G. J.
WYS

THE UNIVERSITY OF CALGARY
DEPARTMENT OF CHEMICAL & PETROLEUM ENGINEERING

ENCH 501
Mathematical Methods in Chemical Engineering

MID-TERM EXAMINATION

Tuesday, October 23, 2001 **Time: 90 Minutes**

Jeje

Attempt All Questions
Use of Electronic Calculators is Permitted
Open Notes, Open Book Examination

Problem #1: (10 Points)

A vented cylindrical vessel which has a diameter of 1.8 m and a height of 2.8 m is normally used for storing potable water. Somehow the water became contaminated with ***atropine sulphate***, a very bitter compound. When the contamination was discovered, the water in the vessel contained 1g in 100 ml water. This is approximately one part in 100. The bitterness threshold is 1:10,000.

It was decided to empty the tank and clean out the tank before refilling. However, the 5 cm diameter discharge line from the tank was located at 0.2 m above the base. (This was to allow any particles in the water to settle at the bottom and not be withdrawn).

The following protocol was followed:

- Stop the flow of fresh water into the tank and open the bottom valve to drain the tank to the level of the outlet.
- Start the supply of fresh water while agitating the liquid in the tank. The solution in the tank leaves the open discharge under the gravity head, ***h***, above the outlet pipe. (See sketch). The supply rate of fresh water is such that the water level rises to a height of 2.5 m above the base of tank before the level stops changing.

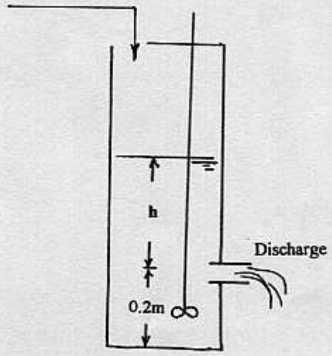
  a) After how long will the concentration of the *atropine sulphate* fall below the bitterness threshold?

  b) How much water will have been used for the flushing?

c) After how long will the water level achieve steady state?

Notes and Data:

1. Treat the discharge port as a borda entrance open to ambient air. Velocity out of the opening is given by $C_d\sqrt{(2gh)}$ Where ***h*** is liquid height above the opening and C_d, the *coefficient of discharge*, is 0.74.
2. The density of water and the solution is 1000 kg/m^3.



Problem #2: (7 Points)

You noticed that a pier submerged in a river is clean at the front, for a distance which varied from 1.4 m (in Spring) to 0.9 m (in Fall) from the sharp leading edge of the side parallel to the river flow. Past these distances, you notice deposits of all kinds of crud and slime on the vertical wall. From a simple laboratory test, you determined that the deposits are sloughed or peeled off when the wall shear stress exceeds 136 mPa.

a) Estimate the range of velocities of water near the surface of the river.
b) What assumptions are involved in your analysis? Justify them.
c) Estimate the boundary layer, the displacement and the momentum thicknesses at the location where crud appears in Spring.

Assume that the water temperature is always at 5°C, the density is 1000 kg/m^3 and the viscosity is 1.55 mPa.s.

Problem #3: (8 Points)

You are about to rent a penthouse apartment in a downtown neighbourhood. The apartment is on the 10^{th} floor. The floor is 36 m above the basement where the furnace which heats up the air blown into your apartment is located. The air is raised by the furnace to a temperature of 72°C and it flows at a linear velocity of 0.5 m/s through a 12 cm diameter galvanized iron duct. The duct passed through a vertical shaft which is circular in cross-section and has a diameter of 48 cm. The shaft is made of massive concrete and its wall temperature is uniform and steady at 15°C. The duct is located eccentrically to the shaft, the centre-to-centre distance being 10 cm. It is held in place by loose fibre insulation.

If you can assume that the air and the galvanized iron duct will have the same temperature at the same horizontal plane, i.e. there are no radial temperature gradients, estimate the temperature of the air which would flow into the apartment.

Data: Properties of air and insulation

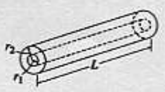
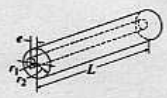
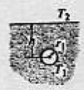
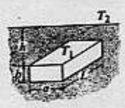
Air: $\rho = 1.088$ kg/m^3

$C_p = 1.006$ kJ/kgK

Insulation $k = 0.028$ W/mK

Conduction Shape Factors are provided in the attached table.

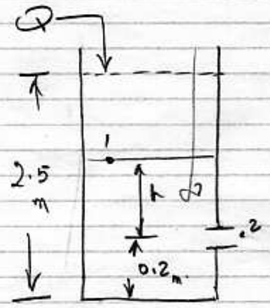
CONDUCTION SHAPE FACTORS

Configuration		Shape Factor
Plane wall 		$\frac{A}{L}$
Concentric cylinders 	$L \gg r_2$	$\frac{2\pi L}{\ln(r_2/r_1)}$ Note there is no steady-state solution for $r_2 \to \infty$, i.e., for a cylinder in an infinite medium.
Concentric spheres 		(*a*) $\frac{4\pi}{1/r_1 - 1/r_2}$ (*b*) $4\pi r_1$ for $r_2 \to \infty$
Eccentric cylinders 	$L \gg r_2$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{r_2^2 + r_1^2 - e^2}{2r_1 r_2}\right)}$
Buried sphere  Medium at infinity also at T_2		$\frac{4\pi r_1}{1 - r_1/2h}$ For $h \to \infty$, the result for item 3(*b*) is recovered
Buried cylinder  Medium at infinity also at T_2	$L \gg r_1$	$\frac{2\pi L}{\cosh^{-1}(h/r_1)}$ $\frac{2\pi L}{\ln(2h/r_1)}$ for $h > 3r_1$ For $h/r_1 \to \infty$, $S \to 0$ since steady flow is impossible
Buried rectangular beam  Medium at infinity also at T_2	$L \gg h, a, b$	$2.756L\left[\ln\left(1 + \frac{h}{a}\right)\right]^{-0.59}\left(\frac{h}{b}\right)^{-0.078}$

