C.J.

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena

Mid-Term Examination, Fall 2009

Instructions: Time: 2:00 to 3:30 pm Oct 27, 2009
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

Problem #1 (15 points)

Weather balloons carry instruments aloft to measure atmospheric pressure, temperature, relative humidity, ozone concentrations and particulate densities. The measuring device, a radiosonde, sends data to the station every 2 seconds and it is expendable. About 900 balloons are released around the globe, every 12 hours, as the primary source of data for computer weather forecast models. The hundreds of aircrafts in the air at anytime provide secondary data. Balloons are generally made of latex rubber which expands as the pressure outside drops during the rise of the balloons. Ultimately, at heights up to 30 km above sea level, the balloons burst. A parachute deploys and the instruments float down. Many radiosonde units are recovered and reused. For this problem, an experimental balloon is being tested. The balloon is a 2 m diameter sphere made of a hard, thin-walled, reinforced plastic shell that would not expand or shrink. It is filled with hydrogen gas at sea level in the "standard atmosphere", i.e. the gas temperature is 15°C, the pressure is 1atm and the density is 0.085 kg/m^3. Other properties of the standard atmosphere and other data are given below.

The shell has a mass of 0.75kg. The gas in the sphere is assumed to be well-mixed always such that there is no temperature gradients inside. The temperature of the shell and the hydrogen gas are also assumed to be the same at any instant. The balloon's rate of rise is controlled and maintained at a steady rate of 12 m/min.

a) How high will the balloon rise above sea level in the standard atmosphere?
b) If an instrument inside the balloon reported the temperature of the gas in the balloon to be -11°C as soon as it attains tha maximum altitude in part (a), estimate the heat transfer coefficient at the exposed surface of the balloon during the ascent.

Data:
For the "standard atmosphere": given a distance z above sea level,
the zone $0<z\leq 11.019$ km is isentropic. Temperature drops linearly with height at a rate of 6.49°C/km.
In this zone, the air pressure (P) and density (ρ) are obtained from the following equations:

$$\frac{P}{P_o} = \left[1 - \frac{gz}{RT_o}\left(\frac{n-1}{n}\right)\right]^{n/_{n-1}} \quad ; \text{and} \quad \rho = \left(\rho_o^n \frac{P}{P_o}\right)^{1/_n}$$

where n = 1.2345, P_o = 1 atm, T_o = 288.15K, R = 287 J/kgK, ρ_o= 1.209 kg/m^3 and g = 9.81 m/s^2.

The heat capacities for hydrogen and the plastic for the shell are respectively 14.2 and 1.6 kJ/kg K.

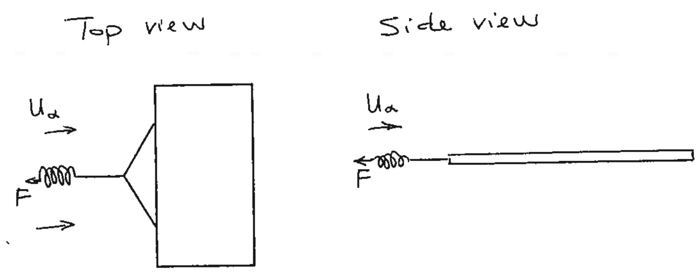
Problem #2 (10 points)

An engineer has been given the task of estimating the run-off rates of water from an oil sands surface mining facility. The water collects and runs as a riverlet in which droplets of bitumen are suspended. The riverlet then discharges into the Athabasca river. The interest is to estimate how much bitumen is being lost and whether the contamination of the Athabasca river is significant. The riverlet runs through a canal with steep side walls. The contour of the riverlet bed and the cross-sectional area are known. The volume rate of the run-off is the average velocity times the cross-sectional area. The average velocity is the flow speed at most of the cross-section, except near the wall.

The engineer has only a submersible spring balance and a rigid, flat sheet of plastic that is 40 cm by 30 cm. The sheet was fully submerged in the flowing stream at 5°C, at a location far from the wall. The water flow was normal to the 40cm side, and the flow is parallel to the surfaces. The spring balance was upstream of the sheet in a horizontal position and attached to the sheet bya loop of a string (see sketch). The engineer recorded a force of 0.912 N.

a) Estimate the volume flow rate if the cross-section of the riverlet is 1.4 m². Use the integral method.
b) What the force that would have been recorded by the spring balance if the 30 cm side had been normal to the flow?

Data:
Properties of the water at 5°C: $\rho = 999.8\ kg/m^3$; $\mu = 1.52$ mPa s



Standard Atmosphere

(a) $y = 0$, $P = 101.325\ kPa$, $T = 15°C$, $\rho = 1.209\ kg/m^3$

(b) $0 < y \le 11.019$ km, isentropic, lapse rate = - 6.49° C /km n = 1.2345

(c) 11.019 km < y ≤ 20.063 km, isothermal, T = -56.5° C

(d) 20.063 km < y ≤ 32.156 km, isentropic, lapse rate = 0.992 ° C/ km n = 0.97177

Example Estimate the pressure and the air density at an elevation of 15 km in the standard atmosphere.

