GJ

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ENCH 501: Transport Processes

Mid-Term Examination, Fall 2005

Instructions: Time: 1:30 to 3:00 pm Oct 25, 2005
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

Problem #1 (15 points)

Diesel fuel is in high and increasing demand for automobiles, commercial trucks, buses, farm and industrial machinery. It is a distillation fraction of crude oil that boils in a range of 150 to 330°C and it is high in alkanes or paraffins (C_9 to C_{30}) which burn readily. At low temperatures, paraffins in diesel fuel start to form wax crystals (at the cloud point) and these tend to plug fuel filters. The cloud point may be lowered by adding heavier crude components such as napthalenes and aromatics, or alkyl esters of long chain fatty acids. As the liquid temperature is further lowered, it gels and stops flowing at the pour point. Thus in cold climates, diesel fuels are often blended with kerosine so that the cloud point is at least 6°C below anticipated lowest ambient temperatures. Diesel fuel is often graded as #1, #2 or #4 in order of decreasing cloud and pour points. If a diesel car travels from a warm to a cold region, the fuel may cool down sufficiently for the vehicle to lose power from restricted flow to the engine. The following problem considers such a situation.

The fuel tank of a car is situated under the trunk space, at the rear of the car. The top half of the tank may be considered insulated while the bottom half is exposed to air flowing over it as the car travels. The tank has a capacity of 80 litres and the exposed area is 0.9m^2. At the start of a journey, the tank is filled with #2 diesel fuel at 12°C from the gas station pump. Properties of the fuel are given below. The ambient temperature on the day is given as -34°C. The vehicle is maintained at a steady speed of 90 km/hr on the highway. The fuel consumption rate (or rate of liquid withdrawal from the tank) is 6 litres/100 km and the convective heat transfer coefficient around the tank (in a partly sheltered location) is given as 3.28 W/m^{2}K. You may assume that the fuel in the tank is well mixed as it sloshes over bumps and around corners.

If the driver noticed that the car suddenly slowed down when the thermocouple in the tank recorded a temperature of -18°C,

a) how far from the start has the car travelled? Show all the steps for you analysis.

b) At this point, how much fuel is left in the car?

c) At what instant from the start would wax crystals start forming in the fuel tank?

Data: Properties of #2 diesel fuel

density = 860 kg/m^3 ; heat capacity = 2.035 kJ/kg K ; cloud point = -15°C ; pour point = -27.8°C

Neglect the mass and thermal capacity of the fuel tank.

Problem #2 (10 points)

The drag on a flat plate held stationary in a fast-flowing stream is to be estimated using the ***integral method***. You are able to identify only three boundary conditions as follows:

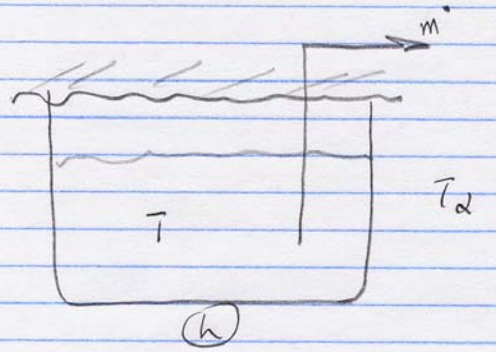
$y = 0$, $u = 0$ (no slip condition) ; $y = \delta$, $u = U_\infty$; and $y = \delta$, $du/dy = 0$ where U_∞ is the free stream velocity above the plate.

Derive expressions for calculating velocity at any (x, y) point within the boundary layer <u>and</u> the drag on the flat plate. Include an explicit relation for the boundary layer thickness $\delta(x)$.

Show all important steps.

ENCH 501 MID TERM EXAM Solution Oct. 25, 2005

#1



At start, $t=0$,

volume of fuel = 80 litres or $0.08\ m^3$

$\therefore$ Initial mass $= 0.08(860)$

$W_0 = 68.8$ kg.

