ENCH 501 Transport Phenomena University of Calgary September 16, 2014

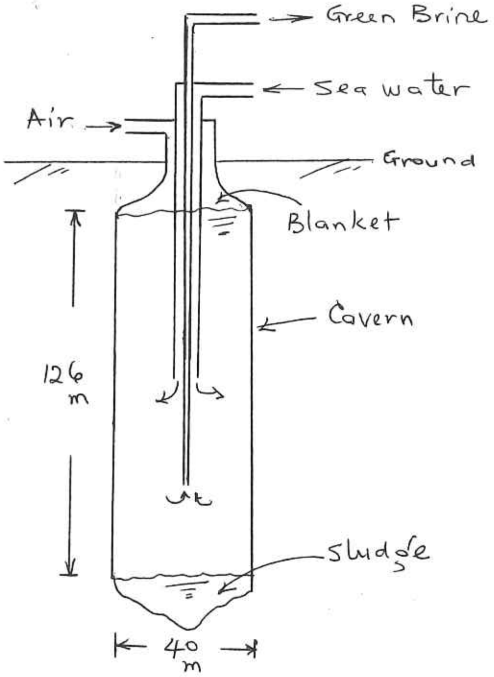
MID-TERM EXAMINATION

Time Allowed: 90 minutes **Open Book and Open Notes Examination.**

Programmable calculators are the only electronic devices permitted.

Question 1. (15 points)

One of the two primary ways to recover salt (NaCl) from underground deposits is solution mining. A well is drilled to the layer of the salt deposit and either fresh water or a weak brine solution is pumped into the well. The water dissolves the salt to form a cavern and a strong brine solution (green brine) is pumped to the surface, as per sketch. Over time, the cavern expands from its original small size as salt is dissolved away. Caverns can be located several hundred meters below ground and may reach and exceed sizes that are 300 m tall by 100 m wide. Sludge or insoluble impurities fall to the bottom and a blanket of compressed air, nitrogen or a light hydrocarbon liquid is created at the top of the cavern so that the salt near the top is not dissolved away to weaken the structure and cause the cavern to collapse, with subsistence of the overburden. Once a cavern has grown to a certain limit, the cavity is frequently used to store hydrocarbon fluids such as natural gas and liquid petroleum gases (LPG) for peak shaving in winter, crude oils as strategic reserves, nuclear or other wastes.



In a salt mining operation, the "working volume" of the cavern from which salt is being dissolved is a cylinder that is 40 m diameter and 126 m high. The bottom has sludge and the top has a blanket of air. For about a month, the operation was shut down with the cavern full of brine at 25°C and there were no flows in or out. At start-up, sea water at 25°C that contained 2.68% by weight of NaCl is the feed into the cavern at a constant rate of 2 m^3/s. Green brine immediately started to flow out at the same volume rate as the feed solution. The density of the first litre of solution produced was measured to be 1,194 kg/m^3. The content of the cavern is assumed to be always well mixed.

(a) If after exactly 20.5 hours of continuous operation, the density of the brine produced has dropped to 1,100 kg/m^3, estimate the mass transfer coefficient for salt dissolution at the vertical wall of the cavern. Show all important steps and state your assumptions.

(b) How much salt would have been recovered from the cavern in the period? If the density of rock salt that is being mined is 2,160 kg/m^3, by how much would the volume of the cavern have changed? Are the assumptions made in part (a) valid?

Data: Weight % of Sodium Chloride dissolved in water versus solution density at 25°C

Wt %	1	2	4	8	12	16	20	24	26	26.3 sat
ρ kg/m^3	1004.09	1011.12	1025.3	1054.12	1083.65	1114.01	1145.33	1177.76	1194.43	1202.01

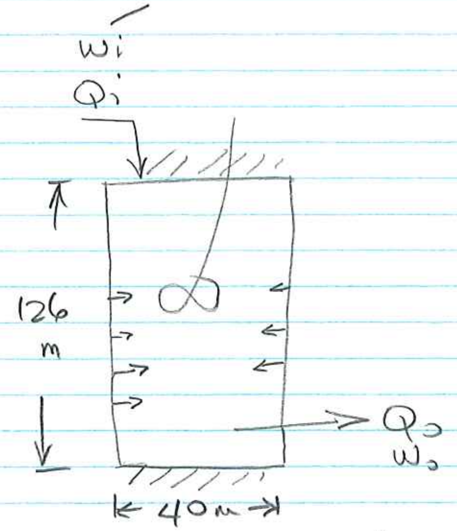
Hint: Choose the solution in the cavern as the control volume and include all input terms.

