

MID-TERM EXAMINATION

Time Allowed: 90 minutes **Open Book and Open Notes Examination.**

Programmable calculators are the only electronic devices permitted.

Question 1. (10 points)

Tanker trucks carry gasoline and other products from refineries to public distribution or retail sites (such as gas stations). Gasoline is usually hot when discharged directly from atmospheric distillation columns into tanks on trucks. The average temperature of the fuel fed into the tank at a refinery is given as 150°C. The ambient air temperature is 5°C. The tank is a steel cylinder with flat ends, and walls are of constant thickness $\delta = 1.5\text{cm}$ all around. The inside diameter D_i of the cylinder is 1.5m and the inside length L_i is 3.3m.

If the internal and external heat transfer coefficients for the tank are very large because the truck driver races, estimate the time required to cool the gasoline that completely filled the tank to 30°C.

(The gasoline should be discharged at the station at a low temperature to avoid flashing and an explosion!)

Data: For a horizontal, cylindrical tank with flat ends, the shape factor $S = \left[\left(\frac{3\pi D_i^2}{2\delta} \right)^n + (7.476 D_i)^n \right]^{1/n}$ with

L_i/D_i	0.4	1.0	2.2
n	1.12	1	1.03

The thermal conductivity of steel is 36 W/m K, density of gasoline is 850 kg/m³ and the specific heat for gasoline is 2.13 kJ/kg K

Question 2. (15 points)

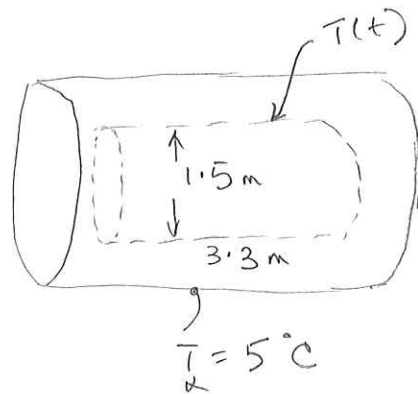
Materials such as food items, cores obtained from drilling, reproductive cells and chemical samples are often preserved by freezing in a chiller or quenching in liquid nitrogen. In the frozen state, the substances do not spoil because of activities of micro-organisms nor do they change composition because volatile components escape. Recovery of the material on thawing is an operation that has to be handled with care. Food items such as turkey, chicken and fish should not be thawed on counter-tops as micro-organisms will grow rapidly on them. They should be thawed in a refrigerator at between 2 and 5°C where growth of bacteria is slow. It is suggested, for example, that turkey should be in the fridge for 24 hours for each 5 lbs or 2.27 kg to defrost. A core obtained from a reservoir should not be in a microwave unless the vapor released is collected and analyzed.

Consider a core obtained from an aquifer to determine the level of contamination from a mining run-off. The core is a 10 cm diameter and 25 cm long cylinder and it is fully saturated with water. It is assumed that the toxic impurities are at low concentrations. Once collected, the sample at 35°C is to be frozen to a temperature of -21°C in a chiller with convective air flow at a constant temperature of -45°C. The sample is then transported, well insulated, to the testing laboratory in 3 hours and 45 minutes. Once it arrives at the laboratory, the sample is placed in a refrigerator maintained at 5°C to thaw. There is a gentle but constant circulation of air in the fridge.

Given the data below, estimate the total elapsed time between collecting the sample in the field and when it is thawed and at 3°C in the laboratory fridge. Show your steps. Assume same properties for water and ice.

Data: The heat transfer coefficient h in the chiller is 6 W/m² K and in the fridge it is 3.2 W/m² K. The thermal conductivity k of the saturated core is 1.56 W/m K. The densities ρ of water and the core solids are 998 and 2,420 kg/m³ respectively. The specific heat values C_p are 4.179 and 0.73 kJ/kg K for water and the solids. The latent heat of fusion for water ΔH_f is 334 kJ/kg. The void fraction ϵ for the core is 0.27.

