direction                                     |
| Pr | Prandtl number (dimensionless)  |
| Q  | heat transfer rate (W)  |
| R  | thermal resistance ( $\text{K/W}$ )   |
| Ra | Rayleigh number (dimensionless)   |
| t  | time (s)  |
| T  | temperature ( $^{\circ}\text{C}$ )  |
| U  | overall heat transfer coefficient ( $\text{W/m}^2\cdot\text{K}$ )                 |
| V  | volume ( $\text{m}^3$ )   |
| x  | coordinate from outer bottle surface inward (m)                                   |

## Greek Symbols

|               |   |
|---------------|---|
| $\alpha$      | thermal diffusivity ( $\text{m}^2/\text{s}$ )               |
| $\beta$       | thermal expansion coefficient ( $1/\text{K}$ )              |
| $\Delta t$    | time step (s)   |
| $\varepsilon$ | total hemispherical emissivity                              |
| $\nu$         | kinematic viscosity ( $\text{m}^2/\text{s}$ )               |
| $\rho$        | density ( $\text{kg/m}^3$ )                                 |
| $\sigma$      | Stefan-Boltzmann constant ( $\text{W/m}^2\cdot\text{K}^4$ ) |
| $\Delta x$    | coordinate spacing in the x-direction (m)                   |

## Subscripts

|          |                                       |
|----------|---------------------------------------|
| amb      | ambient air                           |
| b        | bottle                                |
| cond     | conduction                            |
| i        | inner bottle surface                  |
| init     | initial                               |
| j        | grid point counter                    |
| L        | liquid                                |
| m        | time step counter                     |
| o        | outer bottle surface                  |
| r        | radiation                             |
| s        | surface (inner or outer)              |
| total    | conduction, convection, and radiation |
| $\infty$ | fluid (gas or liquid)                 |

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Metallurgy (1977) from the University of Delaware and an M.S. in Engineering Materials (1972) from the University of Maryland, College Park. From 1969–1985, he was a research engineer at the Naval Surface Warfare Center, White Oak Laboratory, Silver Spring, Maryland. In 1992, he was a Fulbright Scholar at the University of Strathclyde (Glasgow), Department of Pure and Applied Chemistry. From 2001–2003, he was a working visitor at the Smithsonian Center for Materials Research and Education. He is now Professor of Engineering Science and a member of ASM International and the Society of Manufacturing Engineers.

*Saved on  
surface*

## Density Changes with Concentration

Crystalline sodium chloride,  $\text{NaCl(s)}$  has a higher density than water at 2.165 g/mL. The density of any  $\text{NaCl}$  solution will be greater than that of pure water but, as we saw above, the density is close to that of pure water.

The density of a sodium chloride solution increases with the concentration of the salt.

| Per<br>Cent by<br>Weight<br>in<br>Solution | Density of Sodium Chloride Solutions |         |         |         |         |         |         |         |         |         |
|--|--------------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|  | 0°C                                  | 10°C    | 20°C    | 25°C    | 30°C    | 40°C    | 50°C    | 60°C    | 80°C    | 100°C   |
| 1  | 1.00747                              | 1.00707 | 1.00534 | 1.00409 | 1.00261 | 0.99908 | 0.99482 | 0.99000 | 0.97850 | 0.96510 |
| 2  | 1.01509                              | 1.01442 | 1.01246 | 1.01112 | 1.00957 | 1.00593 | 1.00161 | 0.99670 | 0.98520 | 0.97190 |
| 4  | 1.03038                              | 1.02920 | 1.02680 | 1.02530 | 1.02361 | 1.01977 | 1.01531 | 1.01030 | 0.99880 | 0.98550 |
| 6  | 1.04575                              | 1.04408 | 1.04127 | 1.03963 | 1.03781 | 1.03378 | 1.02919 | 1.02410 | 1.01250 | 0.99940 |
| 8  | 1.06121                              | 1.05907 | 1.05589 | 1.05412 | 1.05219 | 1.04798 | 1.04326 | 1.03810 | 1.02640 | 1.01340 |
| 10   | 1.07677                              | 1.07419 | 1.07068 | 1.06879 | 1.06676 | 1.06238 | 1.05753 | 1.05230 | 1.04050 | 1.02760 |
| 12   | 1.09244                              | 1.08946 | 1.08566 | 1.08365 | 1.08153 | 1.07699 | 1.07202 | 1.06670 | 1.05490 | 1.04200 |
| 14   | 1.10824                              | 1.10491 | 1.10085 | 1.09872 | 1.09651 | 1.09182 | 1.08674 | 1.08130 | 1.06940 | 1.05650 |
| 16   | 1.12419                              | 1.12056 | 1.11621 | 1.11401 | 1.11171 | 1.10688 | 1.10170 | 1.09620 | 1.08420 | 1.07130 |
| 18   | 1.14031                              | 1.13643 | 1.13190 | 1.12954 | 1.12715 | 1.12218 | 1.11691 | 1.11130 | 1.09930 | 1.08640 |
| 20   | 1.15663                              | 1.15254 | 1.14779 | 1.14533 | 1.14285 | 1.13774 | 1.13238 | 1.12680 | 1.11460 | 1.10170 |
| 22   | 1.17318                              | 1.16891 | 1.16395 | 1.16140 | 1.15883 | 1.15358 | 1.14812 | 1.14250 | 1.13030 | 1.11720 |
| 24   | 1.18999                              | 1.18557 | 1.18040 | 1.17776 | 1.17511 | 1.16971 | 1.16414 | 1.15840 | 1.14630 | 1.13310 |
| 26   | 1.20709                              | 1.20254 | 1.19717 | 1.19443 | 1.19170 | 1.18614 | 1.18045 | 1.17470 | 1.16260 | 1.14920 |