ENCH 501 Solution - Mid Term Exam. Oct. 23, 2001

Problem #1

[The Basic Jorpee problem is the Walkerton contamination of water supply by e-coli & how long it took to flush out.



First step is to find the steady state rates of water supply and solution discharge.

$$h = (2.5 - 0.2) = 2.3 m$$

$$V_2 = C_d \sqrt{2gh} \quad ; \quad h = 2.3 m, \; g = 9.81 \; m/s^2, \; C_d = 0.74 \qquad (1)$$

$$V_2 = 4.971 \; m/s$$

Opening diameter, D_2 = 5 cm or 0.05 m

$$\text{Area} \; A_2 = \frac{\pi D_2^2}{4} = 0.001963 \; m^2 = A_o \text{ orifice area}$$

$$\therefore \quad Q = V_2 A_2 = 0.0097601 \; m^3/s \qquad (2)$$

Now consider the situation when $h = 0$ at $t = 0$

Perform a material balance on the tank content (ρ = const.)

Input + Generation = Output + Accumulation
(Generation → 0)

$$Q = A_o C_d \sqrt{2g} \, h^{1/2} + A_t \frac{dh}{dt} \qquad (3)$$

Re-arrange

$$\frac{dh}{dt} + \beta h^{1/2} = \gamma \quad ; \quad \beta = \frac{A_o C_d \sqrt{2g}}{A_t} \qquad (4)$$

$$\gamma = Q / A_t$$

subject to the initial condition $t = 0$, $h = 0$

$$\int_0^h \frac{dh}{\gamma - \beta h^{1/2}} = \int_0^t dt \qquad (5)$$

Let $u = h^{1/2}$;

$$\int \frac{2u\,du}{\gamma - \beta u} = 2\left[-\frac{u}{\beta} - \frac{\gamma}{\beta^2}\ln(\gamma - \beta u)\right]_0^u \qquad (6)$$

$\therefore$

$$2\left[-\frac{u}{\beta} - \frac{\gamma}{\beta^2}\ln(\gamma-\beta u)\right] - 2\left[0 - \frac{\gamma}{\beta^2}\ln\gamma\right] = t \qquad (7)$$

$$-\frac{h^{1/2}}{\beta} - \frac{\gamma}{\beta^2}\ln\left(\frac{\gamma - \beta h^{1/2}}{\gamma}\right) = t/2 \qquad (8)$$

Evaluate β and γ

$$\beta = \frac{0.001963\,(0.74)\sqrt{2 \times 9.81}\,.\,4}{\pi\,(1.8)^2} = 2.5285\,(10^{-3})$$

$$\gamma = \frac{Q}{A_t} = \frac{0.009761\,.\,4}{(\pi)(1.8)^2} = 3.8357\,(10^{-3})$$

(c) A plot of h vs t shows that $h = 2.3$m is an asymptote. Hence the liquid level never attains this value. The answer is hence $t = \alpha$.

Now perform a balance on the atropine sulphate, A.

Let the amount of A in the tank at any instant $= x$ (g)

