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The University of Calgary | Department of Chemical & Petroleum Engineering | Oct 30

ENCH 501: Transport Phenomena

Mid-Term Examination, Fall 2012

Instructions: Time: 2 to 3.30 pm
Attempt All Questions. Open Notes & Book. Use of calculators permitted.

Problem #1 (9 points)

A technique for industrial-scale cooking is being adopted for the home. Cooking ***"sous vide"*** or under vacuum at low temperatures is suggested to preserve the taste and nutrients in food. For example, meats cooked at 55°C for hours are tender and tasty. The item (with spices and other ingredients) is tightly wrapped at reduced pressure in a sealed bag and placed in water at a controlled constant temperature. A 1.5 kg piece of beef was cooked for 10 hours with great results.

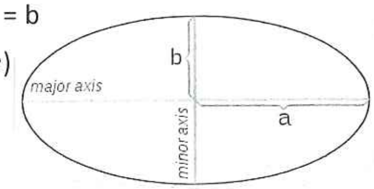
Using the same device and method, a 10 kg piece of roast beef is to be cooked. The density of beef (Jarvis, J. Food Tech. 6, 1971, p383-391) is assumed constant at 1067 kg/m^3. Both pieces of meat are assumed to be shaped as oblate spheroids with the vertical semi-axis equal to half the horizontal semi-axis. The water is pre-heated to 55°C before the meat is immersed. The convective heat transfer coefficient (h) around the meat is not known but is assumed constant and low, irrespective of the size of the item, and h(V/A)/k is less than 0.1. The heat capacity C_p is also not given but assumed to be constant. Both pieces of meat were initially at 5°C and the smaller piece reached a temperature of 50°C when t = 2 hours. The extra time for cooking after the meat reached 50°C was to allow the connective tissue to disintegrate and the meat soften.

a) Estimate the time for the larger piece of meat to cook like the smaller piece. Show your analysis.
b) (**Bonus** - 2 pts) How would your analysis be modified if, instead of meat, you were given lumps of hard oilsands to disintegrate for testing in a laboratory? You may assume the same masses and conditions as above for two pieces. The volume fractions of sand (74%), bitumen (16%), water (6%) and air (4%) are known. The densities and heat capacities of each of the components are available.

Data: Oblate Spheroid – major semi-axis = a; minor semi-axis = b

Volume, $V = \frac{4}{3}\pi a^2 b$; Area, $A = 2\pi a^2 (1 + \{\frac{1-e^2}{e}\} tanh^{-1}\, e)$

where $e^2 = 1 - b^2/a^2$



Problem #2 (8 points)

Surfing is a major international water sport. The surfer normally rides waves near beaches adjacent to large bodies of water such as while standing on board typically made of foam and coated with fibreglass. The boards are of different shapes and sizes. Long boards are ≥ 2.4m and have rounded noses while short boards are 1.5 to 2.1 m long and have pointed noses. Boards are 45 to 60 cm wide.

A good surfer is observed riding a wave at 32 km/hour on a rectangular long board that is 2.4 m long and 50 cm wide. The current in the water under the board is in the same direction at 30 km/hour. If the density and viscosity of the sea water at 10°C at the time is given as 1026 kg/m^3 and 1.88 mPa s respectively, use the integral method (show only important steps and state assumptions) to

a) (5 pts) determine the drag on the board, and
b) (3 pts) estimate the boundary layer, displacement and momentum thicknesses at a distance of half the length of the board from the leading edge.

Problem #3 (8 points)

Bitumen is to be upgraded by passing through a bed of spherical catalyst particles at a high rate. The density of the oil **ρ** is 950 ± 8 kg/m^3 and the viscosity **μ** is 12.6 ± 0.2 mPa s. The bitumen flow rate is to be 550 ± 6 kg/s. The diameter of the catalyst particles $\mathbf{D_p}$ is 6 ± 0.1 mm, the length of the bed **L** is given as 0.9 ± 0.05 m and the diameter of the reactor **D** is 1.2 ± 0.02 m. The bed porosity **ε** is 0.37 ± 0.01. All these measurements were obtained from independent sources.

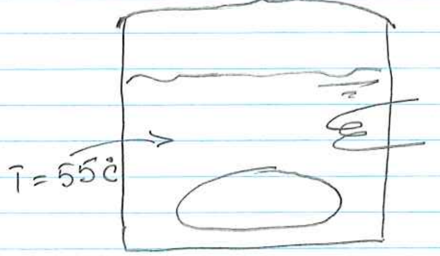
Use the **Blake-Plummer** equation:

$$\left[\frac{\Delta P \rho}{G_o^2}\right]\left[\frac{D_P}{L}\right]\left[\frac{\varepsilon^3}{1-\varepsilon}\right] = 1.75$$

to estimate the pressure drop (**ΔP**) across the bed and the error in the value. The mass velocity $\mathbf{G_o}$ equals $\mathbf{\rho V_o}$ where $\mathbf{V_o}$ is the superficial velocity for the oil (volume rate divided by the cross sectional area of empty reactor). Show your steps.

