ENCH 501 Transport Phenomena University of Calgary October 25, 2016 *aJ*

MID-TERM EXAMINATION

Time Allowed: 90 minutes **Open Book and Open Notes Examination.**

Programmable calculators are the only electronic devices permitted.

Question 1. (12 points)

The hot water extraction process is the primary method for extracting bitumen from oilsands. Mined oilsands is conveyed to an extraction unit and mixed with hot water. Both steam and industrial caustic soda (aqueous sodium hydroxide solution) are continuously supplied to promote separation and disengagement of droplets of oil from sand and clay particles. In a semi-batch operation, oilsands and warm water are charged into a vertical cylindrical vessel to a depth of 2.2m and the suspension (or slurry) is continuously well agitated. The tank, 3m inside diameter and 8m tall, is very well insulated on the external surface. The initial temperature of the suspension in the tank is 38°C. At time t = 0, saturated steam at 118.6°C is steadily fed into the slurry at a volumetric rate of 10.14 m^3/minute. A caustic solution (that is 5.2% by weight NaOH) at 25°C is also steadily supplied at a rate of 0.06 m^3/minute into the slurry. Excellent separation between the oil and the solids is expected to be achieved when the temperature of the slurry reaches 85°C. At this time, both injections of steam and caustic are stopped. Stirring is also stopped so that the oil droplets and the solids particles can disengage. The lighter oil rises and the heavier silica particles settles.

(a) How long will the process take between the start and termination of steam injection? ***Show all your steps.*** (Pay attention to being consistent with your units.)

(b) What would the level of the suspension be in the tank when stirring is stopped?

Data and Assumptions:

- For slurry – assume the specific heat C_p and the density ρ are constants irrespective of the amount of dilution by steam and caustic soda, and the values are 2.45 kJ/kg K and 1014 kg/m^3 respectively.
- No chemical reactions are occurring inside the tank and the 'heat of mixing' (not the sensible heat) of caustic and the slurry is negligible.
- Caustic soda with 5.2 wt % NaOH – density is 1,060 kg/m^3, specific heat is 3.98 kJ/kg K
- Feed steam - temperature is 118.6°C, pressure is 190 kPa, specific volume is 0.929 m^3/kg, heat of evaporation (or condensation) is 2206.2 kJ/kg, specific heat of water (or condensed steam) is 4.186 kJ/kg K.

Question 2. (8 points)

Lookout towers are frequently erected in forested areas so that individuals can monitor the occurrence, or preferably, the initiation of wild fires by lightning or human carelessness. These wood or steel towers, often in elevated terrains, are high above the surrounding trees to provide vantage viewing. A cab (or room) at the top of each tower provides a safe space from potentially dangerous animals. Suppose an observer is located at a height **y** above a flat terrain. Using a binoculars and a sextant, the observer notes a small fire at location A on the plane. The angle of the line-of-sight for position A to the horizon is **α**. Along the same direction on the plane of the terrain, but closer to the watch tower, is another small fire at location B. Point B is at an angle **β** to the horizon. The distance, on the plane and horizontal terrain, from A to B is **x**. Find the functional relationship between separation distance **x** and the parameters **y** and **α ± β** (in any combination of α – β, α + β or β – α).

If y = 20m, α = 5° and β = 32°, what is the value of **x**?

TRIGONOMETRIC IDENTITIES

- **Reciprocal identities**

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u}$$
$$\tan u = \frac{1}{\cot u} \qquad \cot u = \frac{1}{\tan u}$$
$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u}$$

- **Pythagorean Identities**

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

- **Co-Function Identities**

$$\sin(\frac{\pi}{2} - u) = \cos u \qquad \cos(\frac{\pi}{2} - u) = \sin u$$
$$\tan(\frac{\pi}{2} - u) = \cot u \qquad \cot(\frac{\pi}{2} - u) = \tan u$$
$$\csc(\frac{\pi}{2} - u) = \sec u \qquad \sec(\frac{\pi}{2} - u) = \csc u$$

- **Parity Identities (Even & Odd)**

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u$$
$$\tan(-u) = -\tan u \qquad \cot(-u) = -\cot u$$
$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u$$

- **Sum & Difference Formulas**

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

- **Double Angle Formulas**

$$\begin{aligned} \sin(2u) &= 2\sin u \cos u \\ \cos(2u) &= \cos^2 u - \sin^2 u \\ &= 2\cos^2 u - 1 \\ &= 1 - 2\sin^2 u \\ \tan(2u) &= \frac{2\tan u}{1 - \tan^2 u} \end{aligned}$$

- **Power-Reducing/Half Angle Formulas**

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$
$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$
$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