At $t = 0$, the concentration of A in solution is

$$C_0 = x_0/V_0 \quad ; \quad V_0 = \frac{\pi\,(1.8)^2}{4}(0.2)\ m^3$$

ENCH 501 Q1

h, m	f	f*dh	Cum f*dh	exp	kg/m3	time, s
0	0	0			10	0
0.01	3.36E-01	0.00168	0.00168	0.998321	9.50711	2.72764
0.02	4.67E-01	0.004017	0.005697	0.994319	9.038589	5.562724
0.03	5.60E-01	0.005139	0.009155	0.990886	8.615759	8.472726
0.04	6.33E-01	0.005966	0.011104	0.988957	8.240693	11.4463
0.05	6.92E-01	0.006622	0.012587	0.987492	7.899343	14.47706
0.06	7.41E-01	0.007161	0.013782	0.986312	7.58645	17.56077
0.07	7.82E-01	0.007615	0.014776	0.985333	7.298214	20.69443
0.08	8.19E-01	0.008005	0.01562	0.984502	7.031629	23.87579
0.09	8.50E-01	0.008343	0.016347	0.983786	6.784222	27.10308
0.1	8.78E-01	0.008639	0.016982	0.983161	6.553919	30.37489
0.11	9.03E-01	0.008902	0.017542	0.982611	6.338954	33.69009
0.12	9.25E-01	0.009137	0.018039	0.982122	6.137807	37.04775
0.13	9.45E-01	0.009348	0.018485	0.981685	5.949161	40.44707
0.14	9.63E-01	0.009539	0.018887	0.981291	5.771866	43.88741
0.15	9.80E-01	0.009712	0.019251	0.980933	5.604912	47.36822
0.16	9.95E-01	0.009871	0.019584	0.980607	5.447408	50.88905
0.17	1.01E+00	0.010018	0.019889	0.980308	5.298564	54.4495
0.18	1.02E+00	0.010153	0.02017	0.980032	5.157678	58.04927
0.19	1.03E+00	0.010278	0.02043	0.979777	5.024122	61.68807
0.2	1.05E+00	0.010395	0.020673	0.97954	4.897333	65.36569
0.21	1.06E+00	0.010504	0.020899	0.979318	4.776805	69.08195
0.22	1.07E+00	0.010607	0.021111	0.97911	4.662082	72.83671
0.23	1.08E+00	0.010704	0.021311	0.978915	4.552752	76.62986
0.24	1.08E+00	0.010796	0.0215	0.97873	4.44844	80.46133
0.25	1.09E+00	0.010883	0.021679	0.978555	4.348807	84.33106
0.26	1.10E+00	0.010966	0.021849	0.978388	4.253542	88.23904
0.27	1.11E+00	0.011046	0.022012	0.978228	4.162364	92.18525
0.28	1.12E+00	0.011122	0.022168	0.978076	4.075012	96.16972
0.29	1.12E+00	0.011196	0.022318	0.977929	3.991249	100.1925
0.3	1.13E+00	0.011267	0.022463	0.977788	3.910859	104.2536
0.31	1.14E+00	0.011336	0.022603	0.977651	3.833639	108.3532
0.32	1.14E+00	0.011403	0.022738	0.977518	3.759405	112.4913
0.33	1.15E+00	0.011468	0.02287	0.977389	3.687987	116.668
0.34	1.16E+00	0.011531	0.022999	0.977264	3.619225	120.8834
0.35	1.16E+00	0.011593	0.023124	0.977141	3.552975	125.1378
0.36	1.17E+00	0.011654	0.023247	0.977021	3.4891	129.4311
0.37	1.17E+00	0.011714	0.023368	0.976903	3.427474	133.7637
0.38	1.18E+00	0.011773	0.023487	0.976787	3.367979	138.1357
0.39	1.19E+00	0.011831	0.023604	0.976673	3.310507	142.5472
0.4	1.19E+00	0.011888	0.023719	0.97656	3.254956	146.9984
0.41	1.20E+00	0.011945	0.023834	0.976448	3.20123	151.4896
0.42	1.20E+00	0.012002	0.023947	0.976338	3.149241	156.021
0.43	1.21E+00	0.012058	0.024059	0.976228	3.098904	160.5928
0.44	1.21E+00	0.012113	0.024171	0.976119	3.050143	165.2053
0.45	1.22E+00	0.012169	0.024282	0.97601	3.002884	169.8587
0.46	1.23E+00	0.012224	0.024393	0.975902	2.957058	174.5533
0.47	1.23E+00	0.012279	0.024504	0.975794	2.912601	179.2893
0.48	1.24E+00	0.012335	0.024614	0.975686	2.869452	184.067

