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ENCH 501 Transport Phenomena

Final Examination, December 15, 2018

Time Allowed: Noon - 3pm

Instructions: Attempt all questions. Use of electronic calculators allowed but **no other electronic device allowed**. Open Notes, Open Book Examination.

Problem 1 (25 points)

Most houses in Canada have hot water generating and storage systems in the basement. The water heater is most often an insulated tank with the water inside heated by a gas burner at the bottom, or with electric heating coils installed at two or more levels. Typical tanks have capacities of 40 or 50 US gallons (151.4/189.3 litres). Cold water is discharged into the tank through a dip pipe with the exit close to the bottom of the tank. Hot water leaves by a pipe with the inlet close to the top of the tank. As soon as hot water is being withdrawn to go to the kitchen and bathroom sinks, the showers and/or the washing machines, cold water enters the tank at the same volumetric rate as hot water leaves, and either the gas burner or electric heater is turned on. The burner or heater turns off once the average temperature of the water in the tank attains a set-point temperature, typically between 48 and 60°C. The recovery rate for the unit indicates the amount of water (in gallons) that can be raised by 90.25°F from the feed water temperature in one hour, and this is related to the energy supply rate. On some weekends, after many loads of laundry using hot water from the tank, there may be a shortage of hot water for a shower immediately afterwards. Similar problems are observed at small-to-medium scale industrial facilities.

Consider a gas-heated hot water tank of 50 US gallon capacity. The set-point is 56°C and the recovery rate is 40 gallons per hour. Overnight, the tank was full and the water had attained its set-point temperature. In the morning, the clothes and dishwashers were turned on, and hot water was being used in the kitchen. The hot water withdrawal rate from the water heater was constant at 14.2 litres per minute. The feed cold water is at 7°C. Assume the water in the tank is well mixed. You may use 0°C as the reference temperature.

a) Obtain an expression for the temperature of the water in the tank as a function of time.

b) After how long will the temperature of the water leaving the tank be at 18°C (and too cold for a shower)?

Data: Properties of water – Specific heat C_p = 4.814 kJ/kg K; assume the density of water leaving the tank ρ = 991.2 kg/m^3; density of feed water at 7°C ρ_f = 999 kg/m^3

Problem 2 (25 points)

A gel is composed of 98% by weight water and the balance is agar agar. The gel is effectively immobilized water. The gel was made in a deep cylindrical jar of constant cross-section. The gel initially contained no copper sulphate, or solute A. At time zero, a large volume of a solution of copper sulphate in water, with a mole fraction x_α of the solute in the binary mixture, is poured over the gel and is kept agitated so that the coefficient of convective material transfer between the solution and the gel surface is k_a. That is, the molar transfer rate of the solute towards the interface (solution side) is $k_aC(x_\alpha - x_s)$, where C is the molar concentration of the solution (assumed constant) and x_s is the mole fraction of the solute at the interface. The latter (x_s) changes with time. At the gel side of the interface, the flux of water (N_B) is zero and the solute is transported through a stagnant medium. Assume that the solubility of solute A in the gel is low, i.e. x in the gel is << 1.

Obtain an expression for the depth of penetration of solute A (δ_m) into the gel as a function of time. Use the **integral method** and show your derivations.

Problem 3 (25 points)

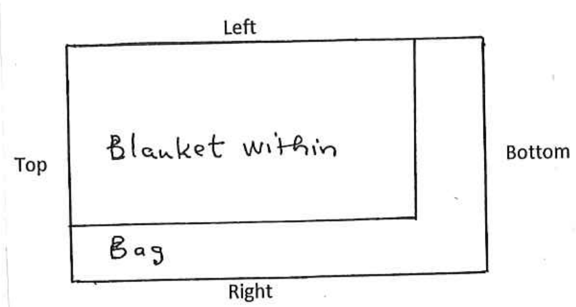
Outdoors camping in winter requires the used of specialized sleeping bags with a shell (or outer fabric) that is tightly woven and impervious to cold air and moisture. The inner layers of insulator and lining must also be designed to be comfortable and conserve heat from the body inside. The effectiveness of a bag depends also on the rates and temperature of the air flowing over the surface.