The fuel withdrawal rate from the tank is

$$6\ \frac{\text{litres}}{100\text{ km}} \cdot 90\ \frac{\text{km}}{\text{hr}} = 5.4\ \text{litres/hr}$$

$$\text{or } \dot{m} = 0.0054\,(860) = 4.644\ \text{kg/hr}$$

Let the mass of fuel in tank at any time t be given by $W(t)$. Then

$$W = W_0 - \dot{m}t \qquad (1)$$

Let T_a be the reference temperature

Perform an energy balance on the fuel in the tank.

$$\cancel{\text{Input}} + \cancel{\text{Generation}} = \text{Output} + \text{Accum.}$$

$$\text{Output} = hA(T-T_a) + \dot{m}C_p(T-T_a)$$

$$\text{Accum} = \frac{d}{dt}\left[WC_p(T-T_a)\right]$$

$$\therefore \frac{d}{dt}\left[WC_p(T-T_a)\right] = -\left[hA + \dot{m}C_p\right](T-T_a) \qquad (2)$$

Since both W and T change with time, the l.h.s. becomes

$$WC_p\frac{d(T-T_a)}{dt} + C_p(T-T_a)\frac{dW}{dt} \quad ; \quad \frac{dW}{dt} = \dot{m}$$

Hence

$$W C_p \frac{d(T-T_a)}{dt} = -\left[hA + 2\dot{m} C_p\right](T-T_a) \quad (3)$$

Substitute (1)

$$(W_0 - \dot{m} t) C_p \frac{d(T-T_a)}{dt} = -\left[hA + 2\dot{m} C_p\right](T-T_a)$$

or

$$\frac{d(T-T_a)}{T-T_a} = -\left[\frac{hA + 2\dot{m} C_p}{C_p}\right]\frac{dt}{W_0 - \dot{m} t}$$

$$\int_{T_0}^{T} d\ln(T-T_a) = \beta \int_0^t d\ln(W_0 - \dot{m} t) \quad (4)$$

where $\beta = \frac{hA}{\dot{m} C_p} + 2$

$$\ln \frac{T-T_a}{T_0 - T_a} = \beta \ln\left(\frac{W_0 - \dot{m} t}{W_0}\right) \quad (5)$$

or

$$\frac{T-T_a}{T_0 - T_a} = \left[1 - \frac{\dot{m} t}{W_0}\right]^{\beta} \quad (6)$$

Given $h = 3.28 \ W/m^2 K$

$A = 0.9 \ m^2$

$\dot{m} = 4.644 / 3600 = 0.00129 \ kg/s$

$c_p = 2035 \ J/kg\,K$

$\therefore \beta = 1.1245 + 2 = 3.1245$

when $T = -18°C$, $T_a = -34°C$ and $T_0 = 12°C$

From (6)

$$\frac{-18-(-34)}{12-(-34)} = \frac{16}{46} = \left[1 - \frac{4.644t}{68.8}\right]^{3.1245}$$

where t is in hours.

$$0.347826 = \left[1 - 0.0675t\right]^{3.1245}$$

$$0.7132 = 1 - 0.0675t$$

$$t = 4.249 \text{ hours}$$

(a)

∴ Distance travelled $= 90(4.249) = 382.4$ km.

(b) Amount of fuel left in tank is

$$68.8 - (4.644)(4.249)$$

$$= 49.07 \text{ kg}$$

or $\frac{49.07(1000)}{860} = 57.06$ Litres

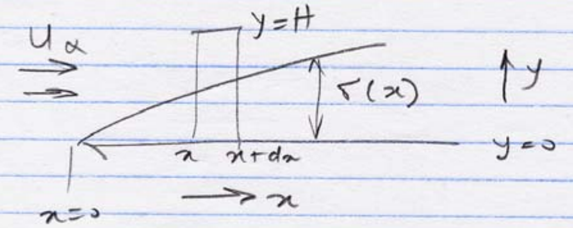
(c) Wax will start to form when $T = -15°C$

$$\therefore \frac{-15-(-34)}{12-(-34)} = \frac{19}{46} = \left[1 - \frac{4.644t}{68.8}\right]^{3.1245}$$

$$0.413 = \left[1 - 0.0675t\right]^{3.1245}$$

$$t = 3.65 \text{ hrs}$$

#2



Given b.c.

$y=0 \quad u=0$

$y=\delta \quad u=U_\infty$

$y=\delta \quad \dfrac{du}{dy}=0$

Perform an integral balance for forces on differential element as in diagram above. This is as in p 85-88 of Notes.

The momentum integral equation (eq. 5.13 Notes) is

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho (U_\infty - u)u\,dy\right]$$

* Choose any function for $u(y)$ which has a maximum of 3 constants - since only 3 b.Cs are known.

