

AJ Dec 11, 2015  
dB

University of Calgary  
Department of Chemical & Petroleum Engineering

ENCH 501 Transport Phenomena

Final Examination, Fall 2015

Time: 8.00 to 11.00 am

Tuesday, December 15, 2015

**Instructions: Automatic 10-point bonus for attempting any but only 3 of 4 Questions. No Electronic device other than a Calculator is Allowed. Open Notes, Open Book Exam.**

**Question #1 (30 points)**

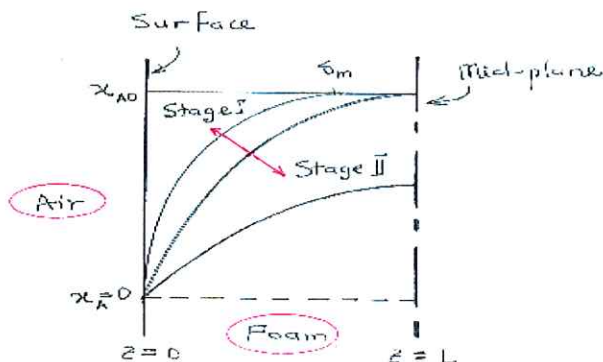
Gel memory foams for pillows, mattress tops and camping pads are normally polyurethanes. When bought new, the foams typically smell of volatile organic compounds (VOC). These compounds may consist of formaldehyde (as preservative), chlorofluoroalkanes or pentane (as blowing agents) and polybrominated diphenyl ether (as fire retardant). Some individuals are sensitive and react to VOC. Such people require that the compounds be allowed to diffuse out of the foams before they can sleep on them. The following problem relates to "off-gassing" of a pillow.

A memory foam pillow is 66 x 51 x 10 cm in size. The pillow has a concentration of VOC (collectively termed substance A) of  $C_{AO}$  or  $C_{XAO}$  on a molar basis, where  $C$  is the molar concentration of all compounds present in the pillow.  $C$  is essentially a constant. The foam (termed substance B) is the medium through which the VOC diffuses. It is stagnant. The sides of the pillow that are 10 cm high are covered with a material impervious to VOC. Thus VOC escapes only through the two large, flat surfaces. At time  $t = 0$ , air that is free of VOC is blown at a high rate on the flat surfaces such that the mass transfer coefficients at the surfaces are assumed to be very large. The mole fraction of the VOC at the surfaces quickly became zero and was maintained at this level. An owner's manual attached to the pillow states that exposure to air for 24 hours is sufficient for eliminating most of the odor. Use the **integral method** for the following and show your steps.

a) If in a laboratory, it was determined that the VOC concentration at the mid-plane of the pillow had dropped to half its original level at  $t = 24$  hours, estimate the **diffusivity** of VOC through the foam.

b) At  $t = 24$  hours, what **fraction** of the total VOC originally in the pillow remained?

**Hint:** The following diagram and doing the analysis in two stages may be useful. Sketch Q.1



## Question #2 (30 points)

Crude oil is accidentally spilled on the water in a gulf. The water was calm and there were no winds or currents in the water. The oils spread as a circular disc floating on the water and it is assumed that the oil did not lose material by evaporation into air or dissolution into the water. A drone was available and it was launched at intervals to measure the radius of the oil disc as a function of time. Three readings were recorded, one in each of the regimes of spreading (a *graph paper is provided*.):

Time	Radius, m
54.6 s	148.4
2 hrs 15 mins 3s	1096.6
13 days 21 hrs 53 mins 20 s	13,359.7

- a) (4 pts) If the thickness of the oil layer when the first measurement was made was 4.2 cm, what was the volume of oil spilled?
- b) (6 pts) What are the radius and thickness of the layer immediately after the spill occurred?
- c) (6 pts) At what times did the transitions between regimes occur?
- d) (4 pts) What was the layer thickness at the time of the third measurement?
- e) (10 pts) If the densities of the water and oil are 1026 and 989 kg/m<sup>3</sup> respectively, and an exact relationship is desired relating the radius  $R$  and time  $t$  in the inertia regime, an  $=$  sign can replace  $\sim$  if the right side of the order-of-magnitude relationship is multiplied by a constant  $\beta$ . What is the numerical value for  $\beta$ ? How and why is this different from the constant neglected in the order-of-magnitude analysis? How good is your new equation for predicting the disc radius at  $t = 403.4$  s?

## Question #3 (30 points)

At a remote camp site for oil prospecting and drilling, a 81 m<sup>2</sup> solar panel is used to heat water that is used by the crew for cleaning and cooking facilities. The circuit for the water flow through the solar panel is open, as shown in the sketch. Water is taken directly from a 120-gallon (or 454.25-litre) tank and pumped at a rate of 30 litres/minute through the solar panel before it is returned into the well-insulated tank. The circulation mixes the tank very well. The sun follows a regular diurnal or daily pattern. The power density or energy absorbed by the solar panel per square meter is given by the expression

$$R(t) = 1060\{\exp(-\alpha[t - 12]^2)\} \text{ W/m}^2; \quad \alpha = 0.136261 \text{ hr}^{-2}$$

where  $t$  is the time of the day in hours (by the 24-hour clock). As the water heats up past 40°C, cold fresh water can be fed into the tank and warm/hot water withdrawn. The volume of water in the tank remains constant at 120 gallons all the time.

If at mid-night ( $t = 0$ ) the temperature of the water in the tank is 10°C, the temperature of the fresh water supply into the tank is always at 15°C, and the minimum temperature of hot or warm water that is required from the tank is 40°C,

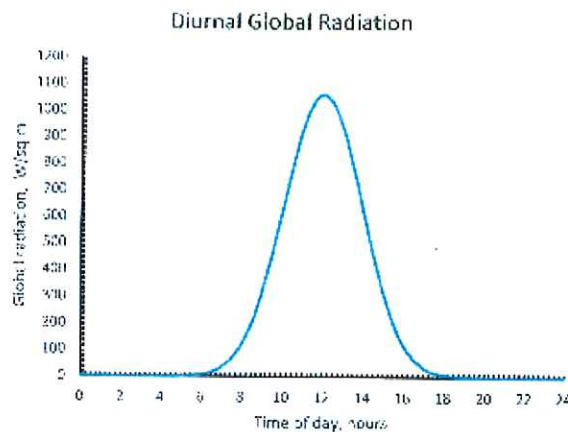