Consider a test on a rectangular bag that 205 cm long and 96 cm wide. An electric blanket, 170 cm long 80 cm wide is placed inside the bag as shown in the sketch (from top view). The blanket simulated a person sleeping within the bag with the feet at the bottom side. The lower surface of the bag is assumed well insulated. If the temperature at the top surface of the bag (only above where the blanket lies) is constant at 16°C, and air at -23°C and 0.2 m/s flows over the surface from the bottom side,

a) Estimate the amount of heat that needs to be generated by the electric blanket to maintain a steady state. Use the **integral method** and show important steps. Treat the part of the bag not above the blanket as an unheated leading edge.

b) If the air flow (at the same velocity) is from the right side of the "sleeper", is the rate of heat generation by the blanket to maintain a steady state different from the result for part (a)?

Data: Properties of air at -23°C - ρ = 1.4119 kg/m^3; μ= 1.606 (10^{-5}) Pa.s; C_p = 1003 J/kg K; k = 0.0226 W/mK.



Problem 4 (25 points)

Give concise but appropriate answers to the following:

a) (4 pts) An open water channel is in the shape of a V with an angle of 60° between the sides of length b. If the channel is full of flowing water, estimate the hydraulic diameter.

b) (5 pts) A gas is contained in a soccer ball and it is suggested the pressure is given by the ideal gas equation, P = nRT/V where n is the number of moles, R is the Universal Gas Constant, T is absolute temperature and V is the volume occupied. If P is to be determined to an accuracy of 2%, and you are not provided with the accuracy of the devices to measure the other variables, what are there allowable errors for each?

c) (8 pts) If the heat transfer coefficient for forced convection through a pipe depends on the diameter of the pipe D, the average flow velocity V, the fluid viscosity μ, the fluid density ρ, the heat capacity of the fluid Cp and the thermal conductivity of the fluid k, what are the dimensionless groups?

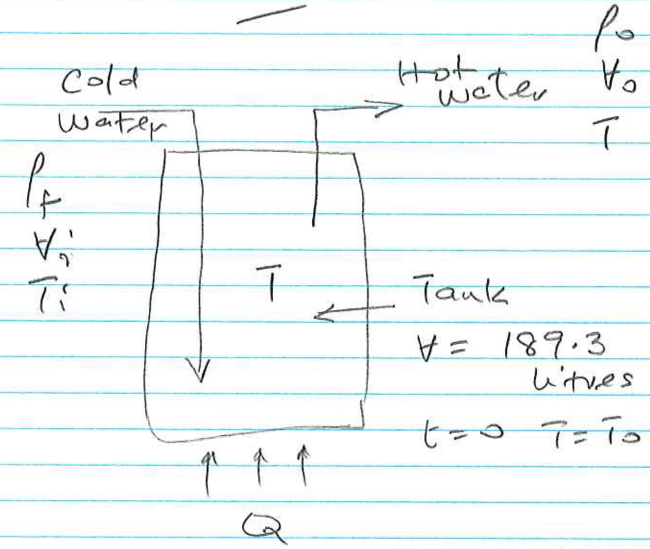
d) (4 pts) For flow over a flat plate, show that the momentum thickness δ_2 may be estimated as 0.1392 δ.

e) (4 pts) Obtain an expression for the rate of distortion in angular deformation. Use sketches to describe.

G. Teja

ENCH 501 Final Exam Solution F18

Problem 1



At any instant, the temperature of water out of the tank is the same as the temp. of water in the tank at that instant.

Estimate the rate of heat input into the tank from the gas heater. This is determined from the recovery rate. It is given that 40 gallons (US) or 151.4 litres feed / can have its temperature increased by 90.25 °F in 1 hour.

$$\therefore \quad Q = \underset{m^3}{(151.4)(10^{-3})}\ \underset{kg/m^3}{(999)}\ \underset{J/kg\,K}{(4814)}\ \underset{K}{\left(\frac{90.25}{1.8}\right)}\ \underset{hr/s}{\frac{1}{3600}}$$

$$= 10140.7 \quad W$$

(a)

Energy Balance on the tank, with 0°C as ref.

