GJ

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Department of Chemical & Petroleum Engineering

ENCH 501 Transport Phenomena

Final Examination, December 20, 2017

Time Allowed: Noon - 3pm

Instructions: Attempt all questions. Use of electronic calculators allowed but **no other electronic device allowed**. Open Notes, Open Book Examination.

Problem 1 (25 points)

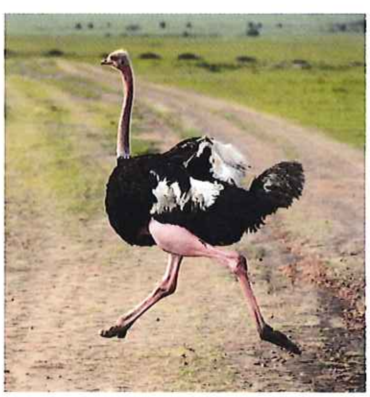
A liquid (ρ = 920 kg/m^3; Cp = 2.2 kJ/kg K; k = 0.13 W/m K; μ = 1.7 mPa s) flows over a horizontal flat plate maintained at a constant temperature of 300°C. The plate is 0.8 m wide and 2.8m long. The flow is parallel to the long side. The free stream velocity of the liquid is 0.31m/s and the bulk liquid temperature is 20°C. At the rear end of the plate, all the liquid within the boundary layer ($0 \le y \le \delta$) is to be collected and diverted into a heat exchanger. It is required that the average temperature of the liquid entering the heat exchanger is 200°C.

Would the design requirement for the heat exchanger be met? Show your analysis. Use the integral method.

Problem 2 (15 points)

Chicken and ostriches are, like humans, bipedal. Both birds normally run from danger than fly away. They are also designed in a comparable manner. The thigh bone (under the feathers) is close to the body. The shin (exposed in the picture) is the "drumstick". Both the thigh and the shin have powerful muscles while the rest of the leg is long, light and bony. Such a design allows both birds to run faster than if they have the human structure. Dimensional quantities that describe normal runs are the speed **u**, the stride length **λ**, the hip-to-ground height **h** and the acceleration of gravity **g**.

Given that an ostrich runs at a steady speed of 50 kph with a stride of 3m and a hip-to-ground height of 1.5m, at what speed and with what stride will a chicken run if the hip-to-ground height is 10cm? Show your analysis.



Problem 3 (3~~0~~5 points)

A hot, near-saturated solution from which a salt is to be crystallized on cooling is to be transferred from one vessel to another. The receiving vessel is a 304-stainless steel, flat-bottomed cylindrical tank, 1.5 m inside diameter, 2m tall and its wall is 5mm thick. It is placed on a glass-wool insulator but its curved wall is exposed to ambient air at 18°C. The initial temperature of the receiving vessel is the same as that for the air. The solution is originally at 92°C and, on cooling it becomes saturated at 37°C. When the solution was poured into the receiving vessel the first time, it was noticed that much of the salt precipitated at and stuck to the tank wall, even with continuous agitation of the solution. This made a recovery of the salt difficult.

A smart engineer on site suggested pre-heating the tank with hot water that is available at 75°C. This hot water is rapidly poured into the tank until it is full. A thin plastic sheet is used to cover the open top of the tank. It is assumed that the thermal mass of the cover is negligible and that the cover is an insulator. The convective coefficient of heat transfer between air and the outer surface of the tank is 9 W/m^{2}K. The hot water is considered well mixed in the tank and the convective coefficient of heat transfer between the water and the inside surface of the tank is 17 W/m^{2}K. Also assume that the tank can be treated as a lumped system, i.e. there are no temperature gradients within the tank wall. After the tank has attained the desired temperature, water is replaced by the solution and the liquid immediately seeded to encourage particle growth within the liquid, not at the wall.

a) To what minimum temperature must the tank be heated to avoid possible salt precipitation at the wall when the heating water is replaced with the solution? Explain.
b) How long will it take for the tank to attain the minimum temperature, and what would the water temperature be at this instant? Obtain expressions for the temperatures of the water and the tank as functions of time.

Data: Material properties

	Density, kg/m^3	Specific heat, kJ/kg K
304 Stainless Steel	8030	0.5
Water	988	4.182

Problem 4 (25 points)

When a thermocouple or a thermometer at room temperature (18°C) is suddenly inserted into a hot pool of a liquid, time must be allowed for the device to attain a temperature close to that of the liquid before a reading is recorded. The problem is about estimating the lag time.

A type-J thermocouple (constantan -iron) is available to measure the temperature of water in a heater. The lead wires are fused at the tip to form a spherical bead with a diameter of 2mm. Each of the lead wires has a diameter of 0.6mm. It is given that 30% by volume of the bead is iron on the side that the iron wire is attached. The constantan wire is attached to the other portion of the bead. A calibrated precision thermometer in the water records a temperature of 72°C. If the thermocouple originally at 18°C is suddenly transferred into the hot such that the bead and 2 cm length of each of the lead wires are immersed, estimate the lag time for the thermocouple to record a temperature of 70°C. The water is gently stirred such that the heat transfer coefficient around the thermocouple is 15 W/m2 K. Show all your steps. Justify assumptions.