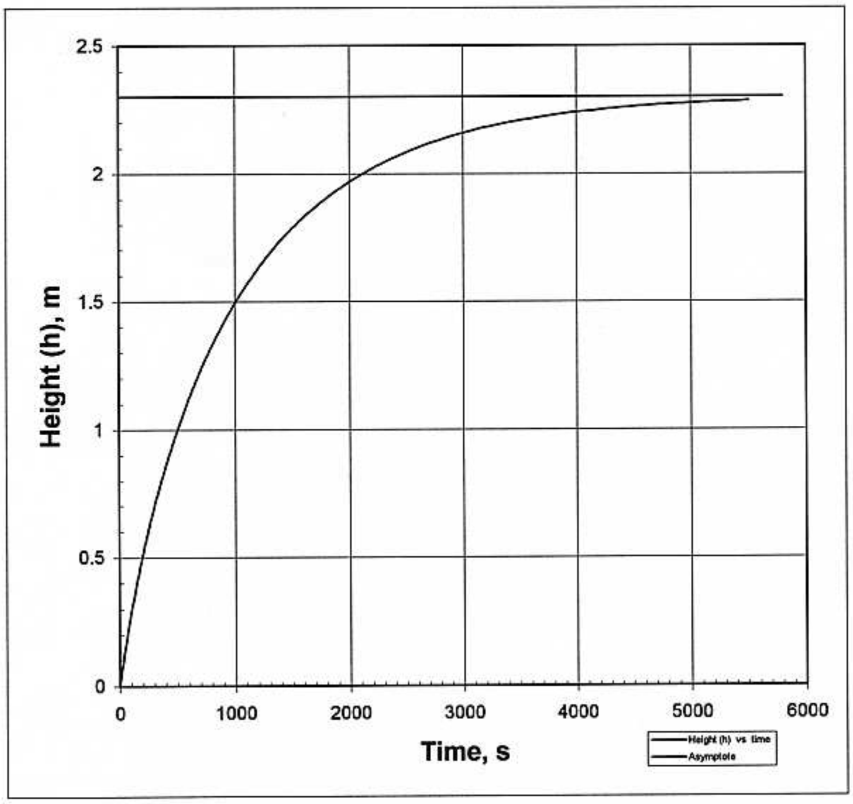
0.49	1.24E+00	0.01239	0.024724	0.975579	2.827553	188.8867
0.5	1.25E+00	0.012445	0.024835	0.975471	2.786851	193.7488
0.51	1.25E+00	0.012501	0.024946	0.975363	2.747296	198.6535
0.52	1.26E+00	0.012556	0.025057	0.975254	2.708838	203.6012
0.53	1.26E+00	0.012612	0.025168	0.975146	2.671432	208.5921
0.54	1.27E+00	0.012668	0.02528	0.975036	2.635037	213.6267
0.55	1.28E+00	0.012725	0.025393	0.974927	2.59961	218.7053
0.56	1.28E+00	0.012782	0.025506	0.974816	2.565115	223.8283
0.57	1.29E+00	0.012839	0.02562	0.974705	2.531513	228.9959
0.58	1.29E+00	0.012896	0.025735	0.974593	2.498771	234.2087
0.59	1.30E+00	0.012954	0.025851	0.974481	2.466855	239.467
0.6	1.30E+00	0.013013	0.025967	0.974367	2.435736	244.7712
0.61	1.31E+00	0.013072	0.026085	0.974252	2.405382	250.1217
0.62	1.32E+00	0.013132	0.026204	0.974137	2.375766	255.5189
0.63	1.32E+00	0.013192	0.026323	0.97402	2.346861	260.9633
0.64	1.33E+00	0.013253	0.026444	0.973902	2.318642	266.4554
0.65	1.33E+00	0.013314	0.026567	0.973783	2.291084	271.9955
0.66	1.34E+00	0.013376	0.02669	0.973663	2.264163	277.5842
0.67	1.35E+00	0.013439	0.026815	0.973541	2.237859	283.222
0.68	1.35E+00	0.013502	0.026941	0.973418	2.212149	288.9092
0.69	1.36E+00	0.013567	0.027069	0.973294	2.187014	294.6465
0.7	1.37E+00	0.013632	0.027198	0.973168	2.162435	300.4343
0.71	1.37E+00	0.013697	0.027329	0.973041	2.138392	306.2732
0.72	1.38E+00	0.013764	0.027462	0.972912	2.114868	312.1637
0.73	1.39E+00	0.013832	0.027596	0.972782	2.091847	318.1063
0.74	1.39E+00	0.0139	0.027731	0.97265	2.069313	324.1017
0.75	1.40E+00	0.013969	0.027869	0.972516	2.047249	330.1503
0.76	1.41E+00	0.014039	0.028008	0.97238	2.025641	336.2529
0.77	1.41E+00	0.01411	0.02815	0.972243	2.004475	342.4099
0.78	1.42E+00	0.014182	0.028293	0.972104	1.983737	348.6221
0.79	1.43E+00	0.014256	0.028438	0.971963	1.963414	354.8899
0.8	1.44E+00	0.01433	0.028585	0.971819	1.943494	361.2142
0.81	1.44E+00	0.014405	0.028734	0.971674	1.923964	367.5955
0.82	1.45E+00	0.014481	0.028886	0.971527	1.904814	374.0345
0.83	1.46E+00	0.014558	0.029039	0.971378	1.886031	380.532
0.84	1.47E+00	0.014637	0.029195	0.971227	1.867605	387.0885
0.85	1.48E+00	0.014716	0.029353	0.971074	1.849526	393.7049
0.86	1.48E+00	0.014797	0.029513	0.970918	1.831784	400.3819
0.87	1.49E+00	0.014879	0.029676	0.97076	1.814369	407.1202
0.88	1.50E+00	0.014962	0.029841	0.9706	1.797272	413.9206
0.89	1.51E+00	0.015047	0.030009	0.970437	1.780485	420.7839
0.9	1.52E+00	0.015132	0.030179	0.970272	1.763999	427.711
0.91	1.53E+00	0.01522	0.030352	0.970104	1.747805	434.7025
0.92	1.54E+00	0.015308	0.030528	0.969934	1.731895	441.7595
0.93	1.54E+00	0.015398	0.030706	0.969761	1.716263	448.8827
0.94	1.55E+00	0.015489	0.030887	0.969585	1.7009	456.0731
0.95	1.56E+00	0.015582	0.031071	0.969407	1.685799	463.3315
0.96	1.57E+00	0.015676	0.031258	0.969226	1.670954	470.6589
0.97	1.58E+00	0.015772	0.031448	0.969042	1.656358	478.0563
0.98	1.59E+00	0.015869	0.031641	0.968855	1.642004	485.5245
0.99	1.60E+00	0.015968	0.031837	0.968665	1.627886	493.0647

