aJ

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501 Transport Phenomena

Final Examination, Fall 2016

Time: 12.00 to 3.00 pm **Wednesday, December 21, 2016**

Instructions: Attempt All Questions. Use of Electronic Calculators Allowed; no other Electronic Device Allowed. Open Notes, Open Book Examination.

Question #1 (30 points)

A company in Sweden, Kihlbergs Steel AB, specializes in forging (shaping) large pieces of hot (yellow-white color) metals recovered from furnaces into various configurations. A circular cylinder of steel may be progressively forged into a rod in a short time. Hot metals are softer and more pliable, and thus require less force by mechanical presses to achieve desirable shapes without creating fractures, stress lines or fault planes.

Consider a steel cylinder that is 50 cm in diameter and 1 m long retrieved from a furnace when it is glowing yellow-white hot. Using mechanical arms, this is to be shaped into a cylindrical rod that has a diameter of 20cm. The initial temperature of the steel is 1300°C and the shaping process is to be completed by the time the temperature of the surface of the material dropped to 850°C (when the metal appears red hot). The operation is carried out in an inert atmosphere of nitrogen at a constant temperature of 26°C. Heat is lost from the metal by both convection and radiation, but both are combined in such a way to indicate an effective convective heat transfer coefficient of 15 W/m^2 K around the rod being made. During the forging, the area of the rod exposed to the ambient gas (A) is observed to approximately follow the relationship

$$A = A_i + (A_f - A_i)(1 - \exp(-\lambda t))$$

where A_i is the initial surface area (including the ends), A_f is the final surface area, the time constant ($1/\lambda$) equals 50s and t is time in seconds. Given the data below,

a) Estimate the maximum time available to complete the forging, based on the operating temperature range.
b) What would the temperature of the rod being shaped be at 10s from the start?

State and justify your assumptions. The coefficient of volumetric expansion (or contraction) is assumed negligible. Assume the metal surface, at any instant, is entirely exposed to the ambient gas.

Data: Properties of hot steel – ρ = 7,840 kg/m^3; k = 27.3 W/m K; and C_p = 0.65 kJ/kg K

Question #2 (30 points)

Snow drifts result in the formation of intricate and interesting sculptures in fields and on or near highways, in the absence of roadside barriers. They are typically formed by the wind moving and depositing snow flakes and small ice particles against stationary objects. Drifts onto roads that sometimes pose dangers for motorists occur because the particle-laden wind flowing across a road start to drop particles from the leading edge presented by the pavement, and such particles are then dragged over the surface into the driving lanes. The particles, given the sizes, will be suspended in air at certain velocities. In the region next to the pavement where the axial velocity (u) is smaller than required to drag the particles through air, the particles would settle. A clear (particle-free zone) is usually observed near the road surface while above it, the air is dense with particles above. The locus of the boundary of this clear zone is to be estimated.

Consider a cold (-23°C) wind blowing at a steady speed of 6 km/hr across a flat road that is raised at the edges. The road (also at -23°C) is bare as any snow deposits are quickly swept off. The size of the "ice" particles are such that an air speed of 2.5 km/hr is required to keep the particles in suspension.

a) Obtain a relationship for the boundary of the clear zone as a function of distance from the leading edge. Show your analysis.
b) If the 3.2 m wide road is swept clean of snow, estimate the minimum shear stress required to remove any snow deposited.

Data: Properties of air at -23°C – viscosity = 1.6 (10^{-5}) Pa s; density = 1.412 kg/m^3

Question #3 (10 points)

Cement kilns, bitumen extraction drums using hot water and caustic and certain industrial drying equipment are large and long cylinders inclined at an angle (between 10 and 30°) to the horizon. Consider one such cylinders. Material to be processed is fed at the higher end at a constant volumetric rate of Q m^3/s and the drum is rotated slowly at ω min^{-1}. The length of the drum is L, its diameter is D, the height of the baffles attached to the inside wall of the drum is δ and the angle of inclination of the cylinder to the horizon is θ. The extent of mixing of the feed at the discharge end of the drum depends, in addition to the foregoing parameters, on the "fluid" viscosity μ, its density ρ and the acceleration of gravity g.
Identify the dimensionless groups that can be used to scale the drum from a pilot to the prototype scale.

