G. J. Oct 20,'03

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ENCH 501: Transport Processes

Mid-Term Examination, Fall 2003

Instructions: Time: 2:00 to 3:30 pm Oct 21, '03
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

Problem #1 (15 points)

Plastic bottles for "natural spring water", soft drinks, milk, juice or shampoo and containers for peanut butter, jelly, yogurt and margarine amongst others are made by molding.

Bottles for water are typically made from polyethylene terepthalate (PET) polymer by injection blow molding. An equipment for this process is illustrated in Figure 1. A molded preform (A) of PET at its softening temperature is fitted on a cap with threads. The cap has a hole through it and it is connected to a reservoir of pressurized air. The cavity for the mold (B), in two halves, is then clamped around the cap. A pulse of low pressure air is admitted through the cap to elongate the preform. This is followed by a burst of high pressure air to inflate the preform against the chilled mold wall to form the bottle. The bottle is ejected from the cavity after the walls have solidified.

In an operation, the cooling system for the mold cavity failed. The bottle had to cool from the softening temperature of 150°C to 98°C before it can be removed without damage from the mold. The wall of the mold may be assumed insulated so that all the heat to be extracted from the bottle must be absorbed by the fixed amount of pressurized air in the cavity. The air was injected at 12°C. The volume of the cavity is 500cm^3. The bottle may be assumed to be a cylinder of a diameter of 6.6cm. The wall of the bottle is thin and there are no temperature gradients within it. The cap diameter is 2.5cm.

(a) What should the injection pressure of the air in the cavity be so that, after a long time, the bottle cools down to 90°C? Assume air is an ideal gas.

(b) Under the conditions for part (a), what is the minimum time that the bottle must stay in the mold before it can be retrieved?

Data:

Mass of molded preform = 16g.

Properties of **PET**: Cp = 1.3 kJ/kg K,
ρ = 1390 kg/m^3, k = 0.3 W/mK

Properties of **air**: Cp = 1.06 kJ/kg K ,
Universal Gas Constant R = 8.314 kJ/kmol K ,
Molar mass = 29.92 kg/kmol

The heat transfer coefficient between the air and the bottle wall is 26 W/m^{2}K

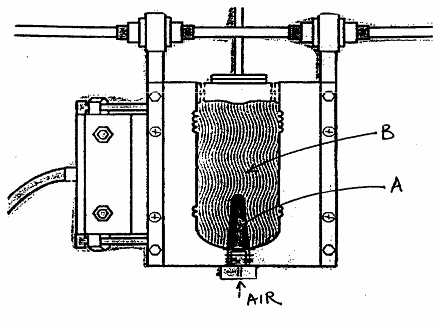


Figure 1

Problem #2 (10 points)

"Containers" are used for moving manufactured goods and other articles of commerce around the world. Packaged goods (such as electronics, textiles, cars...) are placed inside and locked for security. The containers are loaded by cranes onto ocean going vessels for transshipment and on trucks for local distribution to retailers.

The deck of a cargo ship is stacked with standard containers, each 6m long by 2.5m tall by 2.5m wide. The long side is parallel to the sides of the ship. A stack has 6 containers in the vertical direction and 8 containers along the width. Each of 16 stacks is separated from the other by a ½ m gap as illustrated in the sketch below. If the ship is travelling at 15 knots into a head wind of 10 knots,

(a) estimate the total drag of air on the sides and tops of the containers on the deck.

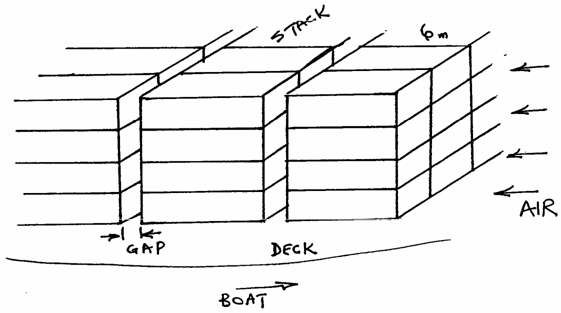
You may assume each stack is tightly packed, the air is well mixed in passing from the surface of one stack to the one behind and the boundary layer on each surface is entirely laminar. Neglect form drag across the faces normal to the air flow.