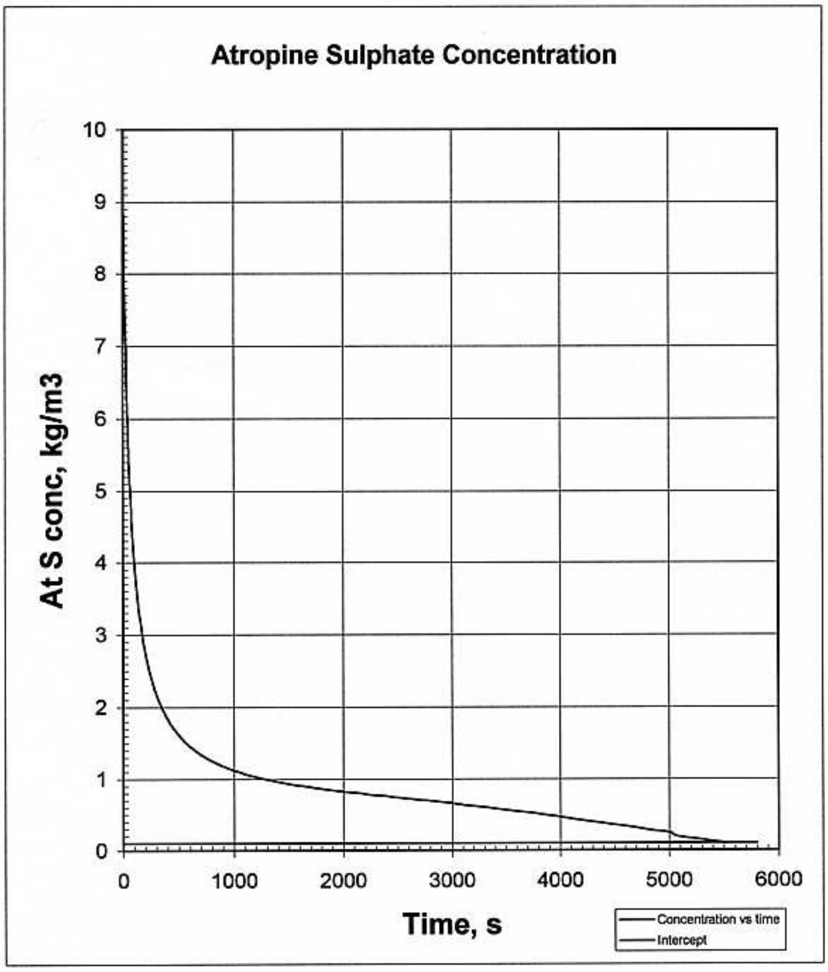
1	1.61E+00	0.016068	0.032036	0.968471	1.613999	500.6778
1.01	1.62E+00	0.016171	0.032239	0.968275	1.600335	508.365
1.02	1.63E+00	0.016274	0.032445	0.968076	1.586891	516.1271
1.03	1.64E+00	0.01638	0.032655	0.967873	1.57366	523.9655
1.04	1.65E+00	0.016487	0.032868	0.967667	1.560636	531.8811
1.05	1.67E+00	0.016597	0.033084	0.967457	1.547816	539.8752
1.06	1.68E+00	0.016708	0.033305	0.967244	1.535193	547.9489
1.07	1.69E+00	0.016821	0.033529	0.967027	1.522763	556.1034
1.08	1.70E+00	0.016936	0.033757	0.966806	1.510522	564.34
1.09	1.71E+00	0.017053	0.033989	0.966582	1.498465	572.66
1.1	1.72E+00	0.017172	0.034225	0.966354	1.486587	581.0647
1.11	1.74E+00	0.017293	0.034466	0.966122	1.474885	589.5554
1.12	1.75E+00	0.017417	0.03471	0.965885	1.463353	598.1335
1.13	1.76E+00	0.017543	0.034959	0.965645	1.451989	606.8005
1.14	1.77E+00	0.01767	0.035213	0.9654	1.440788	615.5576
1.15	1.79E+00	0.017801	0.035471	0.96515	1.429746	624.4066
1.16	1.80E+00	0.017933	0.035734	0.964897	1.41886	633.3488
1.17	1.81E+00	0.018069	0.036002	0.964638	1.408126	642.3859
1.18	1.83E+00	0.018206	0.036275	0.964375	1.397541	651.5195
1.19	1.84E+00	0.018347	0.036553	0.964107	1.387101	660.7512
1.2	1.86E+00	0.01849	0.036836	0.963834	1.376803	670.0827
1.21	1.87E+00	0.018635	0.037125	0.963555	1.366643	679.5158
1.22	1.89E+00	0.018784	0.03742	0.963272	1.35662	689.0523
1.23	1.90E+00	0.018936	0.03772	0.962983	1.346729	698.6941
1.24	1.92E+00	0.01909	0.038026	0.962688	1.336967	708.443
1.25	1.93E+00	0.019248	0.038338	0.962388	1.327333	718.301
1.26	1.95E+00	0.019408	0.038656	0.962082	1.317822	728.2702
1.27	1.97E+00	0.019572	0.03898	0.96177	1.308432	738.3526
1.28	1.98E+00	0.019739	0.039312	0.961451	1.299161	748.5503
1.29	2.00E+00	0.01991	0.03965	0.961126	1.290006	758.8655
1.3	2.02E+00	0.020084	0.039994	0.960795	1.280964	769.3006
1.31	2.04E+00	0.020262	0.040347	0.960456	1.272033	779.8578
1.32	2.05E+00	0.020444	0.040706	0.960111	1.26321	790.5396
1.33	2.07E+00	0.020629	0.041073	0.959759	1.254493	801.3484
1.34	2.09E+00	0.020819	0.041448	0.959399	1.24588	812.2869
1.35	2.11E+00	0.021012	0.041831	0.959031	1.237368	823.3576
1.36	2.13E+00	0.02121	0.042223	0.958656	1.228955	834.5633
1.37	2.15E+00	0.021413	0.042623	0.958273	1.220638	845.9068
1.38	2.17E+00	0.021619	0.043032	0.957881	1.212417	857.391
1.39	2.19E+00	0.021831	0.04345	0.95748	1.204288	869.0189
1.4	2.22E+00	0.022047	0.043878	0.957071	1.19625	880.7936
1.41	2.24E+00	0.022268	0.044315	0.956653	1.1883	892.7184
1.42	2.26E+00	0.022494	0.044763	0.956225	1.180436	904.7965
1.43	2.28E+00	0.022726	0.045221	0.955787	1.172657	917.0314
1.44	2.31E+00	0.022963	0.045689	0.955339	1.16496	929.4266
1.45	2.33E+00	0.023206	0.046169	0.95488	1.157344	941.9859
1.46	2.36E+00	0.023455	0.046661	0.954411	1.149807	954.7129
1.47	2.38E+00	0.02371	0.047165	0.95393	1.142346	967.6117
1.48	2.41E+00	0.023971	0.047681	0.953438	1.134961	980.6864
1.49	2.44E+00	0.024239	0.04821	0.952934	1.127648	993.9412
1.5	2.47E+00	0.024513	0.048752	0.952418	1.120408	1007.38