Let us assume $u = a + by + cy^2$ { Other functions may be used.

$$\frac{du}{dy} = b + 2cy$$

Apply b.c. $y=0,\ u=0 \Rightarrow a=0$

$y=\delta \quad U_\infty = b\delta + c\delta^2$

$y=\delta \quad 0 = b + 2c\delta$) Solve for b and c

$$\therefore \quad \frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Note:

[If one had used $u = a + b\sin(cy)$ with above b.c, $\dfrac{u}{U_\infty} = \sin\left(\dfrac{\pi}{2}\dfrac{y}{\delta}\right)$]

This can be found from: $y=0 \quad u=0 \Rightarrow a=0$

$u = a + b\sin(cy)$

$\frac{du}{dy} = bc\cos(cy)$

$y=\delta \quad U_\alpha = b\sin(c\delta)$

$y=\delta \quad 0 = bc\cos(c\delta)$

Multiply top eq. by c and divide both sides —

$\alpha = \tan(c\delta)$ or $c\delta = n\frac{\pi}{2},\ n=1,3,5 \dots$

use $n=1$, $u = U_\alpha \sin(\pi/2 \cdot y/\delta)$

substitute function into integral equation and simplify.

Let $\frac{u}{U_\alpha} = \phi \quad ; \quad y/\delta = \eta$

The integral eq.

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \mu\frac{U_\alpha}{\delta}\left.\frac{d\phi}{d\eta}\right|_{\eta=0} = \frac{d}{dx}\left[\int_0^\delta \rho U_\alpha^2 (1-\phi)\phi\, dy\right]$$

or

$$\frac{\mu U_\alpha}{\delta}\left.\frac{d\phi}{d\eta}\right|_{\eta=0} = \frac{d}{dx}\left[\rho U_\alpha^2 \delta \int_0^1 (1-\phi)\phi\, d\eta\right]$$

Since velocity profile is:

$$\phi = 2\eta - \eta^2, \quad \frac{d\phi}{d\eta} = 2 - 2\eta \Rightarrow \left.\frac{d\phi}{d\eta}\right|_{\eta=0} = 2$$

$$\int_0^1 (1-\phi)\phi\, d\eta = \int_0^1 (1-2\eta+\eta^2)(2\eta-\eta^2)\,d\eta = \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)\,d\eta$$

$$\therefore \frac{\mu U_\alpha}{\delta}(2) = \frac{d}{dx}\left[\rho U_\alpha^2 \delta \frac{2}{15}\right]$$

$$15\frac{\mu}{\rho}\frac{1}{U_\alpha} = \delta\frac{d\delta}{dx} = \frac{1}{2}\frac{d\delta^2}{dx}$$

$\therefore \quad \frac{d\delta^2}{dx} = 30\frac{\nu}{U_\alpha}$ subject to $x = 0, \delta = 0$

$\delta^2 = 30\frac{\nu x}{U_\alpha} \Rightarrow \delta = 5.477\left(\frac{\nu x}{U_\alpha}\right)^{\frac{1}{2}}$ $\longrightarrow$

$\therefore$ Velocity profile is

$$\frac{u}{U_\alpha} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \quad \text{where } \delta = 5.477\left(\frac{\nu x}{U_\alpha}\right)^{\frac{1}{2}}$$

$\longrightarrow$

The Drag / unit width of plate $\perp$ to flow direction may be obtained from eq. 5.25 (Notes)

$$D = \int_0^L \tau_w\Big|_x \, dx$$

where $\tau_w = \frac{2\mu U_\alpha}{\delta}$; $\delta = \beta x^{\frac{1}{2}}$ and as long as b.l. is laminar. $\beta = 5.477\left(\frac{\nu}{U_\alpha}\right)^{\frac{1}{2}}$

$$D = \int_0^L \frac{2\mu U_\alpha}{\beta} x^{-\frac{1}{2}} dx = \frac{4\mu U_\alpha}{\beta} L^{\frac{1}{2}}$$

$$D = \frac{4}{5.477} \frac{\mu U_\alpha U_\alpha^{\frac{1}{2}}}{\nu^{\frac{1}{2}}} L^{\frac{1}{2}}$$

$$= 0.73\rho\left(\nu U_\alpha^3 L\right)^{\frac{1}{2}} \longrightarrow$$

with $\frac{u}{U_\alpha} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$; $\delta = 4.8\left(\frac{\nu x}{U_\alpha}\right)^{\frac{1}{2}}$