(b) If each container is prevented from sliding by friction between it and the lower surface only, estimate the minimum coefficient of static friction.

Data:
1 knot (kn) is equal 0.51444 m/s.

Properties of air at 1 atm and 20°C: $\rho = 1.2047$ kg/m^3, $\mu = 18.17\ (10^{-6})$ Pa.s

The force required to drag a body over a flat horizontal surface is given by $F = \lambda \cdot N$ where N is the normal force exerted by the body on the surface and λ is the coefficient of static friction. Each container weighs 12 tons. 1 ton = 1016 kg.



ENCH 501 SOLUTION – MID-TERM OCT. 21, 2003

\# 1. Use the preform or the bottle as the control volume.

(a) To determine the injection pressure, it is necessary to calculate the mass of air that would be in the cavity and would be required to cool the bottle down from 150°C to 90°C. In turn, the air would heat up from 12°C to 90°C

That is (if 1 ≡ PET and 2 ≡ air)

$$m_1 C_{p_1} (150-90) = m_2 C_{p_2} (90-12)$$

Given $m_1 = 16\,g$, $C_{p_1} = 1300\ J/kg\,K$, $C_{p_2} = 1060\ J/kg\,K$

$$16(10^{-3})(1300)(60) = m_2 (1060)(78)$$

$$\therefore\ m_2 = 0.01509\ kg$$

Given the molar mass of air = 29.92 kg/kmol

\# moles, $n = 5.0449(10^{-4})$ kmoles

Since air is assumed to be an ideal gas

$$P\forall = nRT \quad \text{where } T = (90+273.15)\,K \text{ in the final state}$$

$$\therefore\ P = \frac{nRT}{\forall} = \frac{5.0449(10^{-4})(8.314)(363.15)}{500(10^{-6})}$$

$$\frac{kmols \cdot kJ/kmol\,K \cdot K}{m^3} \text{ or } kJ/m^3$$

Since J = N.m

$$P = 3{,}046.34\ kPa = \frac{3046.342}{101.325} \text{ or } 30.065\ atm.$$

This is the pressure in the cavity at equilibrium. But the fixed amount of air was injected at 12°C or 285.15 K. Use

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \text{or} \quad \frac{30.065}{363.15} = \frac{P_2}{285.15}$$

$\therefore$ The injection pressure, $P_2 = 23.6075$ atm.

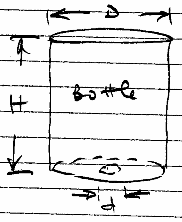
$\longrightarrow$

(b) To determine the time required for cooling the bottle, use the lumped analysis method.

Check validity $\Rightarrow \dfrac{h(\forall/A)}{k} < 0.1$

For the PET as control volume,

Volume, $\forall = \dfrac{\text{mass}}{\text{density}} = \dfrac{0.016}{1390} = 1.151\,(10^{-5})\ m^3$



Area for heat exchange between PET and air are for the bottom, side and top, minus area of cap

i.e. $A = 2\cdot\dfrac{\pi D^2}{4} + \pi D H - \dfrac{\pi d^2}{4}$

$$A = \frac{\pi (6.6)^2 (10^{-4})}{2} + \pi (6.6)(10^{-2}) H - \frac{\pi (2.5)^2 (10^{-4})}{4}$$

H can be determined from the volume of the bottle

$\forall = \dfrac{\pi D^2}{4}\cdot H$ or $H = \dfrac{500(10^{-6})\cdot 4}{\pi (6.6)^2 (10^{-4})} = 0.14615\ m$

Substitute, $A = 0.006842 + 0.030303 - 0.000491$
$= 0.036654\ m^2$

$\therefore \dfrac{h(\forall/A)}{k} = \dfrac{26\left(\dfrac{1.151(10^{-5})}{0.036654}\right)}{0.3} = 0.027 < 0.1$

$\therefore$ valid.

