ENCH 501 Transport Phenomena

Mid-Term Examination, October 18, 2017 **Time Allowed: 9.00 – 10.30 am**

Instructions: Attempt all questions. Use of electronic calculators allowed but **no other electronic device allowed**. Open Notes, Open Book Examination.

Problem 1 (12 points)

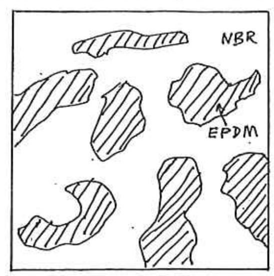
When some different molten metals are well-mixed and cooled, alloys form. Examples are brass from copper and zinc and white gold from gold and platinum or palladium. Since pure gold is soft, platinum, for example, yields a hardened alloy at mass fractions substantially below 25%. In a comparable manner, different polymers can be blended (or mixed) to form products with desirable properties such as improved impact strength, low temperature toughness, resistance to heat distortion, flame retardancy and/or decreased permeability to water and solvents. Polypropylene and polyethylene are two commodity plastics (used as packaging films and blow- and injection-molded items such as bottles) that are semi-crystalline and immiscible. Depending on the relative amounts of the polymers, one will form the continuous phase and the other the dispersed phase as in the sketch below. A blend of polypropylene and the elastomer EPDM (Ethylene-Propylene-Diene terpolymer) is resilient and tough enough for use as car bumpers and dashboards.

Consider a blend of EPDM and NBR (nitrile-butadiene rubber) that is immiscible. Exactly 25% by weight of the blend is EPDM. A flattened but irregular-shaped mass of the blend is to be cooled down from 90°C to 18°C. The volume of the mass is 0.03 m^3 and the exposed surface area is 12.5 m^2. The mass is cooled by air flowing over the surface. The heat transfer coefficient over the entire surface is constant and given as 7.4 W/m^2 K. It has been indicated that how the object is cooled is important to its morphology (or macroscopic structure). In one process, air at a constant temperature of -20°C is suddenly passed over the object from t=0. In the second process, the air temperature is allowed to decrease linearly from 60°C to -20°C from t = 0 to t = 10 minutes and then maintained constant thereafter at -20°C.

a) (**8** pts) For both cooling processes, obtain expressions for the temperature of the blend as a function of time.
b) (**4** pts) What are the elapsed times for the blend to attain the desired final temperature for both processes?

Data and information: Assume that the sum of the volumes of the separate polymers equals the volume of the blend. That is volume change on mixing is negligible.

	k, W/m K	C_p, kJ/kg K	ρ, kg/m^3
EPDM	0.2	2	1000
NBR	0.25	1.35	1300



Problem 2 (8 points)

The steam assisted gravity drainage (SAGD) process, in part, was developed by Dr. Butler while a professor in this department. The process involves the application of two horizontal wells, one above the other, to recover bitumen or heavy oil from a reservoir. Steam is injected into the upper well to be released into the formation from holes along the length of the duct or at the end of the well or the "toe". Latent heat released from condensing steam warms up the bitumen and dramatically decreases its viscosity so that it can drain with water into the lower producing well. The wells are essentially two pipes, at different temperatures, embedded in the same medium.

Consider a SAGD operation with saturated steam injected at 220°C and discharged at the toe. 85% by wt. of the steam injected is returned as water to the surface through the producing well for re-use. The rate of bitumen production is 120 m^3 per day. On a mass basis, the ratio of steam injected to bitumen produced is 2.5. The inside diameter of the steam well is 16 cm and that of the producing well is 12 cm. The wells are separated by a constant distance of 50 cm. The wells are 1250 m long.

a) (**8** pts) If the bitumen and water enter the producer well at 85°C, and they are assumed well dispersed, at what temperature will the mixture exit the well? Show your steps and state your assumptions.
b) (Bonus: **2** pts) What fraction of the injected steam would have condensed at the toe of the steam well?

Data: Heat capacities for water = 4.184 kJ/kg K and for bitumen = 2.093 kJ/kg K. The densities for water and bitumen are 960 and 1020 kg/m^3 respectively. The thermal conductivity of the soil is 1.35 W/m K. The latent heat of condensation of steam is 1857.8 kJ/kg.