ENCH 501 Midterm Exam Solution October 30, 2012

\#1



This is a lumped capacity problem because it is given that h is low and

$$\frac{h\,(\forall/A)}{k} < 0.1$$

Choose the meat as the control volume. It has an initial temperature of T_0. The temperature of the bath is T_α.

Energy balance

$$\text{Input} + \underbrace{\text{Gen}}_{0} = \underbrace{\text{Output}}_{0} + \text{Accum.}$$

$$hA(T_\alpha - T) \qquad\qquad \frac{d}{dt}\left[m c_p (T - T_0)\right]$$

or

$$hA(T_\alpha - T) = m c_p \frac{dT}{dt}$$

$$\int_{T_0}^{T} \frac{dT}{T - T_\alpha} = -\frac{hA}{m c_p}\int_0^t dt$$

$$\ln \frac{T - T_\alpha}{T_0 - T_\alpha} = -\frac{hA}{m c_p}\,t = -\frac{hA}{\rho \forall c_p}\,t$$

Define $\beta = -\dfrac{h}{\rho c_p}$ $\quad\therefore\quad$ $\ln \dfrac{T - T_\alpha}{T_0 - T_\alpha} = -\beta \dfrac{A}{\forall}\, t$

Now $\forall = \dfrac{m}{\rho} = \dfrac{4}{3}\pi a^2 b \; ; \; b = \dfrac{1}{2} a$

$$\therefore \quad \frac{m}{\rho} = \frac{2}{3}\pi a^3$$

For the small piece, $\frac{1.5}{1067} = \frac{2}{3}\pi a^3 \Rightarrow a = 0.087556$ m

or 8.76 cm

$$e^2 = 1 - \tfrac{1}{4} = \tfrac{3}{4} \Rightarrow e = 0.866025$$

$$\tanh^{-1} e = 1.316958$$

$$A = 2\pi (0.087556)^2 \left(1 + \frac{0.25}{0.866025}(1.316958)\right)$$

$$= 0.0664792 \text{ m}^2$$

$$\frac{A}{V} = \frac{0.0664792(1067)}{1.5} = 47.28882 \text{ m}^{-1}$$

Substitute at t = 2 hrs or 7200 s

$$\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = \ln\left(\frac{50-55}{5-55}\right) = \ln\left(\frac{1}{10}\right) = \beta\left(\frac{A}{V}\right)t$$

$$\beta = -6.762772\,(10^{-6})$$

For the large piece,

$$\frac{10}{1067} = \frac{2}{3}\pi a^3 \Rightarrow a = 0.16479 \text{ m.}$$

$$A = 2\pi(0.16479)^2\left(1 + \frac{0.25}{0.866025}(1.316958)\right)$$

$$= 0.23549 \text{ m}^2$$

$$\frac{A}{V} = \frac{0.23549(1067)}{10} = 25.126 \text{ m}^{-1}$$

$$\therefore \quad \ln\left(\frac{1}{10}\right) = \beta\left(\frac{A}{V}\right)t = -6.76277(10^{-6})(25.126)\,t$$

$$t = 13550.8 \text{ s} \quad \text{or} \sim 3.764 \text{ hrs}$$

This is a longer time than for the small piece.

The total time to cook the larger piece is

11.764 hrs or ~ 11 hrs and 46 mins →

The time to soften the meat (8 hours) is the same for both cases.

BONUS Question.

Lumps of hard oil sands.

The basic difference is the accumulation term

This would be

$$\left(m_1 C_{p_1} + m_2 C_{p_2} + m_3 C_{p_3} + m_4 C_{p_4}\right)\frac{dT}{dt}$$

where $m_i = \rho_i V_i = \rho_i \phi_i V$ where ϕ_i is volume fraction

$i = 1, \cdots 4$ and V is total volume

i - are the components of the oilsand lump.

All the other steps for the analysis remain the same.

