GJ

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes

Mid-Term Examination, Fall 2006

Instructions: Time: 2:00 to 3:30 pm Oct 24, 2006
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

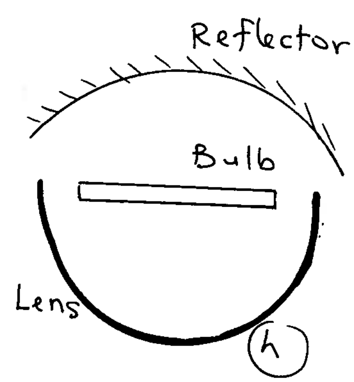
Problem #1 (15 points)

The city of Calgary, in recent years, undertook the "EnviroSmart Streetlights Retrofit Project" to replace fixtures along residential local roads and collector roads. Both lenses and bulbs were changed to reduce power use and decrease light pollution. (Calgary has been shown to lose 10 times as much energy to space compared to Vancouver, Victoria or Seattle.) Fixtures along major roadways and in parks were excluded from modification. 71,000 of 73,000 street lights in Calgary use high-pressure sodium (HPS) lamps. The lamps on highways use 400W to 1000W high intensity discharge bulbs, the "Lumalux/Eco" series. 55.5% of the energy released by the bulb is in the visible range and this pass through the glass lens or cover to illuminate the surrounding. The rest is infrared radiation which does not pass through glass. This energy thus heats up the air around the bulb and the glass lens. Part or all of the heat, in turn, is lost to the ambient. Similar type fixtures are used in large public spaces such as sport arenas and concert halls.

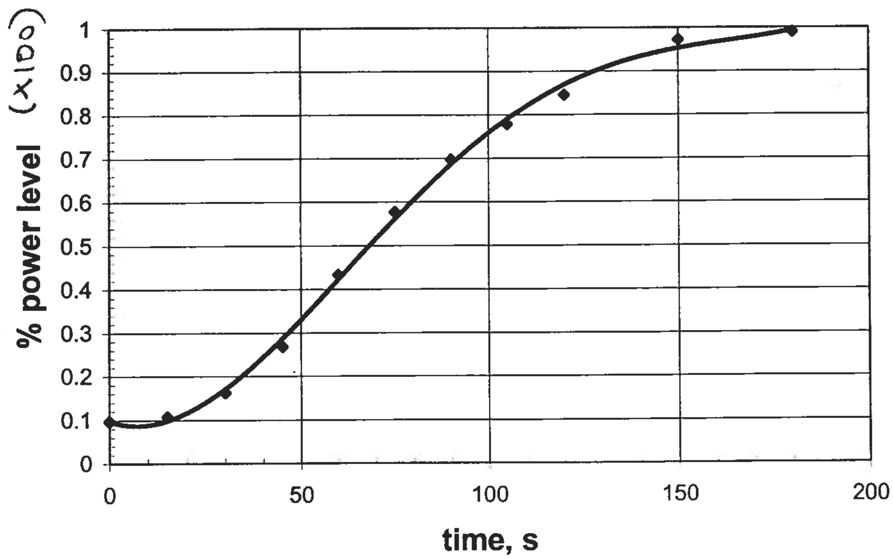
The problem is on one of the 1000W fixtures. The system comprises, as per the sketch below, a bulb, a highly polished aluminum reflector and a borosilicate glass lens. The lens is a hemisphere with a 5mm thick wall and an outer diameter of 30cm. All the heat and light generated by the bulb, at steady state, passes through the lens, i.e. the front part of the fixture.
When the light is turned on, it takes 3 to 4 minutes to reach the maximum intensity. The fraction of the energy used and thus released by the bulb versus time is plotted below. 1000W is released at the maximum level. The transient pattern is fitted with an equation provided below the plot.