- **Sum-to-Product Formulas**

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

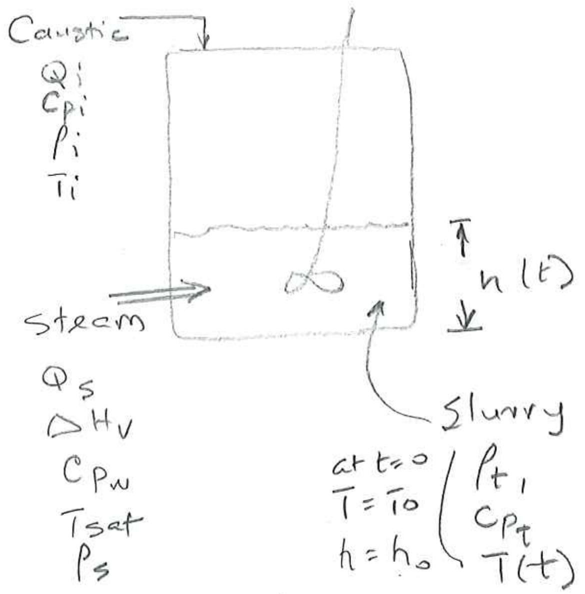
- **Product-to-Sum Formulas**

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

ENCH 501 Mid-Term Exam Solution Oct. 25, 2016

G. Tejo

I Problem # 1



This is a semi-batch operation with both steam and caustic soda continuously added to the slurry.

The steam will heat up both the slurry and the caustic using the both latent heat and sensible heat from condensed water.

An energy balance is carried out on the control volume – the tank.

$$\text{Input} + \underbrace{\text{Generation}}_{0} = \underbrace{\text{Output}}_{0} + \text{Accumulation}$$

→ Input of heat from caustic $= \rho_i Q_i C_{p_i}(T_i - T)$; $T_i < T \therefore$ it extracts heat

from steam $= \rho_s Q_s\left[\Delta H_v + C_{p_w}(T_{sat} - T)\right]$; condenses at T_{sat}

→ Accumulation

$$\frac{d}{dt}\left[h A \rho_t C_{p_t}(T - T_0)\right]$$ where both h and T in the tank are functions of time

Hence

$$\left[\rho_i Q_i C_{p_i} T_i + \rho_s Q_s(\Delta H_v + C_{p_w} T_{sat})\right] - \left[\rho_i Q_i C_{p_i} + \rho_s Q_s C_{p_w}\right]T = A\rho_t C_{p_t}\frac{d\{h(T-T_0)\}}{dt}$$

The energy equation is hence of the form

$$\alpha - \beta T = \gamma \frac{d}{dt}\left[h(T-T_0)\right] \quad ; \alpha, \beta, \gamma \text{ are constants}$$

But both steam and caustic are added to the vessel at constant mass rates, $\Sigma = Q_i \rho_i + Q_s \rho_s$.

If the density of the slurry remains constant at ρ_t, then the rate at which volume is added in the tank $= \dfrac{Q_i \rho_i + Q_s \rho_s}{\rho_t} = \dfrac{dh}{dt} \cdot A$

$$\therefore \; \varepsilon = \frac{dh}{dt} = \frac{Q_i \rho_i + Q_s \rho_s}{\rho_t A} = \text{constant.} \Rightarrow h = a + bt$$

where a and b are constants.

The energy equation is expanded as

$$\alpha/\gamma - \beta/\gamma \, T = \left[(T-T_0)\frac{dh}{dt} + h\frac{dT}{dt}\right]$$

$$= T\frac{dh}{dt} - T_0\frac{dh}{dt} + h\frac{dT}{dt}$$

or

$$\alpha^+ - \beta^+ T = h\frac{dT}{dt} \quad ; \quad \alpha^+ = \alpha/\gamma + T_0\frac{dh}{dt}$$

$$\beta^+ = \beta/\gamma + \frac{dh}{dt}$$

$$h = h_0 + \varepsilon t$$

$$\therefore \quad \alpha^+ - \beta^+ T = (h_0 + \varepsilon t)\frac{dT}{dt}$$

This is an ordinary differential equation subject to the condition $t = 0$, $T = T_0$.

on integration

$$\int_{T_o}^{T} \frac{dT}{\alpha^+ - \beta^+ T} = \int_0^t \frac{dt}{h_o + \varepsilon t}$$

or $$-\frac{1}{\beta^+} \ln(\alpha^+ - \beta^+ T)\Big|_{T_o}^{T} = \frac{1}{\varepsilon} \ln(h_o + \varepsilon t)\Big|_0^t$$

or $$\ln\left[\frac{\alpha^+ - \beta^+ T}{\alpha^+ - \beta^+ T_o}\right]^{-1} = \ln\left[\frac{h_o + \varepsilon t}{h_o}\right]^{\beta^+/\varepsilon}$$

and therefore,

$$\left[\frac{\alpha^+ - \beta^+ T}{\alpha^+ - \beta^+ T_o}\right]^{-1} = \left[\frac{h_o + \varepsilon t}{h_o}\right]^{\beta^+/\varepsilon}$$

This equation relates slurry temperature T to time elapsed t.

Now to substitute values.