#1



for the tank

$$L_i = 3.3 \text{ m}, \quad D_i = 1.5 \text{ m}$$

$$\delta = 0.015 \text{ m}$$

Since both internal and external h values are large, the walls attain the temperatures of the fluids.

$$\text{Rate of heat loss, } Q = k S (\bar{T} - T_{\infty})$$

This equals $-m \dot{C}_p \frac{dT}{dt}$ or heat loss from the gasoline.

$$L_i/D_i = \frac{3.3}{1.5} = 2.2 \quad \therefore n = 1.03$$

$$S = \left\{ \left[\frac{3\pi (1.5)^2}{2(0.015)} \right]^{1.03} + [7.476(1.5)]^{1.03} \right\}^{1/1.03}$$

$$= [860.6218 + 12.0574]^{1/1.03} = 716.4711 \text{ m}$$

$$-\pi R_i^2 L_i \rho C_p \frac{dT}{dt} = k S (\bar{T} - T_{\infty})$$

$$\text{or} \quad -\frac{dT}{dt} = \beta (\bar{T} - T_{\infty}) \quad ; \quad \beta = \frac{k S}{\rho (\pi R_i^2) L_i C_p}$$

$$\text{Condition: } t=0 \quad \bar{T} = T_0 = 150^\circ\text{C}$$

Solve

$$\frac{T - T_a}{T_0 - T_a} = \exp[-\beta t] ; \beta = 2.443 (10^{-3}) s^{-1}$$

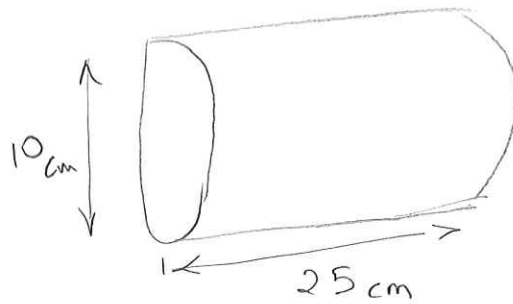
$$T = 30^\circ C, T_a = 5^\circ C, T_0 = 150^\circ C$$

$$\frac{30 - 5}{150 - 5} = 0.172414 = e^{-2.443 (10^{-3}) t}$$

$$t = 719.56 s \quad \text{or} \quad \approx 12 \text{ minutes}$$

→

#2



Check whether lumped analysis is valid, i.e.

$$\frac{h(V/A)}{k} < 0.1$$

Use condition in chiller at higher value for h .

$$\frac{V}{A} = \frac{\pi R^2 L}{2\pi RL + 2\pi R^2} = \frac{RL}{2(L+R)} = \frac{(0.05)(0.25)}{2(0.3)} = 0.02083$$

$$\frac{h(V/A)}{k} = \frac{6(0.02083)}{1.56} = 0.0801 < 0.1$$

\therefore Use lumped analysis.

① Cool from 35°C to 0°C . This is a composite medium — water and solids.

Balance on core

$$\underbrace{\text{Input}}_0 + \underbrace{G_{\text{gen}}}_0 = \underbrace{\text{Output}}_{h_c A (T - T_a)} + \underbrace{\text{Accum}}_{(m_w c_{pw} + m_s c_{ps}) \frac{dT}{dt}}$$

h_c is heat transfer coeff in chiller

The core undergoes several stages —

- ① ° Cool from 35°C to 0°C
- ② ° Freeze the water at 0°C
- ③ ° Cool from 0°C to -21°C

Transport — 3 hrs, 45 mins

- ④ ° Warm from -21°C to 0°C
- ⑤ ° Melt water at 0°C
- ⑥ ° Warm from 0°C to 3°C

The energy equation is

$$-\frac{dT}{dt} = \beta(T - \bar{T}_\infty) ; \quad \beta = \frac{h_c A}{(m_w c_{pw} + m_s c_{ps})}$$

subject to $t=0 \quad T = \bar{T}_0$

and t_1 for $T = \bar{T}_m = 0^\circ\text{C}$

$$\ln\left(\frac{\bar{T}_m - \bar{T}_\infty}{\bar{T}_0 - \bar{T}_\infty}\right) = -\beta t_1 \quad \therefore \quad t_1 = \frac{1}{\beta} \ln\left(\frac{\bar{T}_0 - \bar{T}_\infty}{\bar{T}_m - \bar{T}_\infty}\right)$$