If the fixture was initially at the ambient temperature of 5°C and the maximum safe temperature that the lens is allowed to attain is 250°C,

a) estimate the minimum convective heat transfer coefficient (***h***) external to the surface of the lens. You may assume that the air inside the fixture has a negligible thermal capacity and there are no temperature gradients in the glass lens.
b) If ***h*** is constant at the value you have calculated in part a), estimate the temperature of the lens at 30, 60 and 90 seconds after the light is turned on. Show all your derivations.



Data: Properties of borosilicate glass: density = 2,230 kg/m^3;
specific heat = 0.75 kJ/kg K



$y = 3.8284E\text{-}09x^4 - 1.7192E\text{-}06x^3 + 2.2989E\text{-}04x^2 - 3.0198E\text{-}03x + 9.7489E\text{-}02$

Problem #2 (10 points)

The drag on a sonar transducer is to be estimated through tests with a model in air in a wind tunnel at STP. The sonar is a 30cm diameter sphere and its to be towed in sea water (5°C) at 5 knots (1 knot = 1.852 kph). The model is 15cm diameter.

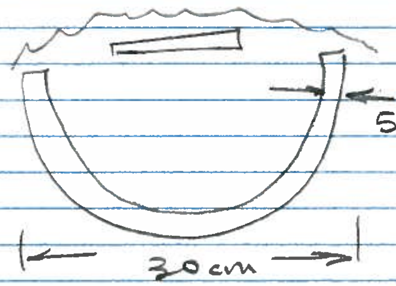
a) Estimate the velocity of air over the model for similarity.
b) If the drag measured for the model is 24.82 N, what will be the drag on the sonar?

For the problem, determine first the dimensionless quantities. The drag **F** depends on the diameter of the sphere **D**, the velocity of travel **V**, the density ρ and the viscosity μ of the fluid. Show your steps.

Data:
Sea water: density = 1025.6 kg/m^3; kinematic viscosity = 1.57(10^{-6}) m^2/s
Air at STP: density = 1.2266 kg/m^3; kinematic viscosity = 1.46(10^{-5}) m^2/s

ENCH 501 MIDTERM Solution Oct. 24, 2006

#1



for sphere,

$$\text{volume} = \frac{4}{3}\pi R^3 = \frac{\pi}{6} D^3$$

$$\text{Area} = 4\pi R^2 = \pi D^2$$

Volume of lens, $\forall = \frac{1}{2}\cdot\frac{\pi}{6}\left(0.3^3 - 0.29^3\right) = 6.8356(10^{-4})\ m^3$

External area of lens, $A = \frac{1}{2}\pi(0.3)^2 = 0.14137\ m^2$

for a 1000W bulb, 445W is heat and this passes through the lens at steady state. (1)

(a) $$445 = h A (T - T_a) = h_{mi}(0.14137)(250 - 5)$$

$$\therefore\ h_{mi} = 12.848\ W/m^2K.$$

(This is a low coefficient that can readily be met in very low wind velocity or by natural convection on the lens. h_{mi} is not changed by much if $T_a = \pm 35°C$)

(b) (During the transient stage, the bulb which contains sodium-mercury amalgam and a small amount of xenon, glows in stages. On application of a high voltage, xenon (starter gas) is ionized to produce a dim, bluish-white glow. This is quickly replaced by a blue, brighter mercury light. As sodium takes over, the light changes to yellow and then golden-white. This is why the power consumption follows the pattern in the plot provided.)

The equation for the rate of heat output by bulb is

$$Q(t) = (0.445)(Q_{i,ss})(y(t)) \quad (2)$$

where $Q_{i,ss} = 1000\,W$

and

$$y(t) = 3.8284(10^{-9})t^4 - 1.7192(10^{-6})t^3 + 2.2989(10^{-4})t^2 - 3.0198(10^{-3})t + 0.09748 \quad (3)$$

in the range $0 \le t \le 180$ s.

(Note that as $t \to \infty$, $y \to 1$ for the problem but this is outside the range of validity for eq. above.)