Input + Gen = Output + Accum.

$$\dot{V}_i \rho_i C_p T_i + Q = \dot{V}_o \rho_o C_p T + \frac{d}{dt}\left(V \rho_o C_p T\right)$$

Let $A = \dfrac{\forall_i \rho_i C_p T_i + Q}{\forall \rho_o C_p}$

and $\beta = \forall_o / \forall$

Then the equation becomes

$$\frac{dT}{dt} + \beta T = A$$

Re-arrange

$$\frac{dT}{A - \beta T} = -\frac{1}{\beta}\frac{d(A-\beta T)}{A - \beta T} = dt$$

$$\therefore \int_{T_0}^{T} d \ln(A - \beta T) = -\beta \int_0^t dt$$

$$\ln\left(\frac{A - \beta T}{A - \beta T_0}\right) = -\beta t$$

or $A - \beta T = (A - \beta T_0) e^{-\beta t}$

$$\therefore T = \frac{A}{\beta} - \left(\frac{A}{\beta} - T_0\right) e^{-\beta t}$$

⟶

(b)

$$\beta = \frac{\forall_o}{\forall} = \frac{14.2/60}{189.3} = 0.00125 \ s^{-1}$$

$$A = \frac{(14.2/60)(10^{-3})(999)(4184)(7) + 10140.7}{(189.3)(10^{-3})(991.2)(4184)}$$

when $T = 18°C$

$$18 = \frac{A}{\beta} - \left(\frac{A}{\beta} - 56\right) e^{-0.00125t}$$

where $\frac{A}{\beta} = 17.3869$

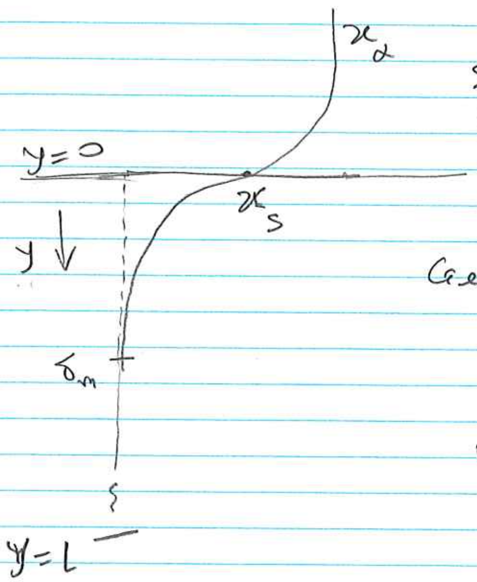
On substitution

$$-4.1427 = -\beta t$$

or $t = 3{,}314.19\,s$ or $\sim$ 55 mins 14 s

$\longrightarrow$

Problem 2



Balance on A within the gel;

Assume a profile

$$\frac{x}{x_s} = \left(1 - \frac{y}{\delta_m}\right)^2$$

which satisfies the conditions

$$y = 0 \qquad x = x_s(t)$$

$$y = \delta_m \qquad x = 0$$

$$y = \delta_m \qquad \frac{dx}{dy} = 0$$

The material balance on A in gel is given by

$$\text{Input} + \underbrace{\text{Gen}}_{0} = \underbrace{\text{Output}}_{0} + \text{Accum.}$$

$$N_A\Big|_{y=0} = \frac{d}{dt}\left[\int_0^L c\,x\,dy\right]$$

The condition at $y=0$ is convective

$$k_a c\,(x_\alpha - x_s) = N_A\Big|_{y=0}$$

and for a binary mixture

$$N_A = -c\,D_{AB}\frac{dx_A}{dy} + x_A(N_A + N_B)$$

in the gel; where $N_B = 0$

$$\therefore \quad N_A = -\frac{c\,D_{AB}}{1-x_A}\frac{dx_A}{dy}$$

The b.c. is hence

$$k_a\,C\,(x_\alpha - x_s) = -\frac{C\,D_{AB}}{1-x_s}\left.\frac{dx}{dy}\right|_{y=0}$$

$$\therefore \quad k_a / D_{AB} = \frac{\frac{1}{1-x_s}\left.\frac{dx}{dy}\right|_{y=0}}{(x_\alpha - x_s)}$$

From the assumed profile

$$\frac{dx}{dy} = \left(-\frac{2}{\delta_m} + \frac{2y}{\delta_m^2}\right)x_s$$

$$\therefore \text{ at } y=0 \quad \frac{dx}{dy} = -\frac{2x_s}{\delta_m}$$

$$\therefore \quad \frac{k_a\,\delta_m}{2\,D_{AB}} = x_s \Big/ (1-x_s)(x_\alpha - x_s)$$

$\longrightarrow$?