Question 2. (10 points)

The rate of heat loss from a heated metal sphere (Q) into a cold fluid has been determined to depend on its diameter (D), its heat capacity (C_p), its thermal conductivity (k), the average speed of the fluid around the sphere (u), the fluid density (ρ) and viscosity (μ), and the temperature difference between the sphere and the fluid (ΔT).
Determine the dimensionless groups that can be used to correlate data obtained from experiments. Show your steps.

SOLUTION – Mid-term Exam ENCH 501 Oct. 16, 2014

#1



The sea brine input has

$$w_i = 0.0248 = \rho_i/\rho$$

From the table, density of sea brine

$$\frac{\rho - 1011.12}{2.68 - 2} = \frac{1026.30 - 1011.12}{4 - 2}$$

$$\rho = 1015.94 \ \mathrm{kg/m^3}$$

$$\therefore \ \rho_i = 27.2272 \ \frac{\text{kg salt}}{\text{m}^3 \text{sol}^n}$$

The green brine effluent at $t=0$ has a density of 1,194 kg/m³. The mass fraction of salt is

$$\frac{w_o - 24}{1194 - 1177.76} = \frac{26 - 24}{1194.43 - 1177.76}$$

or $w_o\% = 25.9484\%$ by wt NaCl

$$\therefore \ \rho_s\Big|_{t=0} = 309.824 \ \frac{\text{kg salt}}{\text{m3 solution}}.$$

After hours, the density of the green brine is 1,100 kg/m³, therefore

$$\frac{w_o - 12}{1100 - 1083.65} \simeq \frac{16 - 12}{1114.01 - 1083.65}$$

$$w_o\Big|_t = 14.1542\% \text{ by wt NaCl}$$

$$\therefore \ \rho_s\Big|_{t=52} = 155.6957 \ \frac{\text{kg salt}}{\text{m3 sol}^n.}$$

Volume of solution in the tank at $t=0$

$= \pi R^2 L$, $R = 20$ m and $L = 126$ m

$= 158{,}336.27$ m³. This volume is assumed unchanged.

ⓐ

Balance on salt in the cavern

$$\text{Input} + \underset{\hookrightarrow 0}{\text{Gen}} = \text{Output} + \text{Accum}$$

$$\rho_i Q_i + k_c A(\rho_s|_{sat} - \rho_s) = \rho_s Q_o + \frac{d}{dt}(\forall \rho_s)$$

The unit for k_c is m/s and $\rho_s(t)$

Re-arrange

$$(\rho_i Q_i + k_c A \rho_s|_{sat}) = (Q_o + k_c A)\rho_s + \forall \frac{d\rho_s}{dt}$$

or $\frac{d\rho_s}{dt} = \beta - \alpha \rho_s$ where $\beta = \frac{\rho_i Q_i + k_c A \rho_s|_{sat}}{\forall}$

and $\alpha = \frac{Q_o + k_c A}{\forall}$

At $t=0$, $\rho_s = 309.824 \frac{kg_s}{m^3 \text{ solution}}$

and at $t = 20.5$ hrs $= 7.38(10^4)$ s

$\rho_s = 155.6957 \frac{kg\ s}{m^3\ \text{sln}}$

Integrate eq.

$$\int_{309.824}^{156.6957} \frac{d\rho_s}{\beta - \alpha\rho_s} = \int_0^{7.38(10^4)} dt$$

$$\ln\left[\frac{\beta - \alpha(155.6957)}{\beta - \alpha(309.824)}\right] = -\alpha(7.38)(10^4)$$

A solution to this is given by $\alpha = 0$

This will give $k_c < 0$. This is the trivial answer.

The value of β is irrelevant.

Now obtain expressions for α and β, given

$Q_i = Q_o = 2\ m^3/s$, $\rho_s\big|_{sat} = 0.263(1202.1) = 316.1286\ \frac{\text{kg salt}}{m^3\ \text{soln}}$

$$\alpha = \frac{2}{158336.27} + \frac{\pi(40)(126)k_c}{158336.27}$$

$$= 1.2631(10^{-5}) + 0.1\,k_c$$

$$\beta = \frac{27.2272(2)}{158336.27} + \frac{\pi(40)(126)(316.128)\,k_c}{158336.27}$$

$$= 3.4392(10^{-4}) + 31.6128\,k_c$$

On substituting these into the equations

$$\ln\left\{\frac{-1.622672(10^{-3}) + 16.0432\,k_c}{-3.569467(10^{-3}) + 0.63028\,k_c}\right\} = -\alpha t$$

where $t = 7.380(10^4)$ s

If $k_c = 3(10^{-5})$ m/s

$$\ln(0.312464) = -1.13487 = -(1.5361)(10^{-5})\,t$$

or $t = 7.388(10^4)$ s

Thus k_c is $\sim 3.002(10^{-5})$ m/s

$\longrightarrow$

ⓑ The amount of salt recovered in 20.5 hrs, W, can be estimated from

$$W = \int_0^t k_c A\left(\rho_s\big|_{sat} - \rho_s(t)\right) dt$$ → what is dissolved. Then add storage change.