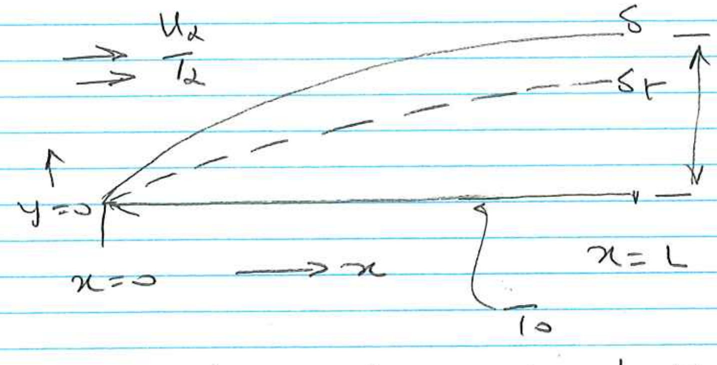
Data:

Material	ρ, kg/m^3	Cp, kJ/kg K	k, W/m K
Constantan	8890	0.39	19.5
Iron	7753	0.486	36

Q. Trejo

ENCH 501 Final Exam solution Dec 20, 2017

Problem #1



Let the reference temperature be T_α and the wall temp = T_0.

The first check is whether the b.l. is laminar.

$$\nu = \mu/\rho = 1.8478(10^{-6})$$

$$Re_L = \frac{U_\alpha L}{\nu} = \frac{(0.31)(2.8)}{(1.8478)(10^{-6})} = 4.6975(10^5)$$

$\therefore$ laminar

from the notes, with 4 b.c.s –

$y=0,\ u=0$; $y=\delta,\ u=U_\alpha$; $y=\delta,\ \frac{du}{dy}=0$; $y=0,\ \frac{d^2u}{dy^2}=0$

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

The momentum balance equation (integral method) is

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho(U_\alpha - u)u\,dy\right]$$

On substitution and solution of resulting o.d.e.,

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

at $L = 2.8$m, $\delta = 0.018956$ m (or 1.896 cm)

Through similar derivations (Notes)

$$\xi = \frac{\delta_t}{\delta} = \frac{1}{1.026} P_r^{-1/3} \quad ; \quad P_r = \frac{C_p \mu}{k} = 28.769$$

$$= 0.31812$$

$$\therefore \delta_t = 6.0303\,(10^{-3})\ m \quad \text{or} \quad 6.03\ mm.$$

The total amount of energy gained by the fluid / flowing through the layer $0 \le y \le \delta$ at $x = L$ is given by

$$Q = W\int_0^{\delta_t} (T - T_\alpha)\, C_p\, \rho u\, dy \quad = W(\rho \bar{u} \delta)\, C_p (\bar{T} - T_\alpha)$$

where W is width of plate. This equation is to be solved for $\bar{T}$. The upper limit is $\delta_t \because T - T_\alpha = 0,\ y > \delta_t$

$$\bar{u} = \frac{1}{\delta}\int_0^{\delta} u\, dy = U_\alpha \int_0^{\delta} \frac{u}{U_\alpha}\frac{dy}{\delta} \quad ; \text{ Let } \eta = \frac{y}{\delta}$$

then

$$\bar{u} = U_\alpha \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right) d\eta = U_\alpha \left[\frac{3}{4}\eta^2 - \frac{1}{8}\eta^4\right]_0^1$$

$$= \frac{5}{8} U_\alpha$$

Now

$$Q = W U_\alpha (T_0 - T_\alpha) \int_0^{\delta} \rho C_p \frac{u}{U_\alpha}\left(\frac{T - T_\alpha}{T_0 - T_\alpha}\right) dy$$

Since (from Notes)

$$\frac{T - T_0}{T_\alpha - T_0} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

$$Q = W\, U_\alpha (T_0 - T_\alpha)\, \rho\, C_p\, \delta \int_0^{\delta_t} \frac{u}{U_\alpha}\left(1 - \frac{T - T_0}{T_\alpha - T_0}\right) \frac{dy}{\delta}$$

$$= \beta \int_0^{\xi} \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left(1 - \left[\frac{3}{2}\frac{\eta}{\xi} - \frac{1}{2}\left(\frac{\eta}{\xi}\right)^3\right]\right) d\eta$$

where $\beta = W\, U_\alpha (T_0 - T_\alpha)\, \rho\, C_p\, \delta$; $\xi = \frac{1}{1.026} Pr^{-1/3}$

$$= \beta \left\{ \frac{3}{20}\xi^2 - \frac{3}{280}\xi^4 \right\} = \frac{3\beta}{20}\left\{ \xi^2 - \frac{1}{14}\xi^4 \right\}$$