Tank cross-sectional area, $A = \frac{\pi D^2}{4} = \frac{\pi 9}{4} = 7.06858$ m^2

$$\gamma = A \rho_t C_{p_t} = (7.06858)(1014)(2450) = 1.756(10^7)$$

$m^2 \quad kg/m^3 \quad J/kg_s\, K.$

$$\alpha = \rho_i Q_i C_{p_i} T_i + \rho_s Q_s (\Delta H_v + C_{p_w} T_{sat}) =$$

$$1060(0.06)(3980)25 + \frac{1}{0.929} 10.14 (2206200 + 4186 \times 118.6)$$

$kg/m^3 \quad m^3/min \quad J/kg_s\, K \quad K \qquad kg/m^3 \quad m^3/min \quad J/kg_s \quad \frac{J}{kg\, K} \cdot K$

$$= 3.5828(10^7)$$

$$\beta = \rho_i Q_i C_{p_i} + \rho_s Q_s C_{p_w} =$$

$$1060(0.06)(3980) + \frac{1}{0.929} 10.14 (4186)$$

$kg/m^3 \quad m^3/min \quad \frac{J}{kg\, K}.$

$$= 2.9882(10^5)$$

$$\varepsilon = \frac{dh}{dt} = \frac{Q_i \rho_i + Q_s \rho_s}{\rho_t A} = \frac{0.06(1060) + \frac{1}{0.929} \times 10.14}{1014(7.06858)}$$

$$= 0.010396 \quad m/min$$

$$\alpha^+ = \frac{\alpha}{\gamma} + \varepsilon T_o = 2.0402 + 0.010396(38)$$

$$= 2.435291$$

$$\beta^+ = \beta/\gamma + \varepsilon = 0.017017 + 0.010396$$

$$= 0.027413$$

Substitute into solution, given T = 94 °C

$$\left[\frac{2.43529 - 0.027413\,(85)}{2.43529 - 0.027413\,(38)}\right]^{-0.379236} =$$

$$\left[\frac{2.2 + 0.010396\,t}{2.2}\right]$$

$$\left(\frac{0.105185}{1.393596}\right)^{-0.379234} = 1 + 0.004725\,t$$

$$t = \frac{2.664234 - 1}{0.004725} \text{ mins}$$

$$= 352.22 \text{ mins}$$

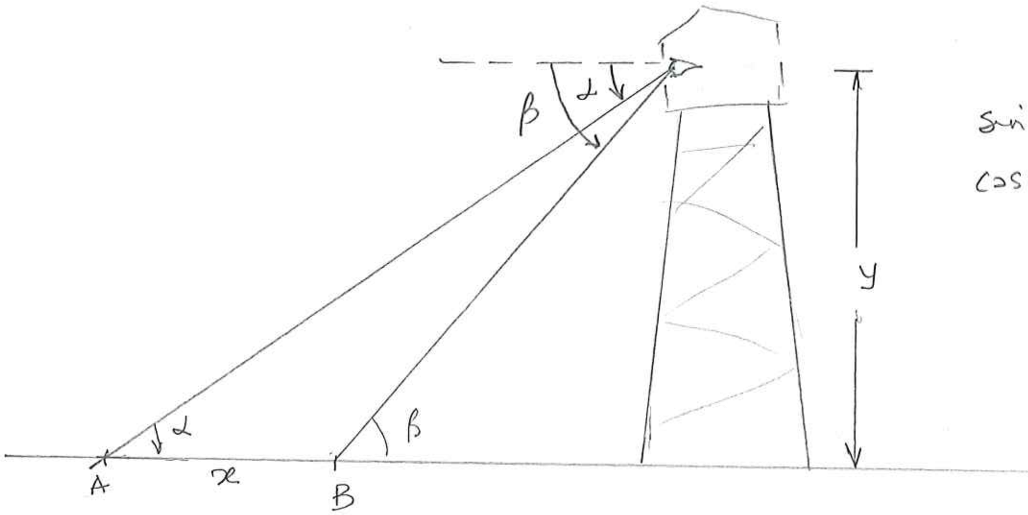
(b) The final height of slurry

$$h_f = h_0 + \varepsilon t = 2.2 + 0.010396\,(352.22)$$

$$= 5.8617 \text{ m}$$

II Problem #2

Reference to sketch



Notes

$\tan\alpha = \sin\alpha / \cos\alpha$

$\sin(\beta-\alpha) = -\sin(\alpha-\beta)$

$\cos(\alpha-\beta) = \cos(\beta-\alpha)$

Use Trigonometric relationships

$$x = \frac{y}{\tan\alpha} - \frac{y}{\tan\beta} = y\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right)$$

$$x = y\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = y\left\{\frac{\frac{\sin\beta\cos\alpha - \sin\alpha\cos\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}\right\}$$

$$\frac{x}{y} = \frac{\frac{1}{2}\left[\sin(\beta+\alpha) + \sin(\beta-\alpha)\right] - \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]}{\frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]}$$

$$x = 2y\sin(\beta-\alpha) / \left[\cos(\beta-\alpha) - \cos(\alpha+\beta)\right]$$

Example, $y = 20$ m, $\alpha = 5°$ and $\beta = 32°$

calc.

$$x = 40\sin(27°) / \left[\cos 27° - \cos 37°\right] = \frac{18.15962}{0.09237} = 196.59 \text{ m}$$