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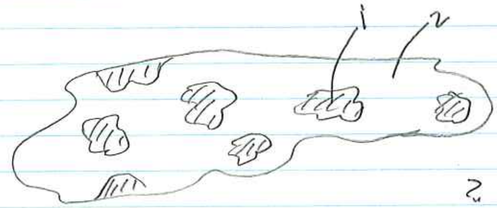
ENCH 501 Mid Term Exam Solution

#1

T_a



$\forall = 0.03\,m^3$

$A = 12.5\,m^2$

? Lumped Capacity method

$$\frac{h\left(\frac{\forall}{A}\right)}{k} < 0.1$$

$h = 7.4\ W/m^2K$

Using the lower value of $k = 0.2\ W/mK$

$$\frac{7.4\left(\frac{0.03}{12.5}\right)}{0.2} = 0.088 < 0.1$$

$\therefore$ Assume that temperature will be uniform in the composite body.

(a) Energy balance on the body

$$\cancel{\text{Input}}^{\,0} + \cancel{\text{Generation}}^{\,0} = \text{Output} + \text{Accumulation}$$

$$hA(T - T_a) \qquad \frac{d}{dt}(E)$$

where E, energy content is $(m_1 C_{p_1} + m_2 C_{p_2})(T - T_a)$

1 ≡ EPDM
2 ≡ NBR

When T_a = constant

$$(m_1 C_{p_1} + m_2 C_{p_2})\frac{d(T - T_a)}{dt} = -hA(T - T_a)$$

But it is given that 25% by wt. of the blend is EPDM (item 1). Let total mass = M kg

$$\frac{0.25M}{1000} + \frac{0.75M}{1300} = 0.03$$

since there is no volume change on blending.

$\therefore \quad M = 36.2791 \ \frac{kg}{s}$

$\therefore \quad m_1 = 9.0698 \ \frac{kg}{s} \ , \quad m_2 = 27.2094 \ \frac{kg}{s}$

Returning to the energy balance

$$\frac{d(T - T_a)}{T - T_a} = d \ln (T - T_a) = - \frac{hA}{m_1 C_{p_1} + m_2 C_{p_2}} dt$$

on integrating,

$$\int_{T_0}^{T} d \ln (T - T_a) = - \beta \int_0^t dt \quad ; \quad \beta = \frac{hA}{m_1 C_{p_1} + m_2 C_{p_2}}$$

(a-1) $$\ln \frac{T - T_a}{T_0 - T_a} = - \beta t \longrightarrow$$

$$\beta = \frac{(7.47)(12.5)}{9.0698(2000) + 27.2094(1350)} = 1.6857(10^{-3}) \ s^{-1}$$

(b-1) $T = 18°C \ , \quad T_a = -20°C \ , \quad T_0 = 90°C$

$$\ln \frac{18 - (-20)}{90 - (-20)} = \ln \frac{38}{110} = -1.0629$$

$$= -1.6857 (10^{-3}) t$$

$\therefore \quad t = 630.54 \ s \quad$ or $\simeq$ 10 mins 31 sec $\longrightarrow$

For the second cooling scheme, T_α changes from $60^\circ C$ to $-20^\circ C$ in 10 mins, i.e.

$$T_\alpha = 60 - \frac{80}{600} t \quad ; \quad t \text{ in seconds}$$

$$= 60 - \frac{4}{30} t \quad \text{or} \quad a - bt$$

The item ultimately cools to $-20^\circ C$ after a long time. Let $T_f = -20^\circ C$. The energy content $E = (m_1 C_{p_1} + m_2 C_{p_2})(T - T_f)$, not $T_\alpha(t)$