With $\xi = 0.31812$,

$$Q = \frac{3}{20}\beta\left(0.1012 - 7.3154\,(10^{-4})\right) = \frac{3}{20}\beta\,(0.10047)$$

Equate the terms, subst. for $\bar{u}$

$$\frac{3}{20}(0.10047)\, \cancel{W}\, U_\alpha (T_0 - T_\alpha)\, \cancel{\rho}\, \cancel{C_p}\, \cancel{\delta} = \cancel{W}(\cancel{\rho}\,\bar{u}\,\cancel{\delta})\,\cancel{C_p}\,(\bar{T} - T_\alpha)$$

$$\frac{3}{20}(0.10047)(300 - 20) = \frac{5}{8}(\bar{T} - 20)$$

$$\bar{T} = 26.75\ ^\circ C$$

This is far below the required inlet temperature for the heat exchanger.

Problem #2

Geometric, dynamic and kinematic similarity is assumed between the ostrich and the chicken.

The variables are λ, u, h and g.

The 2 dimensions are [L] and [t]

$\therefore$ 2 dimensionless gps can be formed by inspection or use of the Buckingham Pi theorem.

The dimensionless gps, by inspection, are

$$\frac{u}{\sqrt{g\lambda}} \quad \text{and} \quad \frac{\lambda}{h}$$

	1	2
u	13.9 m/s	?
λ	3 m	?
h	1.5 m	10 cm

For geometric similarity,

$$\left.\frac{\lambda}{h}\right)_1 = \left.\lambda/h\right)_2 \quad \text{or} \quad \lambda_2 = \frac{3}{1.5}(0.1)$$

ostrich chicken

$$= 0.2\text{ m}$$

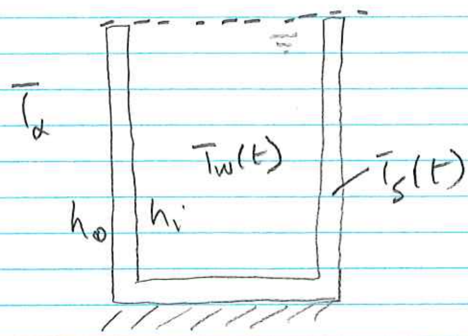
For kinematic similarity,

$$\left.\frac{u}{\sqrt{g\lambda}}\right|_1 = \left.\frac{u}{\sqrt{g\lambda}}\right|_2 \quad \text{or} \quad \frac{13.9}{\sqrt{3}} = \frac{u_2}{\sqrt{0.2}}$$

$$u_2 = 3.59\text{ m/s} \quad (12.92\text{ km/hr})$$

This is more than $\frac{1}{3}$ of the speed of a performance athlete

Problem #3



Let mass of water $= m_w = \forall_w \rho_w$

and mass of tank $= m_s = \forall_s \rho_s$

The energy content of the water, relative to ambient temperature, is

$$m_w C_{p_w} (T_w - T_\alpha)$$

and that of the tank is $m_s C_{p_s} (T_s - T_\alpha)$

$T_s \neq T_w$ except at $t \to \alpha$.

Energy balance on the water as it cools

$$\underbrace{\text{Input}}_{0} + \underbrace{\text{Gen}}_{0} = \text{Output} + \text{Accum.}$$

$$0 = h_i A_i (T_w - T_s) + m_w C_{p_w} \frac{d(T_w - T_\alpha)}{dt}$$

The Energy balance on the tank wall, as it heats up and also loses heat to the ambient

$$\text{Input} + \underbrace{\text{Generation}}_{0} = \text{Output} + \text{Accum.}$$

$$h_i A_i (T_w - T_s) = h_o A_o (T_s - T_\alpha) + m_s C_{p_s} \frac{d(T_s - T_\alpha)}{dt}$$

Let $\theta_w = T_w - T_\alpha$ and $\theta_s = T_s - T_\alpha$

The energy equations are respectively

water $\quad m_w C_{p_w} \dfrac{d\theta_w}{dt} = -h_i A_i (\theta_w - \theta_s)$

and steel $\quad m_s C_{p_s} \dfrac{d\theta_s}{dt} = h_i A_i (\theta_w - \theta_s) - h_o A_o \theta_s$

$$= h_i A_i \theta_w - (h_i A_i + h_o A_o)\theta_s$$

Let $\beta_w = -\dfrac{h_i A_i}{m_w C_{p_w}}$, $\beta_s = \dfrac{h_i A_i}{m_s C_{p_s}}$ and

$$\gamma_s = \frac{h_i A_i + h_o A_o}{m_s C_{p_s}}$$

The energy equations are

(1) $\quad \dfrac{d\theta_w}{dt} = \beta_w (\theta_w - \theta_s)$

(2) $\quad \dfrac{d\theta_s}{dt} = \beta_s \theta_w - \gamma_s \theta_s \quad$ or

$$\frac{d\theta_s}{dt} + \gamma_s \theta_s = \beta_s \theta_w$$

The conditions are $t = 0$, $\theta_w = \theta_{wo}$, $\theta_s = 0$

These are two coupled equations.