Solution $0 < y \le 11.019\ km$, isentropic zone

$$\frac{P}{P_0} = \left[1 - \frac{g\,y}{R\,T_0}\left\{\frac{n-1}{n}\right\}\right]^{\frac{n}{n-1}}$$

$$P_0 = 1atm,\quad n = 1.2345,\quad R = 287\ J/kg\ K \text{ for air},\quad T_0 = 288.15\ K$$

$$P_1 = P\,|_{y=11\,km} = 0.2226\,atm,\quad \rho_1 = \left[\rho_0^n\,\frac{P_1}{P_0}\right]^{1/n} = 0.358\ kg/m^3$$

$11.019\ km < y \le 15\ km$, isothermal zone

$$T = -56.5°C = 216.65\ K$$

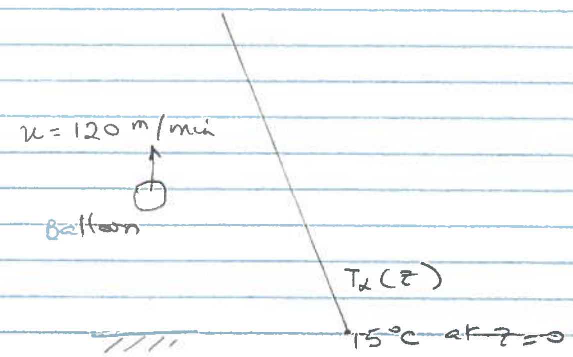
$$\frac{P}{P_1} = \exp\left[-g\,\frac{(y-y_1)}{R\,T}\right],\quad y_1 = 11019\ m$$

$$\frac{P_2}{0.2226} = \exp\left\{-\frac{9.81\ (3981)}{287\ (216.65)}\right\}$$

$$\text{or}\quad P_2 = 0.1188\ atm \text{ and } \rho_2 = \left[\rho_1\,\frac{P_2}{P_1}\right] = 0.191\ kg/m^3$$

ENCH 501 Midterm Solution Oct. 27, 2009

#1



Consider the balloon.

The volume occupied by

$H_2 = \frac{4}{3}\pi R^3$; $R = 1 m$

$V_{H_2} = 4.1888 \ m^3$

The mass of hydrogen

$= \rho_{H_2} V_{H_2}$

$= (0.085)(4.1888)$

$= 0.356 \ kg$

The mass of the shell

$= 0.75 \ kg$

$\therefore$ Total payload to be lifted

$m = 0.356 + 0.75 = 1.106 \ kg$

The weight of the balloon (Force of gravity) $= m \cdot g = 1.106 \times g$ N

At sea level, the buoyancy force on the balloon

$$= \rho_{air_{z=0}} V_{shell} \cdot g = (1.209)(4.1888) g$$

$$= 5.0643 g \quad N \qquad (g = 9.81 \ m/s^2)$$

The buoyancy force >> weight of balloon at sea level and the balloon has lift.

(a) The balloon stops rising at an elevation where the total payload equals the buoyancy:

i.e.,
$$1.106 \cdot \cancel{g} = \rho_{air_z} \cdot (4.1888) \cancel{g}$$

$$\rho_{air_z} = 0.264 \ kg/m^3.$$

Since z is unknown, it is not clear ~~if the~~ balloon stops rising in the isentropic or the isothermal zone. The first step is to calculate the air density at 11.019 km height in the standard atmosphere. Use

$$\frac{P_1}{P_0} = \left[1 - \frac{gz_1}{RT_0}\left(\frac{n-1}{n}\right)\right]^{\frac{n}{n-1}}$$

$R = 287\ J/kg\,K$

$P_0 = 1atm$

$T_0 = 288.15K$

$n = 1.2345$

when $z_1 = 11019\ m$,

$P_1 = 0.2226\ atm$

Substitute into the second equation

$$\rho_1 = \left(\rho_0^n \frac{P_1}{P_0}\right)^{1/n} \quad ; \quad \rho_1 = 0.358\ kg/m^3$$

This is higher than the required density. The balloon will rise into the isothermal zone.