$$A = 2\pi R(L + R) = 2\pi(0.05)(0.3) = 0.09425 \text{ m}^2$$

$$\beta = \frac{6(0.09425)}{m_w(4179) + m_s(730)}$$

$$m_w = \varepsilon V \rho_w$$

$$m_s = (1 - \varepsilon) V \rho_s$$

$$\varepsilon = 0.27, \quad V = \pi R^2 L = \pi(0.05)^2(0.25) = 0.001963 \text{ m}^3$$

$$m_w = (0.27)(0.001963)(998) = 0.5291 \text{ kg}$$

$$m_s = (1 - 0.27)(0.001963)(2420) = 3.4687 \text{ kg}$$

$$\beta = 1.1922(10^{-4}) \text{ s}^{-1}$$

$$t_1 = 8387.939 \ln\left(\frac{35 - (-45)}{0 - (-45)}\right) = 4826.12 \text{ s}$$

(b) The water only freezes

Balance $0 = h_c A (\bar{T}_m - \bar{T}_\infty) + \frac{d(\varepsilon V \rho_w \Delta H_f)}{dt}$

Integrate $t=0 \quad \text{vol water} = \varepsilon V$

$t=t_2 \quad \quad \quad = 0$

$$-\frac{d(\varepsilon V)}{dt} = \beta(\bar{T}_m - \bar{T}_\infty), \quad \beta = \frac{h_c A}{\rho_w \Delta H_f}$$

Integrate

$$t_2 = \frac{\varepsilon V \rho_w \Delta H_f}{h_c A (\bar{T}_m - \bar{T}_a)}$$

$$t_2 = \frac{(0.27)(0.001963)(998)(334)(10^3)}{6(0.09425)(45)} = 6,942.5 \text{ s}$$

(c) Cooling from 0°C to -21°C

This is similar to part (a)

$$\bar{T}_0 = 0^\circ\text{C} \text{ at } t=0$$

$$\bar{T} = -21^\circ\text{C} \text{ at } t_3, \quad \beta = 1.1922(10^{-4}) \text{ s}^{-1}$$

$$\therefore t_3 = 8387.939 \ln \left(\frac{0 - (-45)}{-21 - (-45)} \right) = 5,272.73 \text{ s}$$

Next the thawing stages

(d) Warm from -21°C to 0°C in the fridge.

$$\text{Input} + \underbrace{G_{\text{eff}}}_{\downarrow 0} = \underbrace{\text{Output}}_{\downarrow 0} + \text{Accum.}$$

$$hA(\bar{T}_a - \bar{T}) = (m_w c_{pw} + m_s c_{ps}) \frac{d\bar{T}}{dt}$$

This is the same expression as for part (a)

$$t_4 = \frac{1}{\gamma} \ln \left(\frac{-21 - 5}{0 - 5} \right) \quad \gamma = \frac{h_f A}{m_w c_{pw} + m_s c_{ps}}$$

h_f is heat transfer coeff w/ fridge.

$$\gamma = \beta \left(\frac{3.2}{6} \right) = 6.358 (10^{-5}) s^{-1}$$

$$t_4 = 15,727.227 \ln \left(\frac{26}{5} \right) = 25,928.83 s$$

(e) The ice thaws at $0^\circ C$

$$\begin{aligned} t_5 &= \frac{\varepsilon V \rho_w \Delta H_f}{h_f A (T_m - T_2)} \\ &= \frac{(0.27)(0.001963)(998)(334)(10^3)}{3.2(0.09425)(5)} \\ &= 117,154.7 s \end{aligned}$$

(f) Finally the core warms up from $0^\circ C$ to $3^\circ C$

$$\begin{aligned} t_6 &= \frac{1}{\gamma} \ln \left(\frac{0-5}{3-5} \right) = 15,727.227 \ln \left(\frac{5}{2} \right) \\ &= 14,410.7 s \end{aligned}$$

$$\text{Total time elapsed} = \sum_{i=1}^6 t_i + \underset{\text{transport time}}{13,500 s}$$

$$\begin{aligned} t &= 4826.12 + 6,942.5 + 5,272.73 + \\ &\quad 13,500 + 25,928.83 + 117,154.7 + 14,410.7 \\ &= 188,035.6 s \quad \text{or } \sim 52 \text{ hrs } 14 \text{ mins} \end{aligned}$$