Differentiate equation (2) w.r.t. time

$$\frac{d^2\theta_s}{dt^2} + \gamma_s \frac{d\theta_s}{dt} = \beta_s \frac{d\theta_w}{dt}$$

Substitute this into equation (1)

$$\frac{1}{\beta_s}\left(\frac{d^2\theta_s}{dt^2} + r_s \frac{d\theta_s}{dt}\right) = \frac{\beta_w}{\beta_s}\left(\frac{d\theta_s}{dt} + r_s\theta_s\right) - \beta_w\theta_s$$

On re-arranging, one obtains

$$(3) \qquad \frac{d^2\theta_s}{dt^2} + (r_s - \beta_w)\frac{d\theta_s}{dt} - \beta_w(r_s - \beta_s)\theta_s = 0$$

subject to the conditions

$$t = 0, \qquad \theta_s = 0$$

$$t \to \infty, \qquad \theta_s \to 0$$

The general form of this equation is

$$\frac{d^2\theta_s}{dt^2} + a\frac{d\theta_s}{dt} + b\theta_s = 0$$, a homogeneous equation

or $(D^2 + aD + b)\theta_s = 0$

or $(D - r_1)(D - r_2)\theta_s = 0$

The roots are

$$r_1 = \frac{-a + \sqrt{a^2 - 4b}}{2}$$

$$r_2 = \frac{-a - \sqrt{a^2 - 4b}}{2}$$

and $\theta_s = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

Using the i.c. $t=0$ $\theta_s = 0$

means $0 = C_1 + C_2$ or $C_2 = -C_1$

$$\therefore \theta_s = C_1 \left[e^{r_1 t} - e^{r_2 t} \right] \quad ; \ C_1 \text{ unknown constant}$$

solve equation (1) by method of integrating factor

$$\frac{d\theta_w}{dt} = \beta_w \left(\theta_w - f(t) \right) \quad \text{where } f(t) = \theta_s(t)$$

$$\frac{d\theta_w}{dt} - \beta_w \theta_w = -\beta_w C_1 \left(e^{r_1 t} - e^{r_2 t} \right)$$

$$e^{\int -\beta_w dt} \frac{d\theta_w}{dt} + e^{\int -\beta_w dt} (-\beta_w) \theta_w =$$

$$- e^{\int -\beta_w dt} \beta_w C_1 \left(e^{r_1 t} - e^{r_2 t} \right)$$

or

$$\theta_w e^{-\beta_w t} = \int (-\beta_w) C_1 e^{-\beta_w t} \left(e^{r_1 t} - e^{r_2 t} \right) dt + C_2$$

$$= -\beta_w C_1 \int \left[e^{(r_1 - \beta_w)t} - e^{(r_2 - \beta_w)t} \right] dt + C_2$$

$$= -\beta_w C_1 \left[\frac{e^{(r_1 - \beta_w)t}}{r_1 - \beta_w} - \frac{e^{(r_2 - \beta_w)t}}{r_2 - \beta_w} \right] + C_2$$

At $t=0$, $\theta_w = \theta_{wo} = T_{wo} - T_a$

or $$\theta_{wo} = -\beta_w C_1 \left[\frac{1}{r_1 - \beta_w} - \frac{1}{r_2 - \beta_w} \right] + C_2$$

(Full marks awarded for setting up the equations correctly. Solution obtained using Matlab as follows.)

Final Exam ENCH 501, Dec 20, 2017, Problem 3

1 – water /solution; 2 – wall of the tank;

T_1 - temperature water/solution in the tank, °C;

T_2 - temperature tank wall, °C;

$T_\infty = 18$ °C – ambient temperature;

1. Heat balance for water / solution:

   $$\cancel{Input} + \cancel{Generation} = Output + Accumulation$$

   $$\boxed{0 = h_1 A_1 (T_1 - T_2) + m_1 Cp_1 \frac{dT_1}{dt}};$$

2. Heat balance for the tank wall:

   $$Input + \cancel{Generation} = Output + Accumulation$$

   $$\boxed{h_1 A_1 (T_1 - T_2) = h_2 A_2 (T_2 - T_\infty) + m_2 Cp_2 \frac{dT_2}{dt}};$$

   Let's rewrite 1)-2) as:

   $$\begin{cases} \dfrac{dT_1}{dt} = \dfrac{h_1 A_1}{m_1 Cp_1}(T_2 - T_1); \\ \dfrac{dT_2}{dt} = \dfrac{h_1 A_1 (T_1 - T_2) - h_2 A_2 (T_2 - T_\infty)}{m_2 Cp_2}. \end{cases} \qquad (1)$$