Use the pressure - density relationship

$$\rho_2 = \rho_1 \frac{P_2}{P_1} \Rightarrow 0.264 = 0.358 \frac{P_2}{0.2226}$$

$\therefore P_2 = 0.1642\ atm$

Obtain the elevation at point (2) where balloon stops rising

$$\frac{P_2}{P_1} = exp\left[-g\frac{(z_2 - z_1)}{RT}\right] = \frac{0.264}{0.358} = 0.7374$$

$$z_2 - z_1 = 1930.8m$$

$$\therefore z_2 = 11019 + 1930.8\ m$$

$$= 12949.8m \text{ above sea level.}$$

This is in the isothermal zone →

(b) Energy balance on the sphere - hydrogen (1) and shell (2)

Input + Generation = Output + Accum.

$$\underset{0}{\text{Input}} + \underset{0}{\text{Generation}} = \underset{hA(T-T_\alpha)}{\text{Output}} + \underset{(\sum mC_p)\frac{dT}{dt}}{\text{Accum.}}$$

where, for composite body

$$\sum m C_p = m_1 C_{p_1} + m_2 C_{p_2}$$

and $T_\alpha = T(z)$ and $T(t)$

Note that ~~t~~ is correlated to the location ~~of the~~ balloon, z. Given the steady rise velocity,

$$z = u t \quad \text{or} \quad dz = u\, dt$$

The problem needs to be solved in 2 steps —

① - in isentropic zone $T_\alpha(z) = 15 - \frac{6.49}{1000} z$; z in m

or $T_\alpha(z) = a + bz$

② - in isothermal zone $T_\alpha = -56.5°C$, a constant

In Zone 1

$$-(\sum mC_p)\frac{dT}{dt} = -(\sum mC_p)\, u \frac{dT}{dz} = hA(T - T_\alpha)$$

or

$$\frac{dT}{dz} = -\frac{hA}{(\sum mC_p)u}(T - T_\alpha) \quad ; \quad T_\alpha = a + bz$$

$$\frac{dT}{dz} = -\beta T + \varepsilon + \gamma z$$

where $\beta = \dfrac{hA}{(\sum mC_p)u}$, $\varepsilon = \beta a$; $\gamma = \beta b$

The equation can be solved and there is an integration constant. Also to be evaluated is k in β. Hence 2 conditions are required.

These are
$$\begin{cases} z = 0 & T = 15^\circ C \quad \text{inside sphere} \\ z = 11{,}019\,m, & T = T_{g_1} \quad (\text{unknown}) \end{cases}$$

The equation

$$\frac{dT}{dz} + \beta T = \varepsilon + \gamma z = f(z)$$

Use the integrating factor $e^{\int + \beta\, dz}$

$$e^{\int \beta dz} \frac{dT}{dz} + e^{\int \beta dz} \beta T = e^{\int \beta dz} f(z)$$

$$\therefore \quad T e^{\int \beta dz} = \int e^{\int \beta dz} f(z)\, dz + C_0$$

$$T e^{\beta z} = \int e^{\beta z} (\varepsilon + \gamma z)\, dz + C_0$$

$$= \frac{\varepsilon}{\beta} e^{\beta z} + \frac{\gamma e^{\beta z}}{\beta} \left(z - \frac{1}{\beta} \right) + C_0$$

$$T = \frac{\varepsilon}{\beta} + \frac{\gamma}{\beta} \left(z - \frac{1}{\beta} \right) + C_0 e^{-\beta z}$$

or $$\beta T = \varepsilon + \gamma \left(z - \frac{1}{\beta} \right) + C_0 \beta e^{-\beta z}$$

use b.c

$$\beta(15) = \varepsilon + \gamma\left(-\frac{1}{\beta}\right) + C_0 \beta \qquad \text{at } z = 0$$

$$\beta(T_{g_1}) = \varepsilon + \gamma\left(11019 - \frac{1}{\beta}\right) + C_0 \beta e^{-11019\beta} \qquad \text{at } z = 11019\text{ m.}$$

2 eq + 3 unknowns $\longrightarrow$ β, C and T_{g_1}

For part ②, $T_a = \text{const} = -56.5°C = \psi$

$$-\left(\sum m C_p\right)\frac{dT}{dt} = hA(T - T_a)$$

or
$$\frac{dT}{dz} = -\frac{hA}{\sum m C_p\, u}(T-\psi) \quad ; \quad \text{but } \beta = \frac{hA}{\sum m C_p \cdot u}$$

given $T = T_{g_1}$ at $z_1 = 11019\,m$

Solve
$$\left(\frac{T_2-\psi}{T_{g_1}-\psi}\right) = \exp\left[-\beta(z_2 - z_1)\right]$$

But $T_2 = -11°C$, $\psi = -56.5$, $z_2 = 12949.8\,m$

This is the 3rd eq
$$\left(\frac{-11+56.5}{T_{g_1}+56.5}\right) = \exp\left[-\beta(1930.8)\right]$$