The cooling is in 2 stages $-$ $T_\alpha = a - bt$

$T_\alpha = T_f$, a constant

For the first stage, energy balance on body

$$(m_1 C_{p_1} + m_2 C_{p_2}) \frac{d(T - T_f)}{dt} = - hA(T - T_\alpha)$$

Let $$\beta = \frac{hA}{m_1 C_{p_1} + m_2 C_{p_2}}$$

$$\frac{dT}{dt} = -\beta T + \beta(a - bt)$$

with i.c. $t = 0$ $T = 90^\circ C = T_0$

Using the integrating factor method

$$e^{\int \beta dt} \frac{dT}{dy} + e^{\int \beta dt} \beta T = e^{\int \beta dt} \beta (a - bt)$$

$$T e^{\int \beta dt} = \int e^{\int \beta dt} \beta (a - bt)\, dt + C$$

$$T e^{\beta t} = \beta \int e^{\beta t}(a - bt)\, dt + C$$

$$T = \beta e^{-\beta t}\left[\int a e^{\beta t} dt - \int b t e^{\beta t} dt \right] + C e^{-\beta t}$$

$$= \beta e^{-\beta t}\left[\frac{a}{\beta} e^{\beta t} - b \frac{e^{\beta t}}{\beta}\left(t - \frac{1}{\beta}\right) \right] + C e^{-\beta t}$$

$$T = \left[a - b\left(t - \frac{1}{\beta}\right) \right] + C e^{-\beta t}$$

$$T = \left(a + \frac{b}{\beta}\right) - bt + C e^{-\beta t}$$

$t = 0$, $T = T_0$

$$C = T_0 - \left(a + \frac{b}{\beta}\right)$$

$$T = \left[T_0 - \left(a + \frac{b}{\beta}\right) \right] e^{-\beta t} - bt + \left(a + \frac{b}{\beta}\right)$$

Given $\beta = 1.6857(10^{-3})\ s^{-1}$ $\longrightarrow$

$a = 60$

$b = 4/30$

$$T = T_0 e^{-\beta t} + \left(a + \frac{b}{\beta}\right)\left(1 - e^{-\beta t}\right) - bt$$

$0 \le t \le 600\,s$

at $t = 600_s$

$$\therefore \quad T = 90e^{-1.6857(10^{-3})600}$$

$$+ \left(60 + \frac{4}{30(1.6857)(10^{-3})}\right)\left(1 - e^{-1.6857(10^{-3})600}\right)$$

$$- \frac{4}{30} \cdot 600$$

$$T = 90(0.3637) + (60 + 79.0967)(1 - 0.3637)$$
$$- 80$$

$$= 32.7332 + 88.5072 - 80$$

$$= 41.24 \,^\circ C$$

The object is at this temperature at $t = 10$ mins.

This is the initial temperature for the second phase when the air temperature is constant at $-20\,^\circ C$

Using $\ln \frac{T - T_2}{T_{01} - T_2} = -\beta t$ as previously derived

with $T_{01} = 41.24$

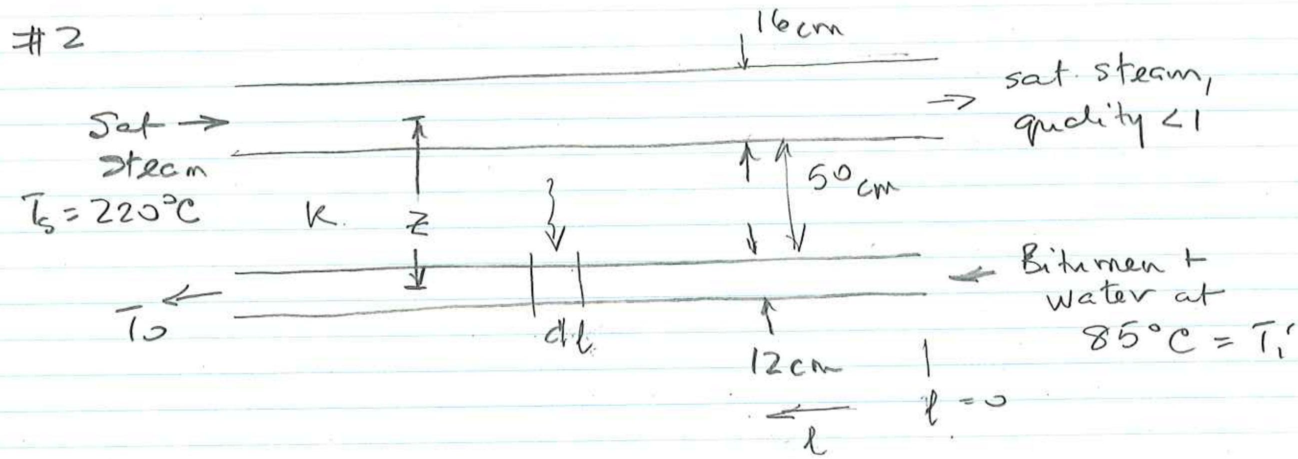
$$\ln \frac{18 - (-20)}{41.24 - (-20)} = \ln \frac{38}{61.24} = -0.4772$$

$$= -1.6857(10^{-3})\, t_{II}$$

$$t_{II} = 283.1 \text{ s}$$

Total time $= 600 + 283.1_s = 883.1s$ →

#2



The steam well will transfer heat to the producing well through the soil.