   After substitute numerical values, we will have

   $$\begin{cases} \dfrac{dT_1}{dt} = 1.31 \cdot 10^{-5} \cdot (T_2 - T_1); \\ \dfrac{dT_2}{dt} = 8.43 \cdot 10^{-4} \cdot (T_1 - T_2) - 3.79 \cdot 10^{-4} \cdot (T_2 - 18). \end{cases} \qquad (2)$$

   System (2) we solve by Matlab with unknowns T_1 and T_2.

```
function dydt = Temper(t,y)
%Temper  Evaluate temperatures water and walls
%
%
%ODE45.

dydt = [1.31e-5*(y(2)-y(1)); 8.43e-4*(y(1)-y(2))-3.79e-4*(y(2)-18)];
```

Run Final Exam ENCH 501, Dec 20, 2017, Problem 3

```
>> [t,y] = ode45(@Temper,[0 60*11.5],[92; 18])
```

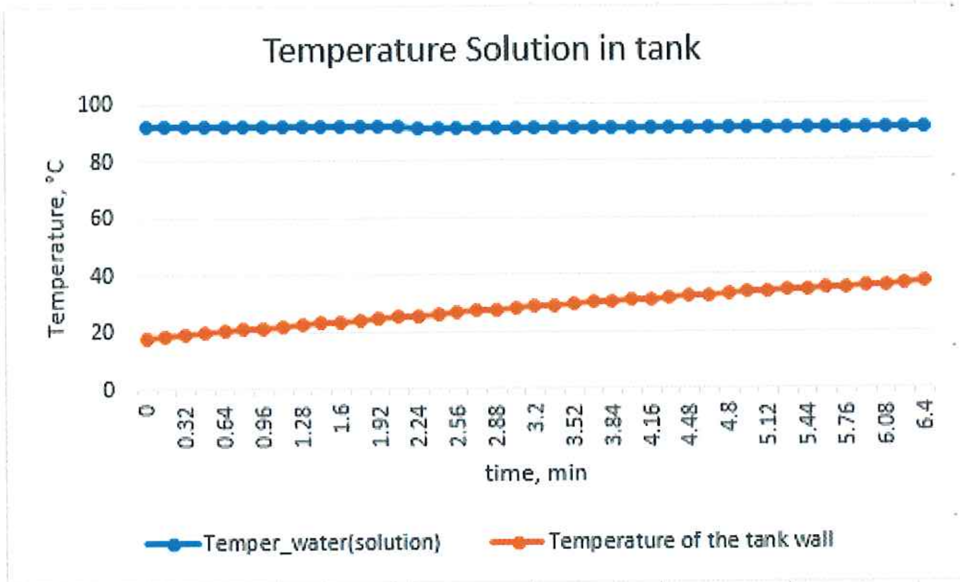
Draw plot:

```
>> plot(t,y(:,1),'-o',t,y(:,2),'-o')

grid on

legend('T_1','T_2')

>> ylabel('Temperature, C');

xlabel('Time t, sec');

>>  title('Pre-heating tank walls by the hot water, T=75 C');
```

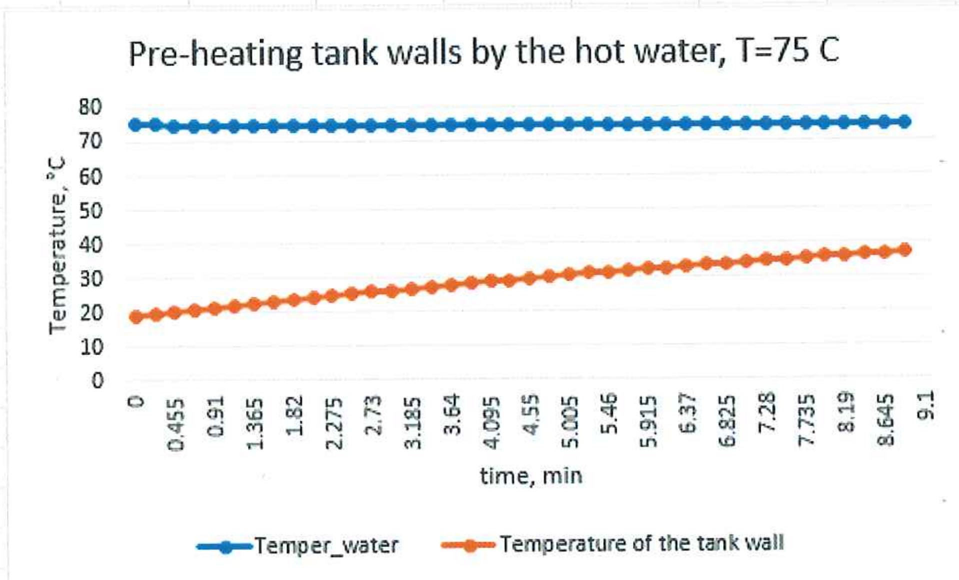
Solution are cooling withot pre-heating			
T_water_0=		92	C
T_tank_0=		18	C
T_outside=		18	C

1. If tank is without pre-heating, it takes 6.4 min to get temperature 37 $^\circ$C from initial 18 $^\circ$C on the wall.