If $x_s << 1$, then

$$\frac{k_a\,\delta_m}{2\,D_{AB}} \simeq \frac{x_s}{x_\alpha - x_s}$$

from the integral equation

$$k_a C\,(x_\lambda - x_s) = \frac{d}{dt}\left[\int_0^{\delta_m} C\, x_s\left(1-\frac{y}{\delta_m}\right)^2 dy\right]$$

$$= \frac{d}{dt}\left[C x_s \delta_m \int_0^1 (1 - 2\eta + \eta^2)\, d\eta\right]$$

where $\eta = y/\delta_m$

$$k_a\,(x_\lambda - x_s) = \frac{d}{dt}\left[x_s \delta_m / 3\right]$$

Divide both sides by x_λ

$$3 k_a \left(1 - \frac{x_s}{x_\lambda}\right) = \frac{d}{dt}\left[\frac{x_s}{x_\lambda}\delta_m\right]$$

One of $x_s(t)$ or $\delta_m(t)$ is to be removed by using the relationship

$$\frac{k_a \delta_m}{2 D_{AB}} = \frac{x_s}{x_\lambda - x_s}$$

$\therefore$

$$3 k_a \left(\frac{1}{1 + \frac{k_a \delta_m}{2 D_{AB}}}\right) = \frac{d}{dt}\left[\frac{\frac{k_a \delta_m}{2 D_{AB}}}{1 + \frac{k_a \delta_m}{2 D_{AB}}}\cdot \delta_m\right]$$

subject to the condition

$t = 0 \quad \delta_m = 0$

This equation is re-arranged as:

$$6 D_{AB}\left(\frac{1}{2D_{AB}+k_a\delta_m}\right) = \frac{d}{dt}\left[\frac{\delta_m^2}{2D_{AB}+k_a\delta_m}\right]$$

$$= \frac{2\delta_m}{2D_{AB}+k_a\delta_m}\frac{d\delta_m}{dt} + \delta_m^2\frac{d}{dt}\left[\frac{1}{2D_{AB}+k_a\delta_m}\right]$$

$$= \frac{2\delta_m}{2D_{AB}+k_a\delta_m}\frac{d\delta_m}{dt} + \delta_m^2\left[\frac{-k_a\, d\delta_m/dt}{(2D_{AB}+k_a\delta_m)^2}\right]$$

$$6 D_{AB} = 2\delta_m\frac{d\delta_m}{dt} - \frac{k_a\delta_m^2}{(2D_{AB}+k_a\delta_m)}\frac{d\delta_m}{dt}$$

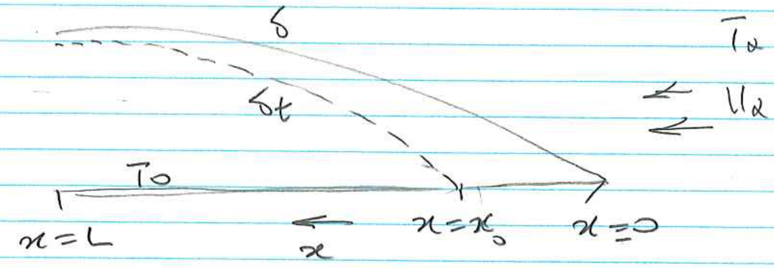
$$6 D_{AB}\int_0^t dt = \int_0^{\delta_m} d\delta_m^2 - \int_0^{\delta_m}\frac{k_a\delta_m^2}{2D_{AB}+k_a\delta_m}\, d\delta_m$$

To yield:

$$6 D_{AB} t = \frac{\delta_m^2}{2} + \frac{2D_{AB}\delta_m}{k_a} - \frac{4D_{AB}^2}{k_a^2}\ln\left[1+\frac{k_a\delta_m}{2D_{AB}}\right]$$

This is the required expression. $\longrightarrow$

Problem 3



Check that the b.l. is laminar

$$Re_L = \frac{L U_\infty \rho}{\mu} = \frac{2.05\,(0.2)(1.4119)}{1.606\,(10^{-5})} = 0.36\,(10^5)$$

$\therefore$ laminar.

Also check that at $x = L$, $\delta_t < \delta$ so $\xi < 1$

From Notes

$$\xi = \frac{1}{1.026}\,(P_r)^{-\frac{1}{3}}\left[1 - \left(\frac{x_0}{L}\right)^{\frac{3}{4}}\right]^{\frac{1}{3}}$$

For flow from bottom to top, $x_0 = 0.35\,m$,
$L = 2.05\,m$, $P_r = 0.7127$

$\therefore \quad \xi \doteq 0.9844 \quad < 1$

For flow from right to left, $x_0 = 0.16\,m$,
$L = 0.96\,m$

$\therefore \quad \xi = 0.9864 \quad < 1$

a   A portion of the bag is not heated.