Solve

(i) $$15\beta = 15\beta - 6.49(10^{-3})\beta\left(-\frac{1}{\beta}\right) + C_0\beta \quad ; \text{ at } z = 0\,m$$

$$\text{or } C_0 = -\frac{6.49(10^{-3})}{\beta}$$

(ii) $$T_{g_1}\beta = 15\beta - 6.49(10^{-3})\left(11019 - \frac{1}{\beta}\right) - \frac{6.49(10^{-3})}{\beta}\cdot\beta e^{-11019\beta} \quad ; \text{ at } z = 11019\,m$$

(iii) $$T_{g_1} = -56.5 + 45.5\exp\left[(\beta)(1930.8)\right]$$

or
$$\left(-56.5 + 45.5\exp(1930.8\beta)\right)\beta = 15\beta - 6.49(10^{-3}) \times (11019 - \frac{1}{\beta}) - 6.49(10^{-3})e^{-11019\beta}$$

(1 eq, 1 unknown)

Re-write as $f(\beta) = 0$

$$f(\beta) = -71.5\beta + 45.5\beta \exp(1930.8\beta) + 6.49(10^{-3})\left\{11019 - \frac{1}{\beta} + e^{-11019\beta}\right\} = 0$$

Choose β, calculate $f(\beta)$ and solve by trial and error. The third term in the function is dominant.

β	$f(\beta)$
10^{-4}	6.6142
$9(10^{-5})$	−0.5947
$9.05(10^{-5})$	−0.1983
$9.07(10^{-5})$	−0.04015
$9.075(10^{-5})$	−0.00073

$$\therefore \quad \beta = \frac{hA}{(\Sigma m C_p)\, u} \doteq 9.075(10^{-5})$$

$$A = 4\pi R^2 = 12.566336 \text{ m}^2$$

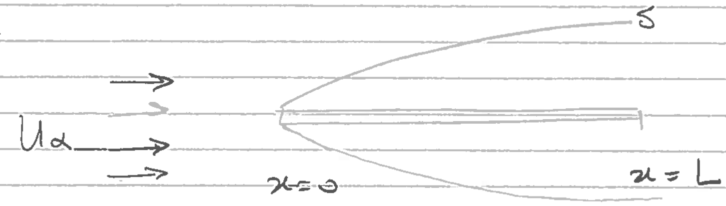
$$\Sigma m C_p = 0.356(14200) + 0.75(1600) = 6255.2 \text{ J/K}$$

$$u = \frac{12}{60} = 0.2 \text{ m/s}$$

$$\therefore \quad h = 9.03(10^{-3}) \text{ W/m}^2\text{K}$$

$\longrightarrow$

#2.



Without knowing U_α, it is not possible to establish whether the b.l. is laminar at $x \le L$. Will check later.

First obtain the equation for drag on the sheet.

From Notes, eq. 5.16, the velocity profile on each side of the plate is given by

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \quad \text{where} \quad \delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

(Eq 5.20)

$$\text{The Drag, } D = \int_0^L \tau_{y=0} \cdot W\,dx, \text{ on one side}$$

$$\text{where} \quad \tau_{y=0} = \mu \frac{du}{dy}\bigg|_{y=0} = \frac{\mu U_\alpha}{\delta}\left(\frac{3}{2}\right)$$

$$= \frac{\mu U_\alpha}{4.64}\left(\frac{U_\alpha}{\nu x}\right)^{\frac{1}{2}}$$

substitute into integral & solve

$$D = 0.6466\,(W)\left(\rho \mu U_\alpha^3 L\right)^{\frac{1}{2}} \quad \text{on one side}$$

Total drag on sheet

$$2D = 0.912 = 2(0.6466)(0.4) \cdot \left(999.8 \times 1.52(10^{-3}) \times U_\alpha^3 \times 0.3\right)^{\frac{1}{2}}$$

$$\therefore \quad U_\alpha = 1.8964 \text{ m/s}$$

$$\therefore \text{ Volume flow rate, } Q = U_\alpha A = 1.8964(1.4)$$

$$= 2.655 \text{ m}^3/\text{s} \longrightarrow$$

(b) Given the same conditions, with sheet rotated 90°

$$\text{Total drag} = 2(0.6466)(0.3)\left(999.8 \times 1.52(10^{-3}) \times 1.8964^3 \times 0.4\right)^{\frac{1}{2}}$$

$$= 0.79 \text{ N} \longrightarrow$$

Need to check whether boundary layers are laminar.

part (a) $\quad Re_x = \dfrac{L U_\infty \rho}{\mu} = \dfrac{(0.3)(1.8964)(999.8)}{1.52(10^{-3})}$

$= 3.74(10^5) \quad < \quad 5(10^{-5})$

part (b) $\quad Re_x = \dfrac{0.4(1.8964)(999.8)}{1.52(10^{-3})}$

$= 4.98(10^5)$

Hence both b.l. are laminar.