or

$$W = \int_0^t Q\rho_s(t)\,dt - Q_i\rho_i t$$ net salt flow, out − in

The general $\rho_s(t)$ equation is obtained from

$$\int_{\rho_{s0} = 309.824}^{\rho_s} \frac{d\rho_s}{\beta - \alpha\rho_s} = \int_0^t dt$$

or

$$\ln\left\{\frac{\beta - \alpha\rho_s}{\beta - \alpha\rho_{s0}}\right\} = -\alpha t$$

$$\rho_s(t) = \frac{1}{\alpha}\left\{\beta - (\beta - \alpha\rho_{s0})e^{-\alpha t}\right\}$$

Using

$$W = Q\left\{\int_0^t \rho_s(t)\,dt - \rho_i t\right\}$$ where $Q = Q_i = Q_o$

Substitute $\rho_s(t) = \delta - \varepsilon e^{-\alpha t}$ with $\delta = \dfrac{\beta}{\alpha}$ and $\varepsilon = \delta - \rho_{s0}$

$$W = Q\left\{\int_0^t (\delta - \varepsilon e^{-\alpha t})\,dt - \rho_i t\right\}$$

$$= Q\left\{\left(\delta t + \frac{\varepsilon}{\alpha}e^{-\alpha t}\right)\Big|_0^t - \rho_i t\right\}$$

$$= Q\left\{\left[\delta t + \frac{\varepsilon}{\alpha}\left(e^{-\alpha t} - 1\right)\right] - \rho_i t\right\}$$

Insert values

$\alpha = 1.5633(10^{-5})$, $\beta = 1.29294(10^{-3})$ with k_c calculated in part (a)

$\delta = 82.7056$, $\xi = -227.1184$ and $\left\{ \begin{array}{l} t = 7.38(10^4)\,s \\ Q = 2\ m^3/s \end{array} \right.$

$W = 28{,}078{,}723.4276$ kg salt mined $\longrightarrow$

The volume of the salt / $= \frac{W}{2{,}160} = 12{,}999.4\,m^3 = \forall_r$ and all of this was derived from the side wall.

The volume, $\forall_r = \pi d h . \eta$ where η is the thickness dissolved.
(This equation is valid only if $\eta << d$)

$d = 40m$, $h = 126m$ $\Rightarrow$ $\eta = 0.821m$ $\longrightarrow$

Check: i. $\eta/d = \frac{0.821}{40} = 0.0205 << 1$

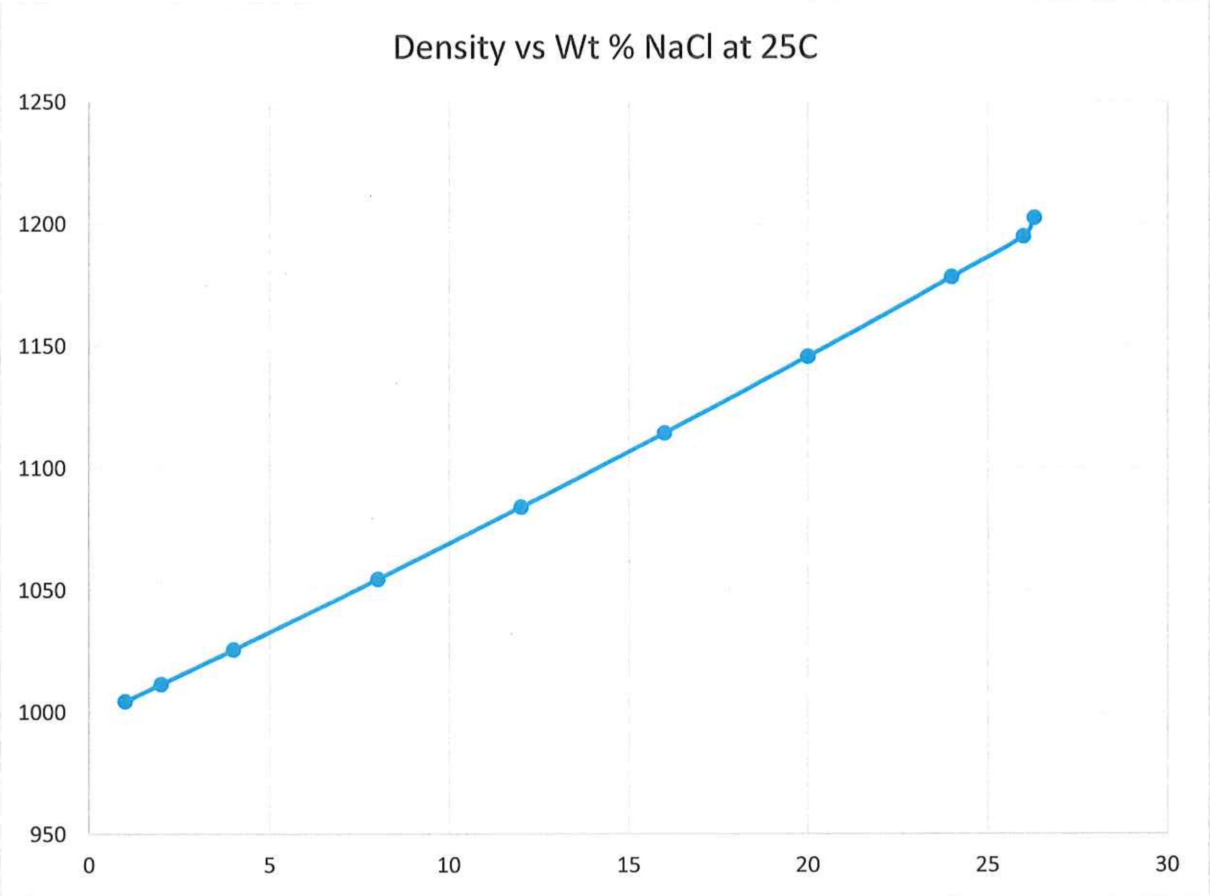
ii. Assumption that cavern volume $\simeq$ const