Question #4 (30 points)

The lithium-ion battery has been reported to explode or spontaneously ignite as a result of overheating when it is either discharged or charged too rapidly. The heat generated within the battery could cause gases (oxygen, carbon dioxide, methane and others) to be produced, which if not vented adequately, become pressurized and cause the battery casing to expand or explode. Accumulated heat can alternately start a fire when the temperature reaches about 500°C at any point within the battery. Thus the control of heat production by the chemical reactions is dependent on the regulation of the flow of current within the battery. Li-ion batteries have configurations that may be cylindrical or rectangular (prismatic). The typical prismatic form consists of layers of $LiCoO_2$ and graphite sheets separated by a 24 μm thick polymer spacer fenestrated with microchannels. The composite sheets are rolled in a spiral from the core outwards and infused with an electrolyte ($LiPF_6$) in a solvent such as ether. Charge and discharge processes occur through the displacement of lithium ions between the anode (graphite) and the cathode ($LiCoO_2$).

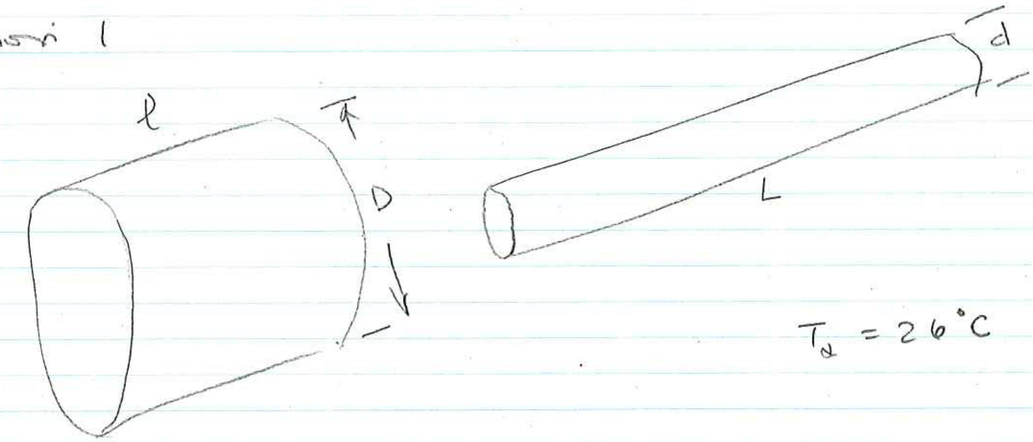
The problem of heat generation and dissipation is to be studied using the **integral method**. A rectangular battery the has a height 2H (z-direction), length 2L (x-direction) and width 2ℓ (y-direction) is freely suspended in air at a constant temperature T_α. The initial temperature of the battery is the same as the ambient. At time t = 0, a rapid discharge of the battery was initiated. Heat was generated at a constant rate of g+ per unit volume and the convective coefficient of heat transfer at all the surfaces of the battery was very large. Assume the battery is homogeneous, that is the material inside is uniformly distributed.

Obtain a relationship for T(x, y, z, t) within the battery. Where in the battery will the temperature be highest? Show your derivations and state any assumptions.

Hints: Identify the planes of symmetry and choose an origin. Heat flux through each exposed surface may vary with the other two directions. For example, $q_{x=L}$ at the surface is expected to vary with (y, z) positions on that exposed surface. Hence heat rate through an area dy dz is $q_{x=L}dydz$. Heat loss rate through the surface normal to the x-direction is hence given by $\iint q_{x=L}dydz$.
The problem is 3-D, unsteady, has a finite domain and the heat transfer coefficient h is infinite.

ENCH 501 Final Examination Solution - Dec. 21, 2016

Question 1



The cylinder is to be shaped into a rod. The first step is to establish if the lumped capacity method is valid. The condition is that

$$\frac{h\left(\frac{V}{A}\right)}{k} < 0.1$$

The highest $\frac{V}{A_i}$ is at $t=0$ when A_i is minimal

$$V = \frac{\pi D^2 l}{4} = \frac{\pi (0.5)^2 1}{4} = 0.19635$$

$$A_i = 2\left(\frac{\pi D^2}{4}\right) + \pi D l = \pi(0.5)\left(\frac{0.5}{2} + 1\right) = 1.963495 \text{ m}^2$$

$$\therefore \quad \frac{15(0.1)}{27.3} = 0.055 < 0.1$$

Hence use lumped capacity method.