1.51	2.49E+00	0.024795	0.049308	0.951888	1.113236	1021.009
1.52	2.52E+00	0.025084	0.049878	0.951345	1.106133	1034.831
1.53	2.55E+00	0.02538	0.050464	0.950788	1.099095	1048.852
1.54	2.58E+00	0.025685	0.051065	0.950217	1.092122	1063.077
1.55	2.62E+00	0.025997	0.051682	0.949631	1.085212	1077.51
1.56	2.65E+00	0.026318	0.052316	0.949029	1.078362	1092.159
1.57	2.68E+00	0.026648	0.052967	0.948412	1.071571	1107.028
1.58	2.72E+00	0.026988	0.053636	0.947777	1.064838	1122.124
1.59	2.75E+00	0.027337	0.054324	0.947125	1.058161	1137.452
1.6	2.79E+00	0.027696	0.055032	0.946455	1.051538	1153.02
1.61	2.83E+00	0.028065	0.055761	0.945765	1.044967	1168.833
1.62	2.86E+00	0.028446	0.056511	0.945056	1.038446	1184.9
1.63	2.90E+00	0.028837	0.057283	0.944327	1.031974	1201.227
1.64	2.94E+00	0.029241	0.058079	0.943576	1.025549	1217.823
1.65	2.99E+00	0.029657	0.058899	0.942802	1.01917	1234.695
1.66	3.03E+00	0.030087	0.059744	0.942006	1.012834	1251.852
1.67	3.08E+00	0.03053	0.060616	0.941184	1.006539	1269.303
1.68	3.12E+00	0.030987	0.061517	0.940337	1.000284	1287.057
1.69	3.17E+00	0.031459	0.062446	0.939464	0.994067	1305.123
1.7	3.22E+00	0.031947	0.063407	0.938562	0.987886	1323.513
1.71	3.27E+00	0.032452	0.064399	0.937631	0.981739	1342.237
1.72	3.32E+00	0.032974	0.065426	0.936668	0.975623	1361.306
1.73	3.38E+00	0.033514	0.066488	0.935674	0.969538	1380.732
1.74	3.44E+00	0.034074	0.067588	0.934645	0.96348	1400.528
1.75	3.49E+00	0.034654	0.068728	0.933581	0.957447	1420.706
1.76	3.56E+00	0.035255	0.069909	0.932479	0.951438	1441.281
1.77	3.62E+00	0.035879	0.071134	0.931337	0.945449	1462.268
1.78	3.69E+00	0.036527	0.072406	0.930153	0.939478	1483.682
1.79	3.75E+00	0.0372	0.073727	0.928925	0.933523	1505.54
1.8	3.83E+00	0.0379	0.0751	0.92765	0.927581	1527.858
1.81	3.90E+00	0.038629	0.076529	0.926326	0.921649	1550.656
1.82	3.98E+00	0.039387	0.078016	0.92495	0.915724	1573.953
1.83	4.06E+00	0.040178	0.079565	0.923518	0.909802	1597.77
1.84	4.14E+00	0.041002	0.08118	0.922027	0.903881	1622.129
1.85	4.23E+00	0.041864	0.082866	0.920474	0.897957	1647.054
1.86	4.32E+00	0.042764	0.084627	0.918855	0.892025	1672.57
1.87	4.42E+00	0.043705	0.086469	0.917164	0.886083	1698.705
1.88	4.52E+00	0.044691	0.088396	0.915398	0.880125	1725.487
1.89	4.63E+00	0.045724	0.090415	0.913552	0.874147	1752.946
1.9	4.74E+00	0.046809	0.092533	0.911619	0.868144	1781.117
1.91	4.85E+00	0.047948	0.094757	0.909594	0.86211	1810.036
1.92	4.98E+00	0.049147	0.097096	0.907469	0.856039	1839.74
1.93	5.11E+00	0.05041	0.099557	0.905238	0.849925	1870.272
1.94	5.24E+00	0.051742	0.102152	0.902892	0.843761	1901.676
1.95	5.39E+00	0.053149	0.104892	0.900422	0.837539	1934.002
1.96	5.54E+00	0.054638	0.107788	0.897818	0.831251	1967.302
1.97	5.70E+00	0.056216	0.110854	0.89507	0.824887	2001.635
1.98	5.88E+00	0.05789	0.114106	0.892163	0.818437	2037.064
1.99	6.06E+00	0.059671	0.117562	0.889086	0.81189	2073.657
2	6.25E+00	0.061569	0.12124	0.885821	0.805232	2111.492
2.01	6.46E+00	0.063595	0.125164	0.882353	0.798449	2150.651

