aJ

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena

Mid-Term Examination, Fall 2008

Instructions: Time: 1:30 to 3:00 pm Oct 28, 2008
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

Problem #1 (15 points)

A large, irregular shaped bronze object is heated to 130°C before it is immersed into water at 96°C to cool. Because the water is at its boiling point in Calgary, bubbles are rapidly formed and released at the surface of the object. The rate of bubble formation and thus the convective heat transfer coefficient around the object varied linearly with the instantaneous temperature difference ($\Delta T = T - T_w$) between the object and the water.

Given the data below, estimate the time required for the temperature of the object to drop to 104°C. How much water is vaporized in the process? Justify and show all your steps.

Data: The volume and surface area of the object are respectively 0.82 m^3 and 1.86 (10^4) m^2. The density of object is 8,666 kg/m^3, the thermal conductivity is 26 W/mK and the heat capacity is 0.343 kJ/kg K. Heat of vaporization of water 2,257 is kJ/kg.

ΔT, °C	h, W/m^2K
30	5.11 (10^4)
8	1.703 (10^4)

Problem #2 (10 points)

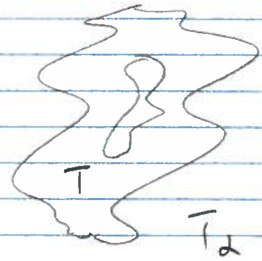
For artificial organs such as the heart or kidney, direct contact between blood and metal or plastic surfaces is to be avoided because the surfaces rapidly damage blood cells and platelets. It has been suggested that the surfaces be coated with a biofilm on which a layer of endothelial cells is grown and maintained. The blood then comes in contact only with biological material and thus stays viable. The fluid flowing over the endothelial cells, however, may shear the cells off. The strength of adhesion of the cells to the surfaces are to be investigated.

You coated the top surface of a 4 X 10 cm glass plate with a biofilm and grew endothelial cells to completely cover the surface. You placed this glass on a flat surface in a chamber through which a nutrient solution (ρ = 1025 kg/m^3; μ = 1.8 mPas) flows in a direction normal to the 4 cm side. The glass plate overhangs slightly on its base support so that the 4 cm side is the leading edge and the flow of liquid is only over the top surface. The free stream velocity is steady at 0.2 m/s. Use the **integral method** to answer the following. Show all important steps.

a) If all the cells from the leading edge to a distance of 3.2 cm are stripped off, some cells remain attached in the region from 3.2 cm to 5.8 cm and all cells adhere to the glass past 5.8 cm, estimate the range of shear stresses that the cells can withstand and remain attached.
b) What force must act to resist the glass plate sliding on its support?
c) What is the maximum free stream velocity at which no cells will be sheared off the glass plate? You may assume that the cells are 20 μm long in the flow direction.

ENCH 501 Midterm Exam Solution Oct. 28, 2008

1

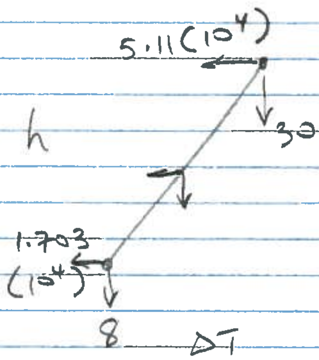


The first step is to determine whether the lumped capacity method is valid.

$$\frac{h\left(\frac{V}{A}\right)}{k} \text{ should be less than } 0.1.$$

In this case, use the highest h applicable.

From the data for h, obtain a linear relationship between h and ΔT



$$\frac{h - 1.703(10^4)}{(5.11 - 1.703)(10^4)} = \frac{\Delta T - 8}{30 - 8}$$

$$h - 1.703(10^4) = 1.5486(10^3)(\Delta T - 8)$$

$$h = 1.5486(10^3)\Delta T + 4.6409(10^3)$$

or $\quad h = a(\Delta T) + b$

For this problem, highest $\Delta T = 34°C$

$$\therefore \quad h_{max} = 5.7293(10^4) \ W/m^2 K.$$

$$\therefore \quad \frac{h\left(\frac{V}{A}\right)}{k} = \frac{5.7293(10^4)\left(\frac{0.82}{1.86(10^4)}\right)}{26} = 0.097$$

The lumped analysis method valid.

Perform an energy balance on the object.

$$\underbrace{\text{Input}}_{0} + \underbrace{\text{Generation}}_{0} = \text{Output} + \text{Accum}$$

$$0 = hA(T - T_\alpha) + mC_p \frac{d(T - T_\alpha)}{dt}$$

where the water temperature (T_α) is a constant.

Let $\theta = T - T_\alpha$ and $h = a\theta + b$

The energy equation becomes

$$0 = A\theta(a\theta + b) + mC_p \frac{d\theta}{dt}$$

or $$\int_{\theta_0}^{\theta} \frac{d\theta}{(a\theta + b)\theta} = \int_{\theta_0}^{\theta} \left\{ \frac{(1/b)}{\theta} - \frac{(a/b)}{a\theta + b} \right\} d\theta = -\frac{A}{mC_p}\int_0^t dt$$