Quick answer *

For the problem, to attain 50°C from 5°C

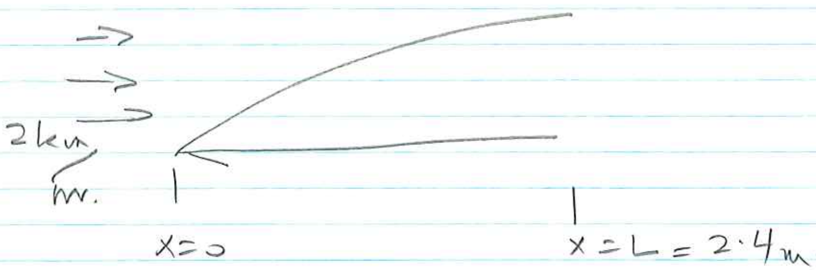
$$\frac{h}{C_p}\left(\frac{A_1}{\rho V_1}\right)t_1\bigg|_{1.5\text{ kg meat}} = \frac{h}{C_p}\left(\frac{A_2}{\rho V_2}\right)t_2\bigg|_{10\text{ kg meat}}$$

since $m = \rho V$

Calc A_1 + A_2

Subst t_1, m_1 + m_2 to get t_2. →

2.



The relative vel. = 2 km/hr or $0.55\dot{5}$ m/s

check: is b.l. laminar?

$$Re_L = \frac{(2.4)(0.556)(1026)}{1.88(10^{-3})} = 7.277(10^5)$$

This is > $5(10^5)$. Hence a portion of the boundary layer will be turbulent. The problem is solved as if entire b.l. is laminar.

(a) From Notes:

$$\text{Drag}, \quad D = \int_0^L W \, \tau\Big|_{y=0} \, dx \; ; \quad \tau\Big|_{y=0} = \mu \frac{du}{dy}\Big|_{y=0}$$

$$\tau = \frac{\mu U_\infty}{\delta} \cdot \frac{3}{2} \; ; \quad \delta = 4.64 \sqrt{\frac{\nu x}{U_\infty}}$$

$$D = \frac{W \mu U_\infty}{\beta} \cdot \frac{3}{2} \int_0^L x^{-\frac{1}{2}} \, dx \; ; \quad \beta = 4.64 \sqrt{\frac{\nu}{U_\infty}}$$

$$= \frac{3}{4.64} \frac{W \mu U_\infty^{3/2} L^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}$$

Substitute values —

$$D = \frac{3}{4.64} \frac{(0.5)(1.88)(10^{-3})(0.5556)^{\frac{3}{2}}(2.4)^{\frac{1}{2}}}{\left[1.8324(10^{-6})\right]^{\frac{1}{2}}}$$

$$= 0.288 \text{ N}$$

(b)

Boundary Layer thickness

$$\delta = 4.64 \sqrt{\frac{\nu x}{U_\alpha}} \quad ; \quad x = 1.2 \text{ m}$$

$$= 4.64 \sqrt{\frac{(1.8324)(10^{-6})(1.2)}{0.5556}}$$

$$= 0.009231 \text{ m} \quad \text{or} \quad 9.23 \text{ mm}$$

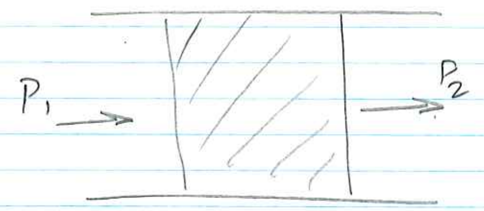
Displacement thickness, $\delta_1 \doteq \frac{3}{8}\delta = 0.00346 \text{ m}$

or 3.46 mm

Momentum thickness, $\delta_2 \doteq 0.1392\,\delta = 0.001285 \text{ m}$

or 1.285 mm

#3



$$\Delta P = P_1 - P_2$$

$$A = \frac{\pi D^2}{4}$$ cross-sectional area of reactor.

Blake - Plummer equation

$$\left(\frac{\Delta P \rho}{G_0^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 1.75 \quad ; \quad G_0 = \rho V_0$$

$$\boxed{G_0 = \frac{\dot{m}}{A}}$$ substitute

$$f = \Delta P = (1.75)\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left(\frac{L}{D_p}\right)\frac{1}{\rho}\frac{\dot{m}^2}{(\pi D^2/4)^2}$$

$$= \frac{(1.75)16}{\pi^2}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\frac{L}{D_p}\frac{1}{\rho}\frac{\dot{m}^2}{D^4}$$

$$\Delta P = 2.836993\left(\frac{1-0.37}{0.37^3}\right)\left(\frac{0.9}{6(10^{-3})}\right)\frac{1}{950}\frac{550^2}{(1.2)^4}$$

$$= 812{,}758.86 \ \text{Pa}$$

or 8.0213 atm. $\left(1\,\text{atm} = 101.325\ \text{kPa}\right)$

Error Estimation - method of propagation, for independent variables.