The control volume is the solid.
This volume is constant since there is no thermal contraction. Hence

$$\frac{\pi D^2 l}{4} = \frac{\pi d^2 L}{4} \quad \text{or} \quad L = \left(\frac{D}{d}\right)^2 l = 6.25 \text{ m}.$$

The final area of the rod is

$$A_f = \pi(0.2)\left(\frac{0.2}{2} + 6.25\right) = 3.989823 \ m^2$$

The equation for how area changes with time is

$$A = A_i + (A_f - A_i)(1 - e^{-\lambda t}) \quad ; \text{Let } \Delta A = A_f - A_i$$

The Energy balance on the rod is

$$\underset{0}{\text{Input}} + \underset{0}{\text{Gen}} = \text{Output} + \text{Accum}$$

$$0 = hA(t)(T - T_\alpha) + \frac{d\left[(\forall\rho)C_p(T - T_0)\right]}{dt}$$

where T_α = temp of ambient and T_0 is the initial temperature of the steel.

$$\frac{dT}{dt} = -\frac{h}{\forall\rho C_p} A(t)(T - T_\alpha)$$

$$\frac{d(T - T_\alpha)}{T - T_\alpha} = -\frac{h}{\forall\rho C_p}\left[A_i + \Delta A(1 - e^{-\lambda t})\right] dt$$

$$\int_{1300}^{850} d\ln(T - T_\alpha) = -\beta\int_0^t (A_f - \Delta A e^{-\lambda t})\, dt$$

$$\ln\frac{1300 - 26}{850 - 26} = \beta\left[A_f t + \frac{\Delta A e^{-\lambda t}}{\lambda}\right]_0^t$$

$$= \beta\left[A_f t + \frac{\Delta A e^{-\lambda t}}{\lambda} - \frac{\Delta A}{\lambda}\right]$$

$$0.435746 = \frac{15}{(0.19635)(7840)}\left[3.989823\,t + \frac{2.026328}{0.102}\left\{e^{-0.102t} - 1\right\}\right]$$

$$44.718726 = 3.989823t - 101.3164\left(1 - e^{-0.102t}\right)$$

Neglecting 2nd term, $t \simeq 11.208$ $\therefore t > 11.2\,s.$

By trial and error, $t \simeq 19.4\,s$

(b) At $t = 10\,s$

$$\ln\left(\frac{1300 - 26}{T - 26}\right) = \frac{15}{0.19635(7840)}\left(3.989823t - 101.3164\left(1 - e^{-0.102t}\right)\right)$$

with $t = 10s$

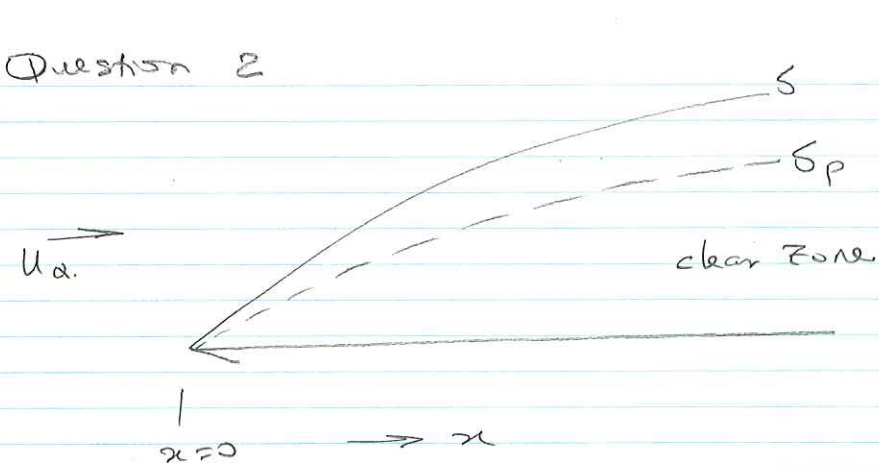
$$= (0.009744)(21.532682)$$

$$= 0.209814$$

$$T = \frac{1300 - 26}{1.233449} + 26$$

$$= 1058.9\ °C \longrightarrow$$

Question 2



The boundary layer, $\delta(x)$, is determined from the momentum balance. The momentum integral equation (6.18, Notes) is

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho(U_\alpha - u)\,u\,dy\right]$$

The conditions for the flow are

$$y=0 \quad u=0$$
$$y=\delta(x) \quad u=U_\alpha$$
$$y=\delta(x) \quad \frac{du}{dy}=0$$
$$\text{and} \quad y=0 \quad \frac{d^2u}{dy^2}=0$$

Assume a profile, eg

$$u = a + by + cy^2 + dy^3 \qquad \text{eq. 6.20}$$

Substitute the conditions to get

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad \text{eq. 6.21}$$

substitute this into the integral equation & solve to get

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}} \qquad \text{eq. 6.25}$$

The free stream velocity $U_\infty = 60$ km/hr and the velocity that would maintain the particles in suspension is 25 km/hr.