2. Answer for Q. a) you need to have initial wall temperature on the wall 37 $^\circ$C (T_min) to avoid salt precipitation on it.
3. Q. b). You need 9.1 min to heat tank wall from 18 to 37 $^\circ$C by water with initial temperature 75 $^\circ$C.


Temperature Solution in tank
Temperature, °C
100
80
60
40
20
0
0
0.32
0.64
0.96
1.28
1.6
1.92
2.24
2.56
2.88
3.2
3.52
3.84
4.16
4.48
4.8
5.12
5.44
5.76
6.08
6.4
time, min
Temper_water(solution)
Temperature of the tank wall

t, sec	t, min	T_water	T_tank_wall
0	0	92	18
9.6	0.16	91.9907	18.5953
19.2	0.32	91.9815	19.1836
28.8	0.48	91.9724	19.765
38.4	0.64	91.9634	20.3395
48	0.8	91.9544	20.9073
57.6	0.96	91.9455	21.4684
67.2	1.12	91.9367	22.0228
76.8	1.28	91.9279	22.5707
86.4	1.44	91.9192	23.1122
96	1.6	91.9106	23.6472
105.6	1.76	91.9021	24.176
115.2	1.92	91.8936	24.6985
124.8	2.08	91.8852	25.2148
134.4	2.24	91.8768	25.7251
144	2.4	91.8685	26.2294
153.6	2.56	91.8603	26.7277
163.2	2.72	91.8521	27.2201
172.8	2.88	91.844	27.7067
182.4	3.04	91.836	28.1876
192	3.2	91.828	28.6628
201.6	3.36	91.8201	29.1324
211.2	3.52	91.8123	29.5964
220.8	3.68	91.8045	30.055
230.4	3.84	91.7967	30.5082
240	4	91.7891	30.956
249.6	4.16	91.7814	31.3985
259.2	4.32	91.7739	31.8359
268.8	4.48	91.7664	32.268
278.4	4.64	91.7589	32.6951
288	4.8	91.7515	33.1171
297.6	4.96	91.7442	33.5341
307.2	5.12	91.7369	33.9462
316.8	5.28	91.7296	34.3535
326.4	5.44	91.7224	34.7559
336	5.6	91.7153	35.1536
345.6	5.76	91.7082	35.5466
355.2	5.92	91.7012	35.9349
364.8	6.08	91.6942	36.3187
374.4	6.24	91.6872	36.6979
384	6.4	91.6803	37.0727



t, sec	t, min	T_water	T_tank_wall
0	0	74.9899	18.6504
13.65	0.2275	74.9799	19.2899
27.3	0.455	74.9699	19.9188
40.95	0.6825	74.9602	20.5371
54.6	0.91	74.9505	21.145
68.25	1.1375	74.9409	21.7429
81.9	1.365	74.9315	22.3307
95.55	1.5925	74.9221	22.9087
109.2	1.82	74.9129	23.477
122.85	2.0475	74.9037	24.0358
136.5	2.275	74.8947	24.5852
150.15	2.5025	74.8857	25.1255
163.8	2.73	74.8769	25.6567
177.45	2.9575	74.8681	26.1791
191.1	3.185	74.8595	26.6927
204.75	3.4125	74.8509	27.1977
218.4	3.64	74.8424	27.6943
232.05	3.8675	74.834	28.1825
245.7	4.095	74.8257	28.6626
259.35	4.3225	74.8175	29.1346
273	4.55	74.8094	29.5988
286.65	4.7775	74.8013	30.0551
300.3	5.005	74.7934	30.5039
313.95	5.2325	74.7855	30.9451