(by partial fractions)

where $\theta_0 = T_0 - T_\alpha$ $(T_0 = 130°C)$

Integrate

$$\frac{1}{b} \ln\left(\frac{\theta}{a\theta + b}\right)\Bigg|_{\theta_0}^{\theta} = -\beta t \quad ; \quad \beta = \frac{A}{mC_p}$$

$$\ln\left(\frac{\theta}{a\theta + b}\right) - \ln\left(\frac{\theta_0}{a\theta_0 + b}\right) = -b\beta t$$

$$\ln \frac{\theta(a\theta_0 + b)}{\theta_0(a\theta + b)} = -b\beta t$$

$$\therefore \quad \frac{\theta(a\theta_0 + b)}{\theta_0(a\theta + b)} = \exp\left[-b\beta t\right]$$

or $$\frac{(T - T_\alpha)(a[T_0 - T_\alpha) + b)}{(T_0 - T_\alpha)(a[T - T_\alpha] + b)} = \exp\left[-b\beta t\right]$$

Substitute values.

$$\beta = \frac{1.86(10^4)}{(0.82)(8666)(343)} = 7.6311(10^{-3})$$

$$b = 4.6409(10^3)$$

$$a = 1.5486(10^3)$$

$T_o = 130\,°C$, $T_\alpha = 96\,°C$

and $T = 104\,°C$

$$\frac{(104-96)(1.5486(10^3)(130-96) + 4.6409(10^3))}{(130-96)(1.5486(10^3)(104-96) + 4.6409(10^3))}$$

$$= 0.7916 = \exp[-35.4152\,t]$$

$t = 0.0066$ s $\longrightarrow$ This is fast

The heat released is

$$m\,C_p\,(T_o - T_{final}) = W(\Delta H_v)$$

where W is mass of water vaporized

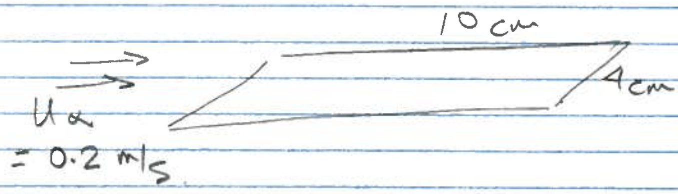
$$(0.82)(8666)(343)(130-104) = W(2.257)(10^6)$$

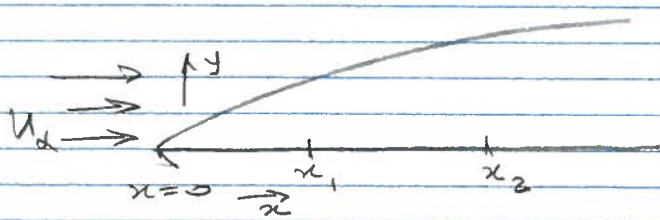
Mass of water vaporized

$$W = 28.078\ \text{kg/s}$$

This is a large value in such a short time. $\longrightarrow$

#2





This is a problem of flow over a flat plate.

From Notes, eq. 5.13, the integral momentum equation is

$$\mu \frac{du}{dy}\bigg|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho\,(U_\alpha - u)\,u\,dy\right]$$

with the boundary conditions

$y = 0 \qquad u = 0$

$y = \delta \qquad u = U_\alpha$

$y = \delta \qquad \frac{du}{dy} = 0$

and $y = 0 \qquad \frac{d^2u}{dy^2} = 0$

one obtains the velocity profile as

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

On substituting this into the integral equation and using the condition, $x = 0$, $\delta = 0$, one obtains

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

(a) The shear stress at the wall is given by

$$\tau_w = \mu \frac{du}{dy}\bigg|_{y=0} = \frac{3}{2}\mu\frac{U_\alpha}{\delta}$$

at $x = x_1 = 0.032\ m$, $\delta = 4.64\sqrt{\frac{\nu(0.032)}{0.2}}$

where $\nu = \mu/\rho = \frac{1.8(10^{-3})}{1025} = 1.756(10^{-6})$

$\therefore \delta = 2.4595(10^{-3})\ m$

$\therefore \tau_w\Big|_{x_1} = 0.2196\ N/m^2$ or Pa | maximum shear

At $x = x_2 = 0.058\ m$, $\delta = 3.3112(10^{-3})\ m$

$\tau_w\Big|_{x_2} = 0.1631\ Pa$ | ~~Minimum shear to detach~~

(b) The drag on the plate is given by

$$D = W\int_0^L \tau_w\, dx = W\int_0^L \mu\frac{du}{dy}\bigg|_{y=0} dx$$

~~where W = 0.04 m & L = 0.1 m.~~

$$D = W\int_0^L \frac{\mu U_\alpha}{\delta}\left(\frac{3}{2}\right)dx$$

$$= 0.6466\, W\left(\rho\mu U_\alpha^3 L\right)^{\frac{1}{2}}$$

$$= 0.6466(0.04)\left(1025(1.8)(10^{-3})(0.2)^3(0.1)\right)^{\frac{1}{2}}$$

$$= 9.937(10^{-4})\ N$$

c/ If no cells are to be detached, the shear stress at $x = 20(10^{-6})$ m from the leading edge must be less than the minimum shear of 0.1631 Pa of part (a).

$$\delta = 4.64\sqrt{\frac{1.756(10^{-6})(20)(10^{-6})}{U_\infty}}$$

But $$0.1631 = \frac{3}{2}\mu\frac{U_\infty}{\delta} = \frac{3}{2}\frac{(1.8)(10^{-3})\ U_\infty^{3/2}}{2.7498(10^{-5})}$$

$$U_\infty^{3/2} = 1.6611(10^{-3})$$

$$U_\infty = 0.014 \text{ m/s}$$

$\longrightarrow$