Choose the lens as the control volume and perform an energy balance – transient stage

Input + Generation = Output + Accumulation
0

$$Q(t) = hA(T - T_a) + mC_p \frac{d(T - T_a)}{dt} \quad (4)$$

where $m = \forall \cdot \rho = 6.8356(10^{-4})(2,230)$

$= 1.5243$ kg

and $C_p = 750$ J/kg K.

Let $\theta = T - T_a$

$$\frac{d\theta}{dt} + \frac{hA}{mC_p}\theta = \frac{Q(t)}{mC_p}$$

or

$$\frac{d\theta}{dt} + \beta\theta = f(t) \quad (5)$$

$$= a + bt + ct^2 + dt^3 + et^4$$

with the initial condition,

$t = 0 \quad \theta = 0$

$$a = \frac{0.097489(0.445)(1000)}{1.5243(750)}$$

$$= 3.7948(10^{-2})$$

$$b = \frac{-3.0198(10^{-3})(445)}{1.5243(750)} = -1.1755(10^{-3})\ s^{-1}$$

$$c = \frac{2.2989(10^{-4})(445)}{1.5243(750)} = 8.9485(10^{-5})\ s^{-2}$$

$$d = \frac{-1.7192(10^{-6})(445)}{1.5243(750)} = -6.692(10^{-7})\ s^{-3}$$

$$e = \frac{3.8284(10^{-9})(445)}{1.5243(750)} = 1.4902(10^{-9})\ s^{-4}$$

and $$\beta = \frac{12.848(0.14137)}{1.5243(750)} = 1.5888(10^{-3})\ s^{-1}$$

Solve equation (5) – use integrating factor $e^{\int \beta dt}$

$$e^{\beta t}\frac{d\theta}{dt} + e^{\beta t}\beta\theta = e^{\beta t} f(t) \qquad (6)$$

$$\therefore \quad \theta e^{\beta t} = \int e^{\beta t} f(t)\,dt + C$$

$$= a\int e^{\beta t}dt + b\int e^{\beta t} t\,dt + c\int e^{\beta t} t^2 dt$$

$$+ d\int e^{\beta t} t^3 dt + e\int e^{\beta t} t^4 dt + C$$

From tables

$$\theta e^{\beta t} = a\frac{e^{\beta t}}{\beta} + b\frac{e^{\beta t}}{\beta}\left(t - \frac{1}{\beta}\right) + c\frac{e^{\beta t}}{\beta}\left(t^2 - \frac{2t}{\beta} + \frac{2}{\beta^2}\right)$$

$$+ d\frac{e^{\beta t}}{\beta}\left(t^3 - \frac{3t^2}{\beta} + \frac{6t}{\beta^2} - \frac{6}{\beta^3}\right) +$$

$$\frac{e\, e^{\beta t}}{\beta}\left(t^4 - \frac{4t^3}{\beta} + \frac{12t^2}{\beta^2} - \frac{24t}{\beta^3} + \frac{24}{\beta^4}\right) + C$$

$\therefore$

$$\theta = T - T_a = \frac{1}{\beta}\left\{ a + b\left(t - \frac{1}{\beta}\right) + c\left(t^2 - \frac{2t}{\beta} + \frac{2}{\beta^2}\right) + \right.$$

$$d\left(t^3 - \frac{3t^2}{\beta} + \frac{6t}{\beta^2} - \frac{6}{\beta^3}\right) +$$

$$\left. e\left(t^4 - \frac{4t^3}{\beta} + \frac{12t^2}{\beta^2} - \frac{24t}{\beta^3} + \frac{24}{\beta^4}\right)\right\} +$$

$$C e^{-\beta t} \qquad (7)$$

Use the initial condition $t = 0$ $T = T_a$ or $\theta = 0$

$$C = -\frac{1}{\beta}\left[a - \frac{b}{\beta} + \frac{2c}{\beta^2} - \frac{6d}{\beta^3} + \frac{24e}{\beta^4}\right]$$

Substitute C into equation (7) to obtain the temperature-time relationship for the lens.

What is left is to substitute the constants and solve at $t = 30, 60$ and $90\,s$ for θ and T.