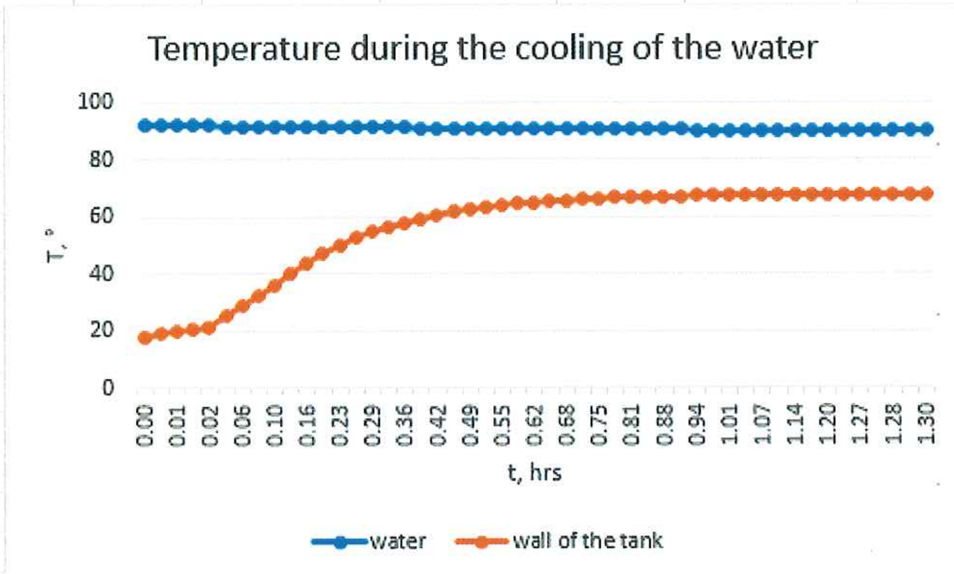
327.6	5.46	74.7777	31.3789
341.25	5.6875	74.77	31.8055
354.9	5.915	74.7623	32.2249
368.55	6.1425	74.7548	32.6373
382.2	6.37	74.7473	33.0428
395.85	6.5975	74.7399	33.4415
409.5	6.825	74.7325	33.8335
423.15	7.0525	74.7252	34.2189
436.8	7.28	74.718	34.5979
450.45	7.5075	74.7109	34.9705
464.1	7.735	74.7038	35.3369
477.75	7.9625	74.6968	35.6972
491.4	8.19	74.6899	36.0514
505.05	8.4175	74.683	36.3997
518.7	8.645	74.6762	36.7421
532.35	8.8725	74.6694	37.0788
546	9.1		

	26-Dec-17		
	Problem 3, Final Exam ENCH 501		
h1=		17	W/m^2K
h2		9	W/m^2K
L=		2	m
D_out=		1.5	m
delta=		0.005	m
ro1=		988	kg/m^3
ro2=		8030	kg/m^4
Cp1=		4182	J/kgK
Cp2=		500	J/kgK
D_int=		1.49	m
L1=		1.995	m
A1=Pi*D_int*L1+ Pi*D_int^2/4=		11.0822037	m^2
A2=Pi*D_outt*L=		9.42477796	m^2
h1*A1=		188.397463	W/K
h2*A2=		84.8230016	W/K
V1=(Pi*D_int^2/4)*L1=		3.47860661	m^3
V2=Pi*D_out^2*L-V1=		0.05568512	m^3
m1=ro1*V1=		3436.86333	kg
m2=ro2*V2=		447.151534	kg
m1*Cp1=		1.44E+07	J/K
m2*Cp2=		2.24E+05	J/K
h1A1/m1Cp1=		1.31E-05	1/sec
h2A2/m2Cp2=		3.79E-04	1/sec
h1A1/m2Cp2=		8.43E-04	1/sec

1. At what time does the tank attain a maximum temperature with the pre-heating water, and what is this temperature?

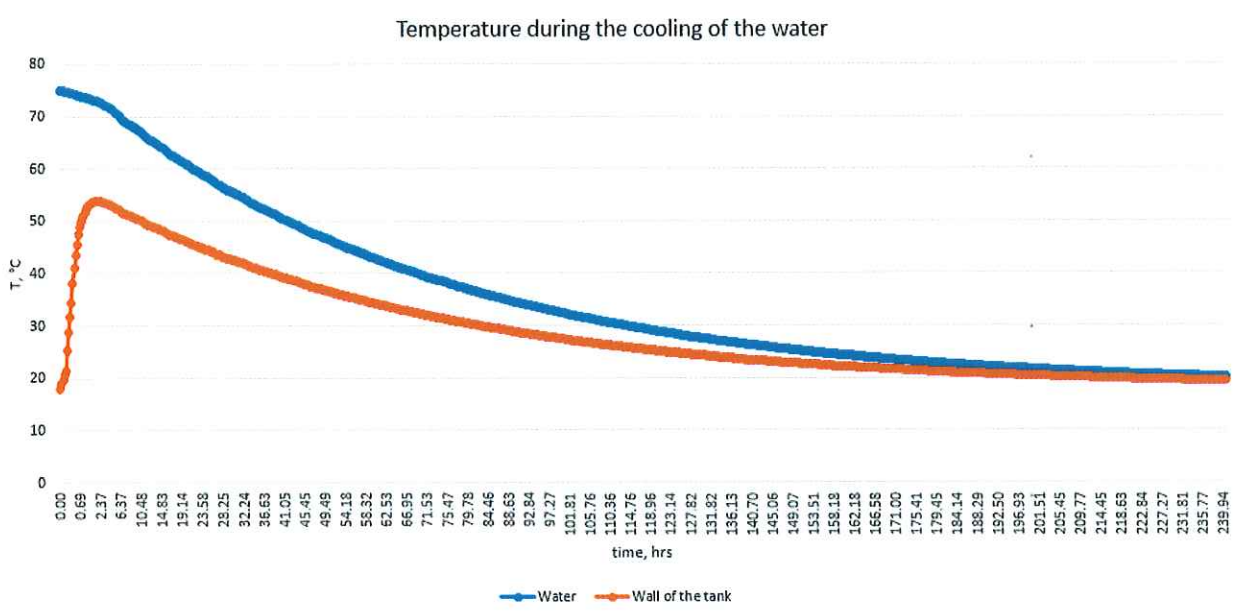
$T_{\max}(water) = 67.73\,^{\circ}C$ — Water temperature when the wall attains its maximum temperature of about 54 °C.

