GJ

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes

Mid-Term Examination, Fall 2007

Instructions: Time: 2:00 to 3:30 pm Oct 23, 2007
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

Problem #1 (15 points)

Inadequate sterilization of fluids or components of medical devices and surgical instruments, and the significant adverse health risks to patients, have been in the news recently. Pathogenic microorganisms need to be destroyed or removed from contact lenses (e.g. *Acanthamoeba*) or from forceps, scapels, components of dialysis machines, clothes and many other surfaces which come in contact with blood in hospitals. The use of both chemical disinfectants (such as bleach and hydrogen peroxide) and heat are encouraged for sterilization. It is also desirable to remove biofilms (contaminated with viruses, mold and bacteria) which might have coated the surfaces. The problem of current interest involves heat sterilization in an autoclave which functions like a pressure cooker. The object to be sterilized has to be raised to a temperature of 121°C and held at or above this temperature for 60 minutes. Temperatures above 100°C are required to destroy *Bacillus* and *Clostridium* spores.

An autoclave for sterilizing steel surgical instruments is to be modelled. The capacity of the autoclave is 30 litres and it is heated through a steam jacket. Saturated steam at 2 atm (abs) and 123.3°C is passed into the jacket at controlled rates. Forty five (45) solid steel cylinders, each 5 cm diameter and 12.5 cm long, are placed in the autoclave which was then filled to capacity with water. At the start, both the cylinders and the water were at 16°C. Then steam was admitted into the jacket in a manner that the temperature of the water in the autoclave (T°C), recorded using a thermocouple, satisfies the following function:

$$T = 16 + \Delta T(1 - e^{-bt})$$

where ΔT is the maximum possible temperature rise for the water in the autoclave (123.3 - 16)°C, t is time in minutes and the exponent b equals 0.2135.

Given the data below and, assuming that the cylinders are stacked in such a way that each is substantially and freely exposed to the water which is well-mixed, the coefficient of convective heat transfer around each cylinder is 320 W/m^2 K,

a) how long will be required to sterilize the cylinders?
b) After how long would the temperature of the water in the autoclave reach 121°C? Comment on the results from parts (a) and (b).
c) If the autoclave is insulated, how much total condensate, at saturated condition, will have been produced from the steam jacket at the instant the cylinders are sterilized, from the start of the process?

State and justify your assumptions. Show all your steps.

Data:
Properties of steel: k = 43 W/mK, ρ = 7,801 kg/m^3, Cp = 0.473 kJ/kg K
Properties of water (avg): k = 0.654 W/mK, ρ = 980 kg/m^3, Cp = 4.179 kJ/kg K, Latent heat of vaporization at 123.3°C = 2,193 kJ/kg

Problem #2 (10 points)

A billboard, 1.5 m high and 2.5 m long, is attached to a wall of a large building. The board is loosely attached to the wall by brackets. You are concerned that a wind flowing parallel to the building at or above 10 km per hour may be strong enough to dislodge the billboard. You may assume that the air is flowing only over the exposed surface of the board in a direction parallel to the longer side.

What force must the brackets be able to withstand under these conditions? Use the ***integral method*** and assume that the velocity profile along the board satisfies

$$u = a + c \sin(by)$$

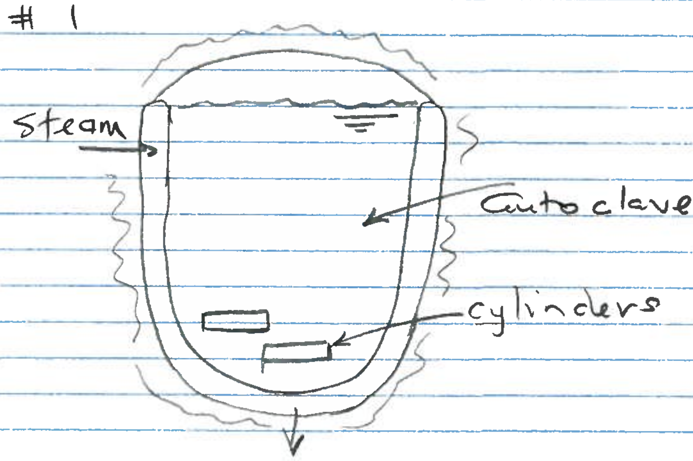
where a, b and c are constants or functions of x (distance from the leading edge along the direction of flow), and y is the direction normal to the board.

Show the important steps in your derivations.

Data:
Properties of air; $\mu = 0.01846$ mPa s; $\rho = 1.1774$ kg/m^3 ; $k = 0.02624$ W/m K ; air temperature = 27°C.

ENCH 501 Mid-Term Solution Oct. 23, 2007

#1



The focus of the problem are the cylinders.

First check whether the Lumped Analysis Method is valid.