$t = 30\,s$	$\theta = 1.26$	$T = 6.26\ ^\circ C$
$60\,s$	4.51	9.51
$90\,s$	10.68	15.68
120	19.15	24.15
150	28.73	33.73
180	38.49	43.49

PROBLEM # 1 MIDTERM 501 OCT-24-2006

	Function Y_1	Function Y_2
a	3.79500E-02	3.79476E-02
b	-1.16770E-03	-1.17546E-03
c	7.78500E-05	8.94846E-05
d	-7.78500E-07	-6.69198E-07
e	1.55700E-09	1.49020E-09
B	1.58880E-03	1.58880E-03
T_{ref} =	5 C	

$$Y_1 = 4*10^{-9}*t^4 - 2*10^{-6}*t^3 + 2*10^{-4}*t^2 - 3*10^{-3}*t + 0.097$$

$$Y_2 = 3.8284*10^{9}*t^4 - 1.7192*10^{6}*t^3 + 2.2989*10^{4}*t^2 - 3.0198*10^{3}*t + 0.09748$$

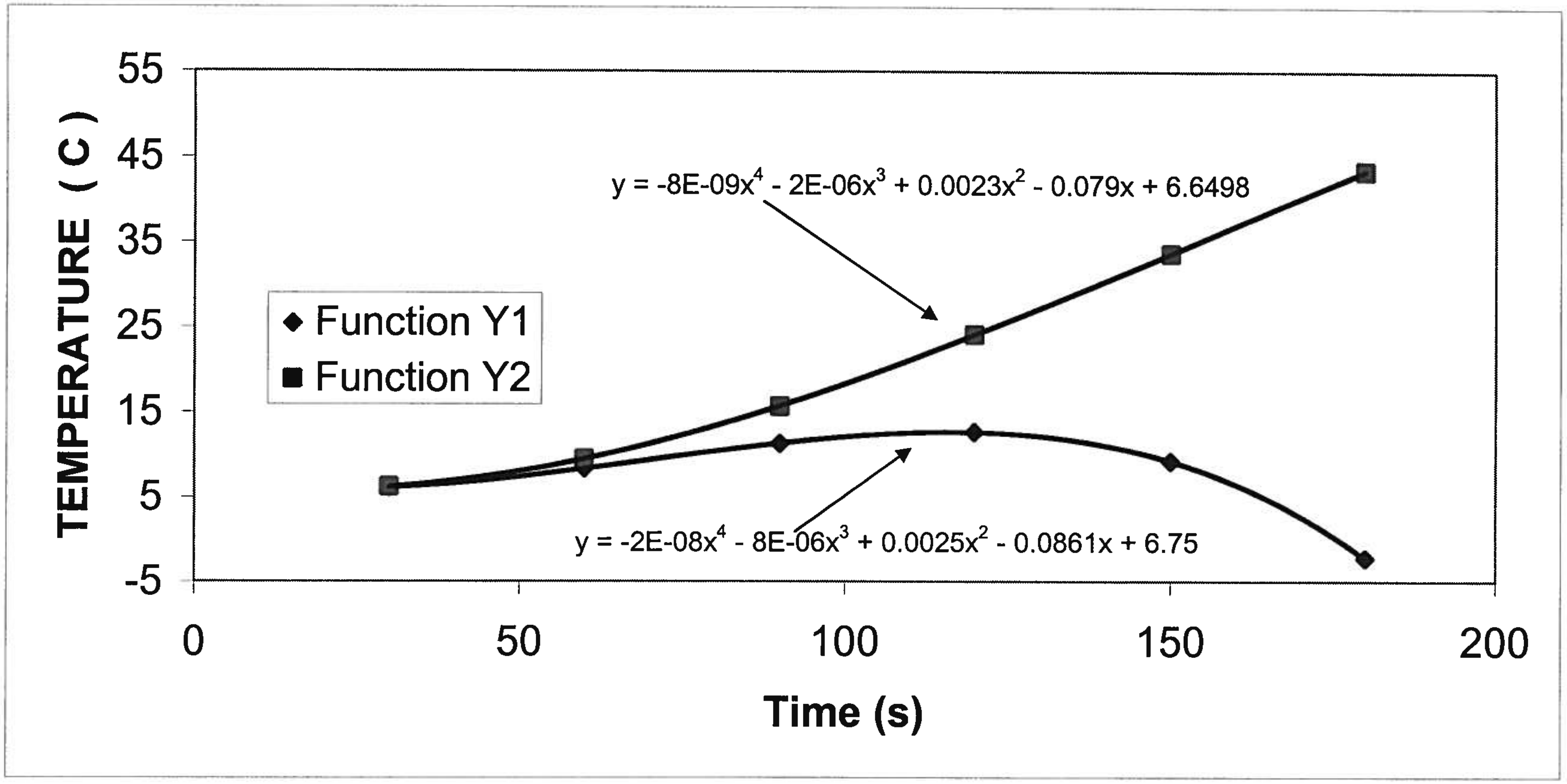
$$\theta = T - T_\alpha = \underbrace{\underset{A}{\frac{1}{\beta}}\left[a + b\underset{B}{\left(t - \frac{1}{\beta}\right)} + c\underset{C}{\left(t^2 - \frac{2t}{\beta} + \frac{2}{\beta^2}\right)} + d\underset{D}{\left(t^3 - \frac{3t^2}{\beta} + \frac{6t}{\beta^2} - \frac{6}{\beta^3}\right)} + e\underset{E}{\left(t^4 - \frac{4t^3}{\beta} + \frac{12t^2}{\beta^2} - \frac{24t}{\beta^3} + \frac{24}{\beta^4}\right)}\right]}_{\text{FIRST TERM}} - \frac{1}{\beta}\underbrace{\left[a - \frac{b}{\beta} + \frac{2c}{\beta^2} - \frac{6d}{\beta^3} + \frac{24e}{\beta^4}\right]}_{C1} e^{-\beta t}$$