From Notes,

The total heat transfer is

$$Q = \int_{x_o}^{L} h_x (T_o - T_\alpha)\, dx \cdot W$$

where $W = 0.8\,m$, $T_o = 16°C$, $T_\alpha = -23°C$

The function h_x is given by

$$h_x = 0.332\, k\, P_r^{1/3} \left[\frac{U_\alpha}{\nu x}\right]^{1/2} \left[1 - \left(\frac{x_o}{x}\right)^{3/4}\right]^{-1/3}$$

where $k = 0.0226\ W/mK$, $P_r = 0.7127$

$\nu = \mu/\rho = 1.1375(10^{-5})\ m^2/s$, $U_\alpha = 0.2\ m/s$

and $x_o = 0.35\,m$

$$h_x = 0.8888\, x^{-1/2} \left[1 - \left(\frac{0.35}{x}\right)^{3/4}\right]^{-1/3}$$

Determine

$$Q = W(T_o - T_\alpha)\int_{x_o}^{L} h_x\, dx \quad \text{numerically.}$$

Prepare a table of h_x vs x and use the trapezoidal rule to estimate the integral.

$$\int_{0.35}^{2.05} h_x\, dx \simeq 2.100683 \ \frac{W}{mK}$$

$$\therefore \quad Q = 0.8(16-(-23))(2.1) = 65.52\ W$$

$\longrightarrow$

(b) for flow of air from right to left

$$Q = \int_{x_0}^{L} h_x (T_0 - T_a)\, dx \cdot W$$

where $W = 1.7\,m$, $x_0 = 0.16\,m$, $L = 0.96\,m$

$$h_x = 0.8888\, x^{-1/2} \left[1 - \left(\frac{0.16}{x} \right)^{3/4} \right]^{-1/3}$$

$$\int_{0.16}^{0.96} h_x\, dx = 1.42895$$

$$\therefore \quad Q = (1.7)(16 - (23))(1.42895) = 94.74\ W$$

It requires 44.6% more heat be generated for flow from right to left than from bottom to top

Question 3 Final F18 ENCH 501

Bottom to top

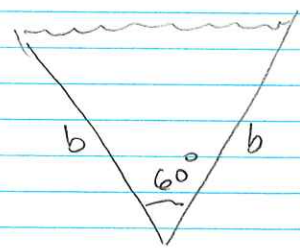
x	hx	Area
0.3501	25.03361	
0.3505	14.64411	0.007936
0.36	5.370305	0.095068
0.37	4.239157	0.048047
0.38	3.683279	0.039612
0.39	3.328814	0.03506
0.4	3.074261	0.032015
0.42	2.720821	0.057951
0.44	2.478508	0.051993
0.46	2.297209	0.047757
0.48	2.154027	0.044512
0.5	2.03671	0.041907
0.6	1.655628	0.184617
0.7	1.434931	0.154528
0.8	1.285033	0.135998
0.9	1.17425	0.122964
1	1.087915	0.113108
1.2	0.960153	0.204807
1.4	0.868626	0.182878
1.6	0.798894	0.166752
1.8	0.743488	0.154238
2.05	0.687975	0.178933
		2.100683 Sum

Right to left fow

0.1601	28.52752	
0.1605	16.68326	0.009042
0.161	13.23598	0.00748
0.162	10.49428	0.011865
0.165	7.705096	0.027299
0.17	6.079074	0.03446
0.18	4.769167	0.054241
0.19	4.120469	0.044448
0.2	3.704657	0.039126
0.22	3.174021	0.068787
0.24	2.833264	0.060073
0.26	2.58792	0.054212
0.28	2.399229	0.049871
0.3	2.247716	0.046469
0.4	1.773698	0.201071
0.5	1.511702	0.16427
0.6	1.339053	0.142538
0.7	1.214215	0.127663
0.8	1.118534	0.116637
0.9	1.042192	0.108036
0.96	1.003183	0.061361
		1.428951 Sum