[Salt Institute](http://www.saltinstitute.org)

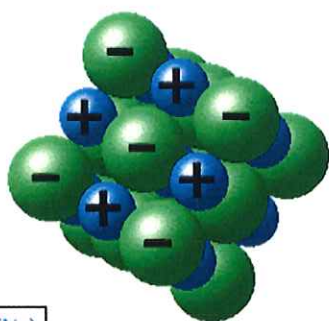
<http://www.saltinstitute.org/About-salt/Physical-properties>

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## Physical Properties

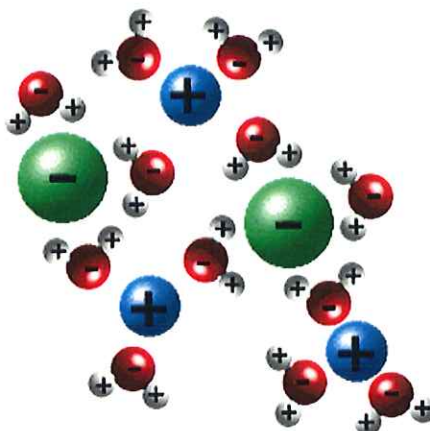
The properties of salt help us understand its beneficial uses.

**NaCl crystal structure**



sodium (Na)  
chlorine (Cl)

**NaCl in water**



The formula for salt, sodium chloride, is 60.663% elemental chlorine (Cl) and 39.337% sodium (Na). Chlorine's atomic weight is 35.4527; for sodium, 22.989768.

Chemical Properties of Pure Sodium Chloride

Solubility of Salt at Various Temperatures

| Temperature °F | Temperature °C | % Salt |
|----------------|----------------|--------|
| -6             | -21.11         | 23.31* |
| 0              | -17.78         | 23.83  |
| 10             | -12.22         | 24.7   |

|       |        |          |
|-------|--------|----------|
| 20    | -6.67  | 25.53    |
| 30    | -1.1   | 26.16    |
| 32    | 0      | 26.29    |
| 32.2  | 0.1    | 26.31**  |
| 40    | 4.44   | 26.33    |
| 50    | 10     | 26.36    |
| 60    | 15.56  | 26.395   |
| 70    | 21.11  | 26.45    |
| 80    | 26.67  | 26.52    |
| 100   | 37.78  | 26.68    |
| 125   | 51.67  | 26.92    |
| 150   | 65.56  | 27.21    |
| 175   | 79.44  | 27.62    |
| 200   | 93.33  | 27.91    |
| 212   | 100    | 28.12    |
| 220   | 104.44 | 28.29    |
| 227.5 | 108.7  | 28.46*** |

\* Eutectic point

\*\* Transition point

\*\*\* Boiling point at one atmosphere pressure



## The New York Times

Green

Energy, the Environment and the Bottom Line

# Toward a Greener Soda Can

By **Matthew L. Wald** June 12, 2012 8:01 am

Of all the materials that are commonly dropped in recycling bins, aluminum is by far the most valuable. New aluminum sells for almost \$2,000 a metric ton, so recycling old cans would seem to be profitable. It takes about 75,000 cans to make a metric ton, so each one should be worth about 2.5 cents.

But recycling the cans turns out to be harder than it looks, because the basic soft drink or beer can is actually made of two kinds of aluminum. The bottom and sides are made from an aluminum sheet that is strong enough to be stamped into a round shape without tearing. For the top, which must be stiff enough to help the can retain its shape and withstand the bending force when it is opened, can makers blend aluminum with magnesium.

When the two parts of the can are melted down, the result is a blend that is suitable for neither purpose, according to Philip Martens, president and chief executive of Novelis, the largest American supplier of aluminum sheet. The solution today, he said, is to mix the recycled material with new aluminum to dilute the magnesium concentration and reduce the metal's stiffness so it can be used for the can bodies. Or, more magnesium can be added so the material can be used for can tops. Last year Novelis used recycled aluminum for 39 percent of its input material.

Nationally, about 50 percent of aluminum cans are recycled. But Novelis would like to raise that to 80 percent by 2020.

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three to four hours of average household use. Using recycled cans brings the energy requirement down to about one-eighth of that. So raising the proportion of recycled material is environmentally advantageous. But to reach that 80 percent goal, Novelis will have to find a way around the alloy problem.