Function Y_1

Time (s)	A	B	C	D	E	First term	C1	C1*EXP(-B*t)	θ	T (C)
30.00	0.03795	0.69993	58.81093	1,110.46	5,591.43	4.2557E+06	4.4634E+06	4.2557E+06	1.14	6.14
60.00	0.03795	0.66490	56.08117	1,058.77	5,331.17	4.0576E+06	4.4634E+06	4.0576E+06	3.37	8.37
90.00	0.03795	0.62986	53.49154	1,009.47	5,083.03	3.8687E+06	4.4634E+06	3.8687E+06	6.31	11.31
120.00	0.03795	0.59483	51.04204	962.44	4,846.44	3.6887E+06	4.4634E+06	3.6887E+06	7.67	12.67
150.00	0.03795	0.55980	48.73267	917.55	4,620.89	3.5170E+06	4.4634E+06	3.5170E+06	4.24	9.24
180.00	0.03795	0.52477	46.56343	874.68	4,405.86	3.3533E+06	4.4634E+06	3.3533E+06	-7.20	-2.20

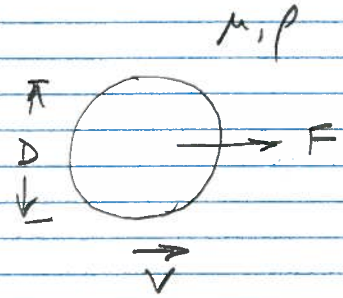
Function Y_2

Time (s)	A	B	C	D	E	First term	C1	C1*EXP(-B*t)	θ	T (C)
30.00	0.0379476	0.704575	67.60018	954.55	5,351.55	4.0121E+06	4.2080E+06	4.0121E+06	1.26	6.26
60.00	0.0379476	0.669312	64.46246	910.11	5,102.46	3.8254E+06	4.2080E+06	3.8254E+06	4.51	9.51
90.00	0.0379476	0.634048	61.48581	867.74	4,864.96	3.6473E+06	4.2080E+06	3.6473E+06	10.68	15.68
120.00	0.0379476	0.598784	58.67023	827.31	4,638.53	3.4776E+06	4.2080E+06	3.4775E+06	19.15	24.15
150.00	0.0379476	0.563521	56.01573	788.73	4,422.65	3.3157E+06	4.2080E+06	3.3157E+06	28.73	33.73
180.00	0.0379476	0.528257	53.52230	751.87	4,216.84	3.1614E+06	4.2080E+06	3.1613E+06	38.49	43.49