The trajectory of the slowest suspended particles is given by

$$\frac{u}{U_\infty} = \frac{2.5}{6} = \frac{3}{2}\left(\frac{y_p}{\delta}\right) - \frac{1}{2}\left(\frac{y_p}{\delta}\right)^3$$

or $$f(\xi) = \xi^3 - 3\xi + 5/6 = 0 \quad ; \quad \xi = y_p/\delta$$

By trial and error

ξ	$f(\xi)$
0.2	0.241
0.3	-0.0396
0.29	-0.0123
0.28	0.0153
0.2856	-0.000171
0.2855	0.000104

$\therefore \xi \simeq 0.28556$

$$\therefore y_p = 0.28556\,\delta \quad \text{is the boundary.}$$

$$= 1.325\sqrt{\frac{\nu x}{U_\infty}}$$

(b) The minimum drag or shear stress is given at $x=L$ by $$\tau_w = \frac{3}{2}\frac{\mu U_\infty}{4.64}\left[\frac{U_\infty}{\nu L}\right]^{1/2}$$

$U_\infty = 6$ km/hr $= 1.667$ m/s

$\mu = 1.6(10^{-5})$ Pa·s

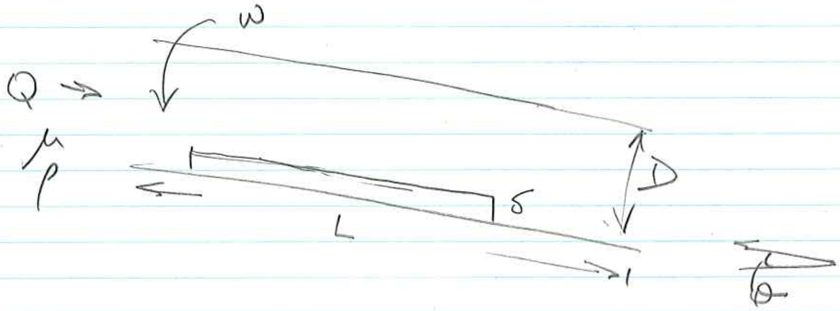
$\rho = 1.412$ kg/m^3) $\nu = 1.1331(10^{-5})$ m^2/s

The shear stress is going to be the lowest at $L = 3.2\,m$

$$\therefore \left.\tau_w\right|_{min} = \frac{3}{2}\frac{(1.6)(10^{-5})(1.667)}{4.64}\sqrt{\frac{1.667}{1.1331(10^{-5})3.2}}$$

$$= 8.2167(10^{-4})\ Pa$$

Question 3



The variables are

	Q	μ	ρ	g	ω	L	θ	D	δ
Units	m^3/s	$Pa.s$ = $\frac{kg}{ms^2}$	$\frac{kg}{m^3}$	$\frac{m}{s^2}$	$\frac{1}{s}$	m		m	m

There are 3 dimensions — L, M, t

$\therefore$ 6 dimensionless groups can be obtained.

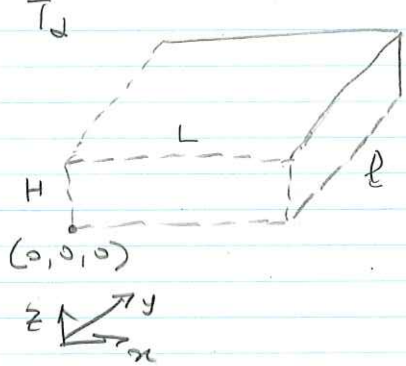
This can be done by the Buckingham Pi theorem or, more rapidly, by inspection.

The dimensionless groups are

θ, D/L and $\frac{\delta}{D}$ — geometric

and $\frac{Q}{\omega L^3}$, $\frac{\rho L^2}{\omega \mu}$ and $L\omega^2/g$ — kinematic.

Question 4



Planes of symmetry through a battery divides the unit into 8 sectors. The analysis is to be done on one sector, as shown in the sketch

This is a 3-D, unsteady state problem in a finite domain. The coefficient h is infinite. Hence the surfaces exposed to air are maintained at T_α.