$$\Delta f = \Delta(\Delta P) = \left[\left(\frac{\partial f}{\partial \varepsilon}\right)^2(\Delta\varepsilon)^2 + \left(\frac{\partial f}{\partial L}\right)^2(\Delta L)^2 + \left(\frac{\partial f}{\partial D_p}\right)^2(\Delta D_p)^2 + \left(\frac{\partial f}{\partial \rho}\right)^2(\Delta\rho)^2 + \left(\frac{\partial f}{\partial \dot{m}}\right)^2(\Delta\dot{m})^2 + \left(\frac{\partial f}{\partial D}\right)^2(\Delta D)^2\right]^{\frac{1}{2}}$$

$$\frac{\partial f}{\partial \varepsilon} = \beta \frac{L}{D_p} \frac{1}{\rho} \frac{\dot{m}^2}{D^4} \frac{d}{d\varepsilon}\left(\frac{1-\varepsilon}{\varepsilon^3}\right) \quad ; \quad \beta = 2.836993$$

$$= \beta\left(\frac{L}{D_p}\right)\frac{1}{\rho}\frac{\dot{m}^2}{D^4}\left(\frac{-\varepsilon^3 - 3(1-\varepsilon)\varepsilon^2}{\varepsilon^6}\right) = \frac{\beta L \dot{m}^2}{D_p \rho D^4}\left(\frac{2\varepsilon - 3}{\varepsilon^4}\right)$$

$$= \frac{(2.836993)(0.9)(550)^2}{6(10^{-3})(950)(1.2)^4}\left(\frac{2(0.37) - 3}{0.37^4}\right) = -7.88\,(10^6)$$

$$\frac{\partial f}{\partial L} = \beta\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\frac{1}{D_p \rho}\frac{\dot{m}^2}{D^4}$$

$$= (2.836993)\left(\frac{1-0.37}{0.37^3}\right)\frac{1}{6(10^{-3})(950)}\frac{550^2}{(1.2)^4}$$

$$= 9.0307\,(10^5)$$

$$\frac{\partial f}{\partial D_p} = \beta\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\frac{L}{\rho}\left(-\frac{1}{D_p^2}\right)\frac{\dot{m}^2}{D^4}$$

$$= (2.836993)\left(\frac{1-0.37}{0.37^3}\right)\left(\frac{0.9}{950}\right)\left(-\frac{1}{6^2(10^{-6})}\right)\frac{(550)^2}{(1.2)^4}$$

$$= -1.3546\,(10^8)$$

$$\frac{\partial f}{\partial \rho} = \beta\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\frac{L}{D_p}\left(-\frac{1}{\rho^2}\right)\frac{\dot{m}^2}{D^4}$$

$$= 2.836993\left(\frac{1-0.37}{0.37^3}\right)\frac{0.9}{6(10^{-3})}\left(-\frac{1}{950^2}\right)\frac{550^2}{(1.2)^4}$$

$$= -8.5552\,(10^2)$$

$$\frac{\partial f}{\partial \dot{m}} = \beta\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left(\frac{L}{D_p}\right)\frac{2\dot{m}}{\rho D^4}$$

$$= 2.836993\left(\frac{1-0.37}{0.37^3}\right)\left(\frac{0.9}{6(10^{-3})}\right)\frac{2(550)}{950(1.2)^4}$$

$$= 2.9555(10^3)$$

$$\frac{\partial f}{\partial D} = \beta\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\frac{L}{D_p}\frac{1}{\rho}\dot{m}^2\left(-\frac{4}{D^5}\right)$$

$$= 2.836993\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left(\frac{0.9}{6(10^{-3})}\right)\frac{1}{950}(550)^2\left(-\frac{4}{(1.2)^5}\right)$$

$$= -2.7092(10^6)$$

$$\therefore \quad \Delta f = \sqrt{\Big\{\underbrace{\left(-7.88(10^6)\right)^2}_{\text{largest}}(0.01)^2 + \left(9.0307(10^5)\right)^2(0.05)^2}$$

$$+ \left(-1.3546(10^8)\right)^2(10^{-4})^2 + \left(-8.5552(10^2)\right)^2(8)^2$$

$$+ \left(2.9555(10^3)\right)^2(6)^2 + \left(-2.7092(10^6)\right)^2(0.02)^2\Big\}$$

$$= \sqrt{1.172898(10^{10})}$$

$$\Delta(\Delta P) = 1.083(10^5)\ \text{Pa} \quad \text{or} \quad 1.0688\ \text{atm}$$

$$\therefore \quad P = 8.0213 \pm 1.0688\ \text{atm}$$