Using the shape factor method, 2 parallel pipes,

$$S = \frac{2\pi \ell}{\cosh^{-1}\left(\frac{z^2 - r_1^2 - r_2^2}{2 r_1 r_2}\right)}$$

$$z = 50 + 8 + 6 = 64 \text{ cm}$$

$$r_1 = 8 \text{cm} \quad \text{and} \quad r_2 = 6 \text{ cm}$$

Consider a differential element $d\ell$

Rate of heat supply from the steam well is

$$dQ = k\, dS\, (T_s - T)$$

This is equal to the rate at which the bitumen/ water mixture gains heat

$$dQ = (m_1 C_{p_1} + m_2 C_{p_2})\, dT$$

where
1 ≡ bitumen
2 ≡ water

$$\therefore \quad k\beta(T_s - T)\,dl = (m_1 C_{p_1} + m_2 C_{p_2})\,dT$$

where $\beta = 2\pi / \cosh^{-1}\left(\dfrac{z^2 - r_1^2 - r_2^2}{2 r_1 r_2}\right)$

$$\text{or} \quad \frac{k\beta}{m_1 C_{p_1} + m_2 C_{p_2}}\,dl = \frac{dT}{T_s - T} = -d\ln(T_s - T)$$

$$\lambda \int_0^L dl = -\int_{T_i}^{T_o} d\ln(T_s - T)$$

$$\ln \frac{T_s - T_i}{T_s - T_o} = \lambda L \quad \text{or} \quad \frac{T_s - T_i}{T_s - T_o} = e^{\lambda L}$$

$$T_s - T_o = \frac{T_s - T_i}{e^{\lambda L}} \quad \text{or} \quad T_o = T_s - \frac{(T_s - T_i)}{e^{\lambda L}}$$

Now to get the parameters $\longrightarrow$

Rate of flow of bitumen = $120\ m^3/day$

$$\text{or} \quad \frac{120}{24 \times 3600} = 1.389(10^{-3})\ m^3/s$$

The steam-to-oil ratio is 2.5

$\therefore$ Rate of steam injection =

$$(2.5)(1.389)(10^{-3})(1020)\ kg/s$$

$$= 3.5417\ kg/s$$

85% of this is recovered. ∴ mass rate of water in producer well is

$$m_2 = (0.85)(3.5417) = 3.0104 \text{ kg/s}$$

Rate of bitumen

$$m_1 = (1.389)(10^{-3})(1020) = 1.4168 \text{ kg/s}$$

$$p = \frac{2\pi}{\cosh^{-1}\left(\frac{64^2 - 8^2 - 6^2}{2 \times 8 \times 6}\right)} = \frac{2\pi}{4.4217}$$

$$= 1.42099$$

$$\lambda = \frac{1.35(1.42099)}{1.4168(2093) + 3.0104(4184)} \quad m^{-1}$$

$$= \frac{1.91833}{15,560.876} = 1.2328(10^{-4})$$

since $L = 250$ m, on substitution

$$T_o = 220 - \frac{(220-85)}{e^{1.2328(10^{-4})(250)}} = 104.28°C$$

→

(b) The total heat gained by the bitumen and water in the producer well is

$$(m_1 C_{p_1} + m_2 C_{p_2})(104.28 - 85)$$

This equals the latent heat extracted in condensing steam within the steam well.

Let the mass rate of steam condensation = W kg/s

$$[(1.4168)(2093) + 3.0104(4184)](19.28)$$

$$= W\,(1857.8)(10^3)$$

$W = 0.1615$ kg/s condensate rate

The fraction of injected steam condensed is

$$\frac{0.1615}{3.5417} = 0.0456$$

That is 4.56% of the steam condenses in the pipe.