$\longrightarrow$

This part of the problem is similar to the problem of heat transfer into a finite reservoir in Notes!

The energy balance equation for the PET bottle is:

$$-m_1 c_{p_1} \frac{dT}{dt} = h A (T - T_a)$$

where T_a is the temp. for air and it is also a function of time.

An energy balance for the system, at any time, yeilds that

$$\underbrace{m_1 C_{p_1} (150 - T)}_{\text{heat loss by PET}} = \underbrace{m_2 C_{p_2} (T_\infty - 12)}_{\text{heat gain by air.}}$$

Using the data in part (a), $\dfrac{m_1 C_{p_1}}{m_2 C_{p_2}} = 1.30037$

and $T_\infty = 207.05608 - 1.30037\, T$

or $T_\infty = a - bT$ (in algebraic form)

Substitute this into the energy balance equation and re-arrange

$$-\frac{dT}{dt} = \frac{hA(1+b)}{m_1 C_{p_1}} \left[T - \frac{a}{1+b} \right] = \beta \left[T - \gamma \right]$$

Substitute values $\beta = 0.105397$ and $\gamma = 90.01$

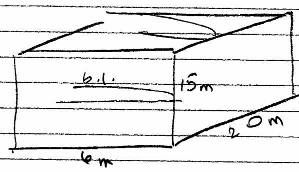
Integrate equation

$$\frac{T_f - \gamma}{T_0 - \gamma} = \exp\left[-\beta t\right] \quad \text{where} \quad T_0 = 150°C, \; T_f = 98°C$$

$$\frac{98 - 90}{150 - 90} = \exp\left[-0.105397\, t\right]$$

$t = 19.12$ s $\longrightarrow$

#2 Consider a stack. Boundary layers are formed on three surfaces – sides + top



Relative velocity

$U_\infty = 25$ knots

or $25(0.51444)$
$= 12.861$ m/s.

(a) The problem is the same as for drag on a flat plate.

From p. 91, Class Notes – eq. 5.25, Drag per unit width is given as

$$D = \int_0^L W \tau_w\big|_x dx \quad ; \quad W = 1, \quad L = 6m$$

where $\tau_w\big|_x = \beta x^{-\frac{1}{2}}$; $\beta = \frac{3}{2}\frac{\mu U_\infty}{4.64}\left(\frac{U_\infty}{\nu}\right)^{\frac{1}{2}}$

For the stack, the width is $2(15m) + 20 = 50m = W$

$$D = 50\beta \int_0^L x^{-\frac{1}{2}} dx$$

$$= 50\beta\left(2L^{\frac{1}{2}}\right)$$

Use data

$$\beta = \frac{3}{2} \cdot \frac{18.17(10^{-6})(12.861)}{4.64}\sqrt{\frac{12.861 \times 1.2047}{18.17(10^{-6})}}$$

$$= 0.069759$$

$$D = (50)(0.069759)(2)(6^{\frac{1}{2}}) = 17.087 \text{ N}$$

drag on a stack

Total drag on 16 stacks $= 273.4$ N

$\longrightarrow$

(b) The containers which experience the highest skin drag and lowest normal force at the base are the containers at the top edge. These experience shear at one vertical side and the top.

The total drag on this container is:

$$D_c = \int_0^L W_c \, \tau_w\big|_x \, dx \quad ; \quad W_c = 2.5m + 2.5m$$

$$= 5\beta\,(2L^{\frac{1}{2}}) = 1.7087 \text{ N}$$

But $D_c = \mu \cdot N$ where $N = 12(1016)(9.81)$ Newtons

Hence $1.7087 = \mu(12)(1016)(9.81)$

$$\therefore \mu = 1.4286\,(10^{-5})$$