$$\frac{\Delta\forall}{\forall_0} = \frac{\forall_r}{V_0} = \frac{12{,}999.4}{158{,}336.27} = 0.0821 < 1$$

The assumptions are OK for first calculations but the answers can be refined.

The rate of dissolution and recovery is probably much higher than the commercial operation. Convection currents would not be too strong in the large volume & attempts will be made to regulate the salt production rates.

% Wt Salt	Density.kg m-3
1	1004.09
2	1011.12
4	1025.3
8	1054.12
12	1083.65
16	1114.01
20	1145.33
24	1177.76
26	1194.43
26.3	1202.01



\#2 $Q = f(D, c_p, k_s, u, \rho, \mu, \Delta T)$

$\exists$ 8 variables and 4 dimensions – M, L, t and T
Hence there are 4 dimensionless groups.

- By inspection, 2 are $\frac{c_p \mu}{k_s}$ and $\frac{Du\rho}{\mu}$ $(= Re)$
- Remove 2 variables – eg c_p and u, one from each dimensionless groups identified.

$$Q = g(D, k_s, \rho, \mu, \Delta T)$$

units	$\frac{J}{s}$	m	W/mK	kg/m^3	Pa.s	K
Dimensions	$\frac{ML^2}{t^3}$	L	$\frac{ML}{t^3 T}$	$\frac{M}{L^3}$	$\frac{M}{Lt}$	T

Choose repeating variables – $D, \rho, k_s, \Delta T$

$\Pi_1 = D^a \rho^b k_s^c \Delta T^d \mu^e$ and $\Pi_2 = D^f \rho^g k_s^h \Delta T^i Q^j$

$$\Pi_1 = L^a \left[\frac{M}{L^3}\right]^b \left[\frac{ML}{t^3 T}\right]^c [T]^d \left[\frac{M}{Lt}\right]^e$$

$$\left.\begin{array}{lll} L & 0 = & a - 3b + c - e \\ M & 0 = & b + c + e \\ t & 0 = & -3c + e \\ T & 0 = & -c + d \end{array}\right\} \begin{array}{l} a = 2c \\ b = 2c \\ d = c \\ e = -3c \end{array} \qquad \Pi_1 = \left[\frac{D^2 \rho^2 k_s \Delta T}{\mu^3}\right]^c$$

$$\Pi_1 / Re^2 = \left[\frac{k_s \Delta T}{\mu u^2}\right]$$

$$\Pi_2 = L^f \left[\frac{M}{L^3}\right]^g \left[\frac{ML}{t^3 T}\right]^h [T]^i \left[\frac{ML^2}{t^3}\right]^j$$

$$\left.\begin{array}{lll} L & 0 = & f - 3g + h + 2j \\ M & 0 = & g + h + j \\ t & 0 = & -3h - 3j \\ T & 0 = & -h + i \end{array}\right\} \begin{array}{l} f = h \\ b = 0 \\ j = -h \\ i = h \end{array} \qquad \Pi_2 = \left[\frac{D k_s \Delta T}{Q}\right]^h$$

$$Q / D k_s \Delta T = F\left(\frac{c_p \mu}{k}, \frac{Du\rho}{\mu}, \frac{k_s \Delta T}{\mu u^2}\right) \longrightarrow$$