The energy balance on the sector - the control volume is given by

$$\underbrace{\text{Input}}_{0} + \underbrace{\text{Generation}}_{g^+ L \ell H} = \text{Output} + \text{Accum}$$

Output terms are

$$\int_{z=0}^{H}\int_{y=0}^{\ell} q_{x=L}\, dy\, dz + \int_{z=0}^{H}\int_{x=0}^{L} q_{y=\ell}\, dx\, dz +$$

$$\int_{y=0}^{\ell}\int_{x=0}^{L} q_{z=H}\, dx\, dy$$

and the accumulation term is

$$\frac{d}{dt}\left[\int_{z=0}^{H}\int_{y=0}^{\ell}\int_{x=0}^{L} \rho C_p (T - T_\alpha)\, dx\, dy\, dz\right]$$

The integral energy equation is

$$\iint_{z\,y} \left.\frac{\partial T}{\partial x}\right|_{x=L} dy\,dz + \iint_{z\,x} \left.\frac{\partial T}{\partial y}\right|_{y=l} dx\,dz + \iint_{y\,x} \left.\frac{\partial T}{\partial z}\right|_{z=H} dx\,dy$$

$$- \frac{1}{\alpha}\frac{\partial}{\partial t}\left[\iiint_{z\,y\,x} (T-T_\alpha)\,dx\,dy\,dz\right] + \frac{g^+}{k} L l H = 0$$

The conditions are

$$x = 0 \qquad \frac{\partial T}{\partial x} = 0 \qquad\qquad x = L \qquad T = T_\alpha$$

$$y = 0 \qquad \frac{\partial T}{\partial y} = 0 \qquad\qquad y = l \qquad T = T_\alpha$$

$$z = 0 \qquad \frac{\partial T}{\partial z} = 0 \qquad\qquad z = H \qquad T = T_\alpha$$

Let $\theta = T - T_\alpha = (L^2 - x^2)(l^2 - y^2)(H^2 - z^2)\,\Gamma(t)$

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=L} = -2L\,(l^2-y^2)(H^2-z^2)\,\Gamma$$

$$\left.\frac{\partial \theta}{\partial y}\right|_{y=l} = -2l\,(L^2-x^2)(H^2-z^2)\,\Gamma$$

$$\left.\frac{\partial \theta}{\partial z}\right|_{z=H} = -2H\,(L^2-x^2)(l^2-y^2)\,\Gamma$$

Substitute into the integral equation

$$-\frac{8}{9} L l^3 H^3 \Gamma - \frac{8}{9} l L^3 H^3 \Gamma - \frac{8}{9} H l^3 L^3 \Gamma - \frac{1}{\alpha}\frac{8}{27} L^3 l^3 H^3 \frac{\partial \Gamma}{\partial t}$$

$$+ \frac{g^+}{k} L l H = 0$$

$$-\frac{8}{9}\left\{\ell^2 H^2 + L^2H^2 + \ell^2 L^2\right\}\Gamma - \frac{1}{\alpha}\frac{8}{27}\left(L^2\ell^2H^2\right)\frac{d\Gamma}{dt} + \frac{g^+}{k} = 0$$

or

$$\frac{d\Gamma}{dt} + \beta\Gamma - \varepsilon = 0$$

where

$$\beta = \frac{3\alpha\left(\ell^2H^2 + L^2H^2 + \ell^2L^2\right)}{L^2\ell^2H^2}$$

$$= 3\alpha\left(\frac{1}{L^2} + \frac{1}{\ell^2} + \frac{1}{H^2}\right)$$

and

$$\varepsilon = \frac{27}{8}\frac{g^+}{k}\frac{\alpha}{L^2\ell^2H^2}$$

Subject to the condition $\Gamma(0) = 0$

$$\frac{d\Gamma}{dt} = -(\beta\Gamma - \varepsilon)$$

$$\frac{d\Gamma}{\beta\Gamma - \varepsilon} = -dt$$

$$d\ln(\beta\Gamma - \varepsilon) = -\beta\, dt$$

Integrating with i.c. gives.

$$\ln\left(\frac{\beta\Gamma - \varepsilon}{-\varepsilon}\right) = -\beta t$$

$$1 - \frac{\beta}{\varepsilon}\Gamma = \exp(-\beta t)$$

$$\Gamma = \frac{\varepsilon}{\beta}\left(1 - e^{-\beta t}\right)$$

(a) The approximate temperature profile is

$$T(x, y, z, t) = T_a + \left(L^2 - x^2\right)\left(\ell^2 - y^2\right)\left(H^2 - z^2\right) \times \frac{\varepsilon}{\beta}\left(1 - e^{-\beta t}\right)$$

(b) The highest temperature is at (0, 0, 0)

$\longrightarrow$