For a cylinder

$$\frac{h\,(\forall/A)}{k} \text{ should be } < 0.1$$

$$\forall = \frac{\pi d^2}{4}\cdot h = \frac{\pi (25)(10^{-4})}{4}(12.5)(10^{-2})$$

$$= 2.4544\,(10^{-4})\ m^3$$

$$A = \pi d h + 2\frac{\pi d^2}{4} = \pi(5)(10^{-2})(12.5)(10^{-2}) + \frac{\pi(25)(10^{-4})}{2}$$

$$= 2.3562\,(10^{-2})\ m^2$$

$$\therefore \frac{320}{43}\left(\frac{2.4544}{2.3562}\right)(10^{-2}) = 0.0775 < 0.1$$

☑ $\therefore$ Assume no temp. gradients in cylinders.

Total volume occupied by cylinders

$$= 45(\forall) = 0.011045\ m^3$$

Since total volume = 30 litres or $0.03\ m^3$,

the volume of water = $0.018955\ m^3$

and its mass = $980(0.018955) = 18.5761$ kgs

With respect to each cylinder, the autoclave water is the ambient at T_α where

$$T_\alpha = 16 + (123.3 - 16)(1 - e^{-0.2135t})$$

$$\boxed{\checkmark} \quad T_\alpha(t) = 123.3 - 107.3\, e^{-0.2135t} \rightarrow \text{temp. of water in autoclave}$$

$$= a + c\,e^{-bt}$$

Use a cylinder as the control volume, energy balance is: Input = Accum

$$\text{or} \quad hA(T_\alpha - T) = mc_p \frac{dT}{dt}$$

or, on substitution for T_α

$$hA(a + ce^{-bt} - T) = mC_p \frac{dT}{dt}$$

Let $\beta = hA/mC_p$

$$\therefore \quad \beta(a + ce^{-bt}) - \beta T = \frac{dT}{dt}$$

$$\text{or} \quad \frac{dT}{dt} + \beta T = \cancel{Q(t)} = \beta(a + ce^{-bt})$$

Use integration factor $e^{\int \beta dt}$ to obtain

$$T e^{\int \beta dt} = \int e^{\int \beta dt} Q(t)\, dt + C_o$$

$$= \int e^{\beta t}(\beta a + \beta c e^{-bt})\, dt + C_o$$

$$T e^{\beta t} = a e^{\beta t} + \frac{\beta c}{\beta - b} e^{\beta t} e^{-bt} + C_o$$

$$\therefore \quad T = a + \frac{\beta c}{\beta - b} e^{-bt} + C_o e^{-\beta t} \rightarrow \text{temperature of cylinder}$$

with the unknown constant C_o determined using the condition — $t = 0$, $T = 16\,°C$

$$\beta = \frac{hA}{mc_p} = \frac{320(2.3562)(10^{-2})}{2.4544(10^{-4})(7801)(473)}$$

$$= 8.3254(10^{-3})\ s^{-1} \text{ or } 0.4995\ min^{-1}$$

$a = 123.3\ °C$

$c = -107.3\ °C$

Substitute, $C_o = +80.1088$

Hence, temperature of cylinder (°C)

$$T(t) = 123.3 - 187.409\, e^{-0.2135t} + 80.1088\, e^{-0.4995t}$$

t, min	T, °C
0	16
18	119.29 → 123.3 − 4.016 + 0.009976
25	122.4
22	121.59
21	121.18
20.5	120.95 → 123.3 − 2.355 + 0.00286
20.6	120.997 → 123.3 − 2.3052 + 0.002722

∴ total time required to sterilize cylinders

= 20.6 + 60 = 80.6 or

1 hour, 20 minutes and 36 s.

(b) From

$$T_\alpha = 123.3 - 107.3\, e^{-0.2135t}$$

When $T_\alpha = 121\ °C$, $t = 18$ mins.

Comment: We note that the water in the

autoclave attains 121°C faster than the cylinders. If it had been assumed that the water and the cylinders will always be at the same temperature, one would have specified 78 minutes for the operation. It is obvious, under given conditions, that some of the microorganisms may survive!

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(c) At $t = 80.6$ minutes

water temp, $T_{\alpha_f} = 123.3 - 107.3\, e^{-0.2135(80.6)}$

$\doteq \underline{123.3}$ °C

cylinder temp, $T_f \doteq 123.3$ °C

The mass of condensate, m, equals

$$m = \frac{m_w C_{pw}(T_{\alpha f} - 16) + m C_p (T_f - 16)}{\Delta H_v}$$

$$= \frac{18.5761(4179)(123.3-16) + 45(2.4544)(10^{-4})(7801)(473)(123.3-16)}{(2193)(10^3)}$$

$$= \frac{8{,}329{,}647.7 + 4{,}372{,}894.47}{2193(10^3)}$$

$$= 5.7923 \text{ kg}$$

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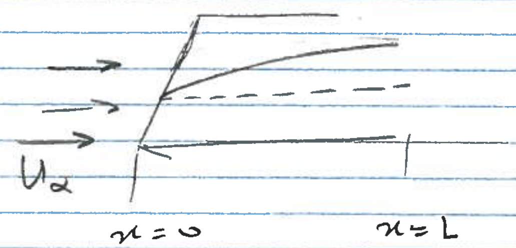
#2



Given $U_\alpha = 10$ km/hr

$= 2.778$ m/s

Check whether entire b.l. is laminar.