TEMPERATURE (C)
55
45
35
25
15
5
-5
0
50
100
150
200
Time (s)
Function Y1
Function Y2
y = -8E-09x^4 - 2E-06x^3 + 0.0023x^2 - 0.079x + 6.6498
y = -2E-08x^4 - 8E-06x^3 + 0.0025x^2 - 0.0861x + 6.75

2



$$F = f(D, V, \mu, \rho)$$

Use the Buckingham's Pi theorem to obtain dimensionless gps.

	F	$=$	$f($ D	V	μ	$\rho)$
units	N or $\frac{kg \cdot m}{s^2}$		m	$\frac{m}{s}$	$Pa \cdot s$ or $\frac{N}{m^2} \cdot s$	kg/m^3
Dimensions	$\frac{ML}{t^2}$		L	$\frac{L}{t}$	$\frac{M}{Lt}$	$\frac{M}{L^3}$

Fundamental dimensions — M, L, t or 3 dimensions

\# of variables $= 5$ $\therefore$ \# of dimensionless gps $= 2$

Let $\Pi_1 = D^a V^b \rho^c \mu$

$\Pi_2 = D^a V^b \rho^c F$

Then $\underbrace{\Pi_1 = M^0 L^0 t^0}_{\text{dimensionless}} = L^a \left(\frac{L}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt}$; Collect coefficients

$$\left.\begin{array}{lll} \text{Mass} & 0 = & c + 1 \\ \text{Length} & 0 = & a + b - 3c - 1 \\ \text{time} & 0 = & -b - 1 \end{array}\right\} \quad c = -1,\ b = -1;\ a = -1$$

$\therefore$ $\Pi_1 = \dfrac{\mu}{DV\rho}$ or its inverse $\dfrac{DV\rho}{\mu}$ (Re #)

And $\Pi_2 = L^a \left(\frac{L}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{ML}{t^2}$

$$\left.\begin{array}{lll} \text{Mass} & 0 = & c + 1 \\ \text{Length} & 0 = & a + b - 3c + 1 \\ \text{time} & 0 = & -b - 2 \end{array}\right\} \quad c = -1,\ b = -2;\ a = -2$$

$\therefore$ $\Pi_2 = \dfrac{F}{D^2 V^2 \rho}$, force coefficient.

$\longrightarrow$

a For similarity, the dimensionless gps must have same values for both model and prototype.

$$\therefore \left(\frac{DV\rho}{\mu}\right)_p = \left(\frac{DV\rho}{\mu}\right)_m \quad ; \quad \text{kinematic viscosity, } \nu = \mu/\rho$$

$$V_p = \frac{5(1.852)(1000)}{3600} = 2.572 \text{ m/s}$$

$$\frac{(0.3)(2.572)}{1.57(10^{-6})} = \frac{(0.15)V_m}{1.46(10^{-5})}$$

$$\therefore V_m = 47.84 \text{ m/s} \longrightarrow$$

vel. of air in wind tunnel

(b) For the force

$$\left(\frac{F}{D^2V^2\rho}\right)_p = \left(\frac{F}{D^2V^2\rho}\right)_m$$

$$\frac{F}{(0.3^2)(2.572)^2(1025.6)} = \frac{24.82}{(0.15)^2(47.84)^2(1.2264)}$$

$$\therefore F = 239.94 \text{ N} \longrightarrow$$

Force on Sonar.