$t|_{T_{\max}} = 78\,\text{min} = 1.3\,hrs$

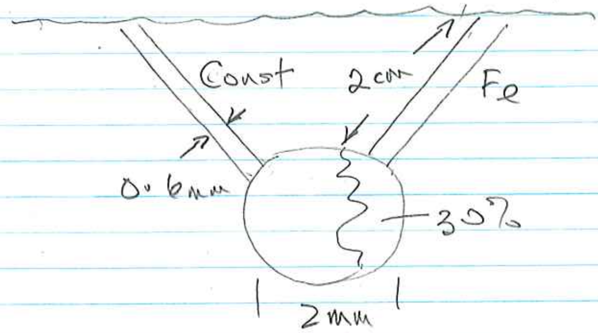


2. If the pre-heating water is not discharged from the tank and both the water and the tank wall continue to cool down together, what is the pattern of temperature decay with time (to about 20C)?

$$t\big|_{T=20\,^{\circ}C} = 9.18\,days$$



Problem # 4.



This is a composite body system

neglect any heat generated by electrical currents

Control volume — spherical bead and wires

Let Constantan = 1 + Iron = 2

Energy balance

$$\text{Input} + \cancel{\text{Gen}} = \cancel{\text{Output}} + \text{Accum}$$

$$hA(T_\infty - T) \qquad\qquad \text{Gen} \to 0 \qquad \text{Output} \to 0$$

Note that accumulation is relative to the amount of heat gained from t=0 →

$$(m_1 C_{p_1} + m_2 C_{p_2}) \frac{d(T - T_0)}{dt}$$

$$M_1 = \rho_1 V_1 = \rho_1 \left(0.7\left(\tfrac{4}{3}\pi R_s^3\right) + \pi R_1^2 L \right)$$

$$M_2 = \rho_2 V_2 = \rho_2 \left(0.3\left(\tfrac{4}{3}\pi R_s^3\right) + \pi R_2^2 L \right)$$

$$A = \left(4\pi R_s^2 - \pi R_1^2 - \pi R_2^2\right) + 2\pi R_1 L + 2\pi R_2 L$$

Total exposed area

- Check whether lumped analysis method is applicable. This is given by

$$\varepsilon = \frac{h\,(V/A)}{k} < 0.1$$

Assume the entire body is constantan (as it has the lower k and ∴ gives a higher value for ε.)

$$\frac{4}{3}\pi R_s^3 = 1.333\pi(10^{-9}) \ m^3$$

$$\pi R_1^2 L \equiv \pi R_2^2 L = 1.8\pi(10^{-9}) \ m^3$$

$$\therefore \ \forall_1 = 8.587(10^{-9}) \ m^3$$

$$\forall_2 = 6.9115(10^{-9}) \ m^3$$

$$\Sigma V = 1.5499 (10^{-8}) \ m^3$$

$$A = 8.74(10^{-5}) \ m^2$$

$$h = 15 \ W/m^2 K$$

$$k = 19.5 \ W/mK.$$

$$\therefore \ \frac{h\left(\frac{\forall}{A}\right)}{k} = \frac{15\left(\frac{1.55(10^{-8})}{8.74(10^{-5})}\right)}{19.5} = 1.36(10^{-4}) << 0.1$$

$\therefore$ lumped method is valid.

The energy balance equation becomes

$$\frac{dT}{dt} = \frac{hA}{m_1 C_{p_1} + m_2 C_{p_2}} (T_a - T)$$

$$\int_{T_0}^{T} \frac{dT}{T_a - T} = \beta \int_0^t dt \ ; \quad \beta = \frac{hA}{m_1 C_{p_1} + m_2 C_{p_2}}$$

$$-\int_{T_0}^{T} \frac{d(T_a - T)}{T_a - T} = \beta \int_0^t dt$$

$$\ln\left(\frac{T_a - T}{T_a - T_0}\right) = -\beta t$$

$$p = \frac{15\,(8.74)(10^{-5})}{(8890)(8.587)(10^{-9})390 + 7753\,(6.9115)(10^{-9})(486)}$$

$$= \frac{0.001311}{0.029772 + 0.02604} = 0.023489$$

Given $T_2 = 72\,°C$, $T = 70\,°C$, $T_0 = 18\,°C$

$$\ln \frac{72 - 70}{72 - 18} = -0.023489\, t$$

$$t = 140.3 \ s$$

→