2.02	6.69E+00	0.065763	0.129358	0.87866	0.791526	2191.227	
2.03	6.93E+00	0.068088	0.133851	0.87472	0.784444	2233.322	
2.04	7.19E+00	0.070589	0.138677	0.870509	0.777182	2277.052	
2.05	7.47E+00	0.073285	0.143874	0.865997	0.769718	2322.542	
2.06	7.77E+00	0.076201	0.149486	0.86115	0.762023	2369.936	
2.07	8.10E+00	0.079364	0.155565	0.855931	0.754068	2419.395	
2.08	8.46E+00	0.082808	0.162172	0.850295	0.745817	2471.102	
2.09	8.85E+00	0.086571	0.169379	0.844189	0.737228	2525.264	
2.1	9.29E+00	0.0907	0.177271	0.837552	0.728252	2582.119	
2.11	9.76E+00	0.095251	0.185951	0.830314	0.718833	2641.942	
2.12	1.03E+01	0.100292	0.195543	0.822388	0.708902	2705.052	
2.13	1.09E+01	0.105907	0.206199	0.813671	0.698378	2771.822	
2.14	1.16E+01	0.1122	0.218107	0.80404	0.687162	2842.692	
2.15	1.23E+01	0.1193	0.2315	0.793343	0.675135	2918.187	
2.16	1.32E+01	0.127376	0.246676	0.781394	0.662148	2998.941	
2.165	1.36E+01	0.13407	0.261446	0.769937	0.651061	3041.524	
2.17	1.42E+01	0.139031	0.273102	0.761015	0.642159	3085.725	
2.175	1.47E+01	0.144378	0.28341	0.753211	0.634236	3131.669	
2.18	1.53E+01	0.150158	0.294537	0.744877	0.6259	3179.497	
2.185	1.60E+01	0.156426	0.306584	0.735957	0.617109	3229.368	
2.19	1.67E+01	0.163245	0.31967	0.726389	0.607811	3281.461	
2.195	1.75E+01	0.170692	0.333937	0.716099	0.59795	3335.98	
2.2	1.83E+01	0.178859	0.349551	0.705005	0.58746	3393.16	
2.205	1.93E+01	0.187854	0.366713	0.693009	0.576264	3453.271	
2.21	2.03E+01	0.197811	0.385665	0.679998	0.564272	3516.624	
2.215	2.15E+01	0.208892	0.406703	0.665842	0.551381	3583.586	
2.22	2.28E+01	0.221301	0.430193	0.650383	0.537467	3654.588	
2.225	2.43E+01	0.23529	0.456591	0.63344	0.522385	3730.144	
2.23	2.60E+01	0.251182	0.486471	0.614792	0.505964	3810.87	
2.235	2.79E+01	0.269394	0.520576	0.594178	0.487995	3897.521	
2.24	3.02E+01	0.290476	0.55987	0.571283	0.46823	3991.027	
2.245	3.29E+01	0.315165	0.605641	0.545724	0.446367	4092.557	
2.25	3.60E+01	0.344475	0.659641	0.517037	0.42204	4203.607	
2.255	3.99E+01	0.379842	0.724317	0.484655	0.394802	4326.133	
2.26	4.47E+01	0.423365	0.803207	0.44789	0.364111	4462.768	
2.265	5.09E+01	0.478246	0.901611	0.405915	0.329318	4617.166	
2.27	5.90E+01	0.549627	1.027873	0.357767	0.289668	4794.615	
2.275	7.02E+01	0.646328	1.195955	0.302415	0.244358	5003.187	
2.276	7.30E+01	0.716298	1.362626	0.255988	0.20676	5049.665	
2.277	7.60E+01	0.74521	1.461508	0.231886	0.187218	5098.027	
2.278	7.93E+01	0.77656	1.52177	0.218325	0.176198	5148.431	
2.279	8.28E+01	0.81067	1.587229	0.204491	0.164967	5201.058	
2.28	8.67E+01	0.847921	1.658591	0.190407	0.153543	5256.111	
2.281	9.10E+01	0.88877	1.736691	0.176102	0.14195	5313.825	
2.282	9.57E+01	0.933763	1.822533	0.161616	0.130221	5374.47	
2.283	1.01E+02	0.983566	1.917329	0.146999	0.118396	5438.358	
2.284	1.07E+02	1.038994	2.02256	0.132316	0.106527	5505.855	91.76425 ← t in mins.
2.284001	1.07E+02	1.068257	2.107251	0.121572	0.097876	5505.924	
2.284002	1.07E+02	1.068319	2.136577	0.118058	0.095048	5505.994	
2.284003	1.07E+02	1.068381	2.1367	0.118044	0.095036	5506.063	
2.284004	1.07E+02	1.068443	2.136824	0.118029	0.095024		