The b.l. becomes turbulent when

$$Re_x = \frac{U_\alpha x \rho}{\mu} = 5(10^5)$$

$$\therefore \quad 5(10^5) = \frac{(2.778)\,x\,(1.1774)}{1.846(10^{-5})}$$

or $x = 2.822$ m. This is longer than the side of the billboard. Hence the b.l. will be laminar. $\longrightarrow$

The momentum integral equation is (Eq. 5.13, Notes)

$$\mu \frac{du}{dy}\bigg|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho(U_\alpha - u)u\,dy\right]$$

with the conditions

$y=0 \quad u=0$, no slip ①

$y=\delta \quad u = U_\alpha$ ②

$y=\delta \quad \frac{\delta u}{\delta y} = 0$ ③

$y=0 \quad \frac{\partial^2 u}{\partial y^2} = 0$ ④

Given a profile –

$$u = a + c\sin by$$

$$du/dy = bc\cos by$$

$d^2u/dy^2 = -b^2 c \sin by \rightarrow$ satisfies b.c ④

Use b.c. (1) $\Rightarrow$ $a = 0$

(3) $0 = bc\cos b\delta \Rightarrow b\delta = \frac{\pi}{2} \left(\text{or } \frac{3\pi}{2} \ldots \right)$

$$b = \frac{\pi}{2}\frac{1}{\delta}$$

(2) $U_\alpha = c\sin\frac{\pi}{2}$ or $c = U_\alpha$

$\therefore$ Velocity profile is given by

$$u = U_\alpha \sin\frac{\pi}{2}\frac{y}{\delta} \quad \text{or} \quad \frac{u}{U_\alpha} = \sin\frac{\pi}{2}\frac{y}{\delta}$$

Let $\phi = \frac{u}{U_\alpha}$ and $\eta = y/\delta$

The momentum integral equation becomes

$$\frac{\mu U_\alpha}{\delta}\left.\frac{d\phi}{d\eta}\right|_{\eta=0} = \frac{d}{dx}\left[\rho U_\alpha^2 \delta \int_0^1 (1-\phi)\phi \, d\eta\right]$$

where

$$\int_0^1 \left(1 - \sin\frac{\pi}{2}\eta\right)\sin\frac{\pi}{2}\eta \, d\eta = \int_0^1 \sin\frac{\pi}{2}\eta \, d\eta - \int_0^1 \sin^2\frac{\pi}{2}\eta \, d\eta$$

$$= (-1)\frac{2}{\pi}\cos\left(\frac{\pi}{2}\eta\right)\Big|_0^1 - \frac{2}{\pi}\left[\frac{1}{2}\frac{\pi}{2}\eta - \frac{1}{2}\sin\frac{\pi}{2}\eta\cos\frac{\pi}{2}\eta\right]_0^1$$

$$= \frac{2}{\pi}\left[1 - \pi/4\right\} = 0.13662$$

and

$$\left.\frac{d\phi}{d\eta}\right|_{\eta=0} = \frac{\pi}{2}\cos\frac{\pi}{2}\eta\Big|_{\eta=0} = \pi/2$$

Substitute

$$\frac{\mu U_\alpha}{\delta}\cdot\frac{\pi}{2} = \rho U_\alpha^2 (0.13662)\frac{d\delta}{dx}$$

or $\frac{\nu}{U_\alpha}(11.4976) = \delta \frac{d\delta}{dx}$

subject to → $x = 0$, $\delta = 0$

Solve $\delta^2 = 22.9952 \frac{\nu x}{U_\alpha}$

$$\delta = 4.7953\sqrt{\frac{\nu x}{U_\alpha}}$$

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Local wall shear stress, $-\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$

$$= \frac{\mu U_\alpha}{\delta}\left(\frac{\pi}{2}\right)$$

Drag on billboard, $D = \int_0^L (-\tau_w) W\,dx$; $W = 1.5\text{ m}$

$$= W\int_0^L \frac{\mu U_\alpha}{4.7953} \cdot \frac{\pi}{2}\sqrt{\frac{U_\alpha}{\nu}}\, x^{-\frac{1}{2}}\,dx$$

$$= \frac{W\mu U_\alpha \pi}{4.7953(2)}\sqrt{\frac{U_\alpha}{\nu}}\int_0^L x^{-\frac{1}{2}}\,dx$$

$$= \frac{W\mu U_\alpha \pi}{4.7953(2)}\sqrt{\frac{U_\alpha}{\nu}} \cdot 2L^{\frac{1}{2}}$$

Substitute values,

$$D = \frac{(1.5)(1.846)(10^{-5})(2.778)\pi}{4.7953}\sqrt{\frac{2.778(2.5)(1.1774)}{1.846(10^{-5})}}$$

$$= 0.03354 \text{ N}$$

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