This week, at a convention of aluminum industry executives organized by American Metal Market, a source of news and data about metals, Mr. Martens will be showing off a can made from a single alloy. The trick, he said, is to anneal the metal, treating it with heat so that it becomes strong enough to withstand stamping to become a can body.

“Ultimately you want to get this to be a closed-loop system, where you are working end to end, starting with the consumer and ending with the consumer,” he said. A can could come off a supermarket shelf one day and travel to the consumer’s kitchen, the recycling bin, the smelter and then the can manufacturer, returning to the supermarket in 45 to 60 days, he said.

The changes in the can would have to be invisible to the consumer and be fairly cheap; the beverage business is quite competitive. But the can with the higher recycled content could be “a differentiator” that would boost supermarket sales, Mr. Martens said.

“We do believe in the issue of business sustainability,” and producing a material “where the consumer will say, ‘Why not?’ ” he said. He said he was in discussions with several companies that buy his aluminum and make the cans.

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**ENCH 501 Transport Phenomena****Mid-Term Examination, October 23, 2018****Time Allowed: 1.30 – 3.00 pm**

**Instructions:** Attempt all questions. Use of electronic calculators allowed but no other electronic device allowed. Open Notes, Open Book Examination.

**Problem 1 (15 points)**

One of the colligative properties of liquids is that their freezing points are depressed (or boiling points elevated) by addition of a solute to a solvent. This phenomenon has many useful applications. Glycols (ethylene or propylene) prevent the freezing of water in vehicular radiators in winter and reduce boil-off during car operations. Water-glycol solutions are rated often from -25°C to -45°C, based on the composition. Sodium or calcium chloride are spread on roads to remove ice by depressing the freezing point of water. Typical commercial road salt is effective down to ground surface temperatures of about -10°C, beyond which gravel is spread instead of salt. Another practical use is to quickly chill liquids such as water, soft drinks and beer in cans or soda-glass bottles below room temperature for functions and events. The problem is on the latter.

Ice is available from a freezer. A large vessel is also available. This was filled with equal masses of the ice at -15°C and distilled water at a temperature of 0°C. Salt (NaCl) is then added to the water, while stirring continuously, until the salt concentration reached 16% by weight. The relationship for freezing point depression ( $\Delta T$ ) is given by

$\Delta T = K_f \cdot \phi \cdot i$  where  $K_f$  is the cryoscopic constant for the solvent,  $\phi$  is the molality of the solution (moles of solute per kilogram of solvent), and  $i$  is the van't Hoff factor (number of ions in solution for each dissolved solute molecule). For water  $K_f$  equals 1.853 kg.K/mol and NaCl has  $i$  equal 2. The molar mass of NaCl is 58.44 g/mol, and 18.016 g/mol for H<sub>2</sub>O.

A soda-glass bottle of beer (weighing 536.8g full and 202.3g empty) and an aluminum can of ginger ale (weighing 379.1g full and 15.8g empty) were both at 21°C when they were fully immersed in the salt solution that is being stirred such that the convective heat transfer coefficient around the containers is 18 W/m<sup>2</sup>K. Given the data as below,

- a) Estimate the time required for each of the full bottle and can to reach 5°C. Show and justify your steps.
- b) If the vessel is metallic, and the rate of heat gain from the ambient is significant such that ice melted to dilute the salt solution to 10% by weight of salt in 30 minutes, how long would it take to cool the full bottle and the can to 5°C? You may assume that the temperature of the salt solution changed linearly with time.

| <i>Material</i>   | <i>Density,<br/>kg/m<sup>3</sup></i> | <i>Heat capacity,<br/>J/kg K</i> | <i>Thermal conductivity,<br/>W/mK</i> | <i>Ext. Surface area,<br/>m<sup>2</sup></i> |
|-------------------|--------------------------------------|----------------------------------|---------------------------------------|---|
| Soda glass bottle | 2502                                 | 750                              | 1.4                                   | 0.0348                                      |
| Aluminum can      | 2700                                 | 903                              | 237                                   | 0.0292                                      |
| Beer              | 1008                                 | 4157                             | 0.635                                 | -   |
| Ginger Ale        | 1032.6                               | 3950                             | 0.566                                 | -   |

**Problem 2 (10 points)**



The energy balance on the can of ale

$$\text{Input} + \text{Gen} = \text{Output} + \text{Accum.}$$

$$\downarrow_0 \quad \downarrow_0 \quad hA(T - T_\infty) \quad \frac{d}{dt} \left[ (m_1 c_{p1} + m_2 c_{p2})(T - T_{ref}) \right]$$

or

$$\frac{dT}{dt} = - \frac{hA}{m_1 c_{p1} + m_2 c_{p2}} (T - T_\infty); \quad \begin{matrix} 1 \equiv \text{can} \\ 2 \equiv \text{ale} \end{matrix}$$

or

$$\frac{dT}{dt} = \beta (T - a - bt)$$

$$\frac{dT}{dt} = \beta T - \beta a - \beta b t$$

subject to  $t=0, T=T_0 = 21^\circ\text{C}$