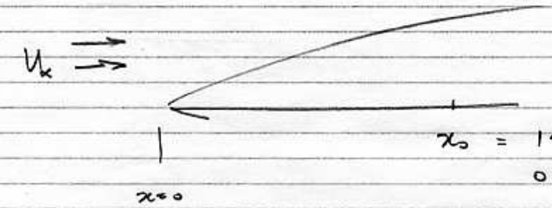
2.284005	1.07E+02	1.068505	2.136948	0.118014	0.095012	
2.284006	1.07E+02	1.068567	2.137072	0.118	0.095001	
2.284007	1.07E+02	1.068629	2.137195	0.117985	0.094989	
2.284008	1.07E+02	1.068691	2.137319	0.117971	0.094977	
2.284009	1.07E+02	1.068753	2.137443	0.117956	0.094965	
2.28401	1.07E+02	1.068814	2.137567	0.117941	0.094953	
2.284011	1.07E+02	1.068876	2.137691	0.117927	0.094942	
2.284012	1.07E+02	1.068938	2.137815	0.117912	0.09493	
2.284013	1.07E+02	1.069	2.137939	0.117898	0.094918	
2.284014	1.07E+02	1.069062	2.138062	0.117883	0.094906	
2.284015	1.07E+02	1.069124	2.138186	0.117868	0.094894	
2.284016	1.07E+02	1.069186	2.13831	0.117854	0.094883	
2.284017	1.07E+02	1.069248	2.138434	0.117839	0.094871	
2.284018	1.07E+02	1.06931	2.138558	0.117825	0.094859	
2.284019	1.07E+02	1.069372	2.138682	0.11781	0.094847	
2.28402	1.07E+02	1.069434	2.138806	0.117795	0.094835	
2.284021	1.07E+02	1.069496	2.13893	0.117781	0.094824	
2.3					0.1	0
2.3					0.1	5800


2.5
2
1.5
1
0.5
0
Height (h), m
0
1000
2000
3000
4000
5000
6000
Time, s
Height (h) vs time
Asymptote



Atropine Sulphate Concentration
At S conc, kg/m3
10
9
8
7
6
5
4
3
2
1
0
0
1000
2000
3000
4000
5000
6000
Time, s
Concentration vs time
Intercept

Problem # 2



$x_0 = 1.4$ m Spring
0.9 m Fall

local wall shear stress, $\tau_w = 0.136$ Pa

(a)

Use integral method and the results in the Notes, p 91, equation 5.24

$$\tau_w = \frac{3}{2}\frac{\mu U_\infty}{\delta} = \frac{3}{2}\frac{\mu U_\infty}{4.64}\left[\frac{U_\infty}{\nu x}\right]^{\frac{1}{2}}$$

Spring, $x_0 = 1.4$ m

$$\tau_w = \frac{3}{2} \cdot \frac{1.55(10^{-3})\, U_\infty}{4.64}\left[\frac{U_\infty}{1.55(10^{-6})(1.4)}\right]^{\frac{1}{2}} = 0.136$$

$$U_\infty^{3/2} = 0.399818, \qquad U_\infty = 0.543 \text{ m/s}$$

Fall, $x_0 = 0.9$ m

$$U_\infty^{3/2} = 0.320569, \qquad U_\infty = 0.468 \text{ m/s}$$

(b) Assumption that boundary layer is laminar.

Proof: $Re_x = \dfrac{U_\infty x_0 \rho}{\mu} \le 5(10^5)$

Spring is more critical, $Re_x = 4.902(10^5)$ ∴ OK.

(c) b.l. thickness $\delta = 4.64 \sqrt{\frac{\nu x_0}{U_a}}$ — eq. 5.20 Notes.

displacement thickness, $\delta_1 = \frac{3}{8}\delta$ — p 94, Notes

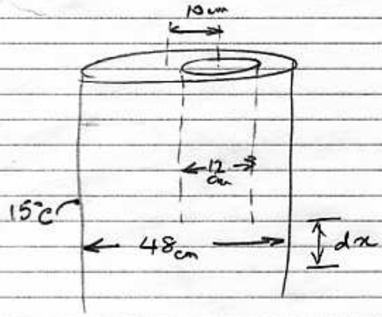
momentum thickness, $\delta_2 = 0.1392\,\delta$ — p 95 Notes

$\delta = 0.00928\text{ m}$ or 9.3 mm →

$\delta_1 = 0.00348\text{ m}$ →

$\delta_2 = 0.001291\text{ m}$ →

Problem #3



This is a shape factor problem.

$$dS = \frac{2\pi\, dx}{\cosh^{-1}\left(\frac{r_2^2 + r_1^2 - e^2}{2 r_1 r_2}\right)} = \beta\, dx$$

$$dQ = k(dS)(T - T_\alpha)$$

where $T_\alpha = 15°C$

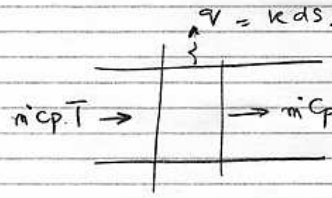
and $k = 0.028$ W/mK

$r_1 = 24$ cm , $r_2 = 6$ cm , $e = 10$ cm

$$\therefore \beta = \frac{2\pi}{\cosh^{-1}\left(\frac{6^2 + 24^2 - 10^2}{2(6)(24)}\right)} = \frac{2\pi}{1.177928} = 5.3341$$

(1.7778)

$q = k\, dS\, \Delta T$



Hence $\dot{m} C_p\, dT = -k\, dS (T - T_\alpha)$ (-ve because it is a loss)

$= -k\beta (T - T_\alpha)\, dx$

or $$\frac{dT}{T - T_\alpha} = -\frac{k\beta}{\dot{m} C_p}\, dx$$

$\dot{m} = \rho \dot{V} = \rho u \frac{\pi d^2}{4}$

$= 0.006152$ kg/s

Solve $$\ln(T - T_\alpha)\Big|_{T=72}^{T} = -\frac{k\beta}{\dot{m} C_p} x_0 \qquad \frac{T-15}{72-15} = 0.7144$$

when $x_0 = 36$ m , $T = 38.9122$ °C ⟶