Problem 4

(a)



x-section area of triangle is

$\frac{1}{2}$ base x height =

$(b \sin 30)(b \cos 30) =$

$0.433\, b^2$

Hydraulic diam

$$D_H = \frac{4\,(\text{x-section})}{\text{wetted perimeter}} = \frac{4\,(0.433)\,b^2}{2b}$$

$$= 0.866\, b \longrightarrow$$

(b) $P = \dfrac{nRT}{V}$ Assume correlated data.

$$\frac{\Delta P}{P} = \pm\left(\frac{\Delta n}{n} + \frac{\Delta T}{T} + \frac{\Delta V}{V}\right) = 0.02$$

Using the principle-of-equal-effects,

$$\frac{\Delta n}{n} = \frac{\Delta T}{T} = \frac{\Delta V}{V} = \frac{0.02}{3}$$

$\therefore$ each can have 0.67% error $\longrightarrow$

(c) $h = f\,(D, V, \mu, \rho, C_p, k)$

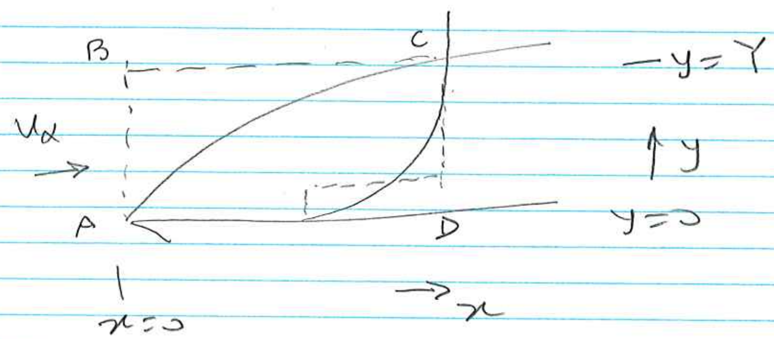
7 variables + 4 dimensions | L, M, t, T

$\therefore$ 3 groups.

Using the Pi-theorem or other methods, the dimensionless groups are

$$\frac{hD}{k}, \quad \frac{DV\rho}{\mu} \quad \text{and} \quad \frac{C_p\mu}{k}$$

(d)



From Notes — momentum in AB $\int_0^Y \rho_\alpha U_\alpha^2 \, dy$

momentum out CD $\int_0^Y \rho u^2 \, dy$

Momentum out BC $U_\alpha \int_0^Y (\rho_\alpha U_\alpha - \rho u) \, dy$

Combined momentum loss

$$= \int_0^Y \rho u (U_\alpha - u) \, dy$$

This equals momentum in of thickness δ_2

so $\delta_2 U_\alpha^2 \rho_\alpha = \int_0^Y \rho u (U_\alpha - u) \, dy$

$$\therefore \delta_2 \simeq \int_0^\delta \frac{\rho u}{\rho_\alpha U_\alpha} \left(1 - \frac{u}{U_\alpha}\right) dy$$

If $\rho_a = \rho$ = constant and

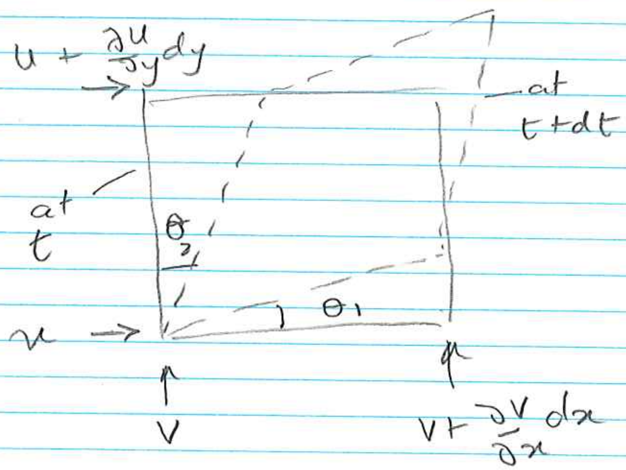
$$\frac{u}{U_a} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$ from Notes

Substitute & solve

$$\delta_2 = 0.1392\ \delta$$

$\longrightarrow$

(e)



Rate of distortion = $\frac{1}{2}$ (rate of angular deformation)

$$\theta_1 \simeq \frac{\frac{\partial v}{\partial x} dx \cdot dt}{dx}$$

$$\theta_2 = \frac{\frac{\partial u}{\partial y} dy \cdot dt}{dy}$$

$\therefore$ Rate of distortion

$$= \frac{1}{2} \frac{d(\theta_1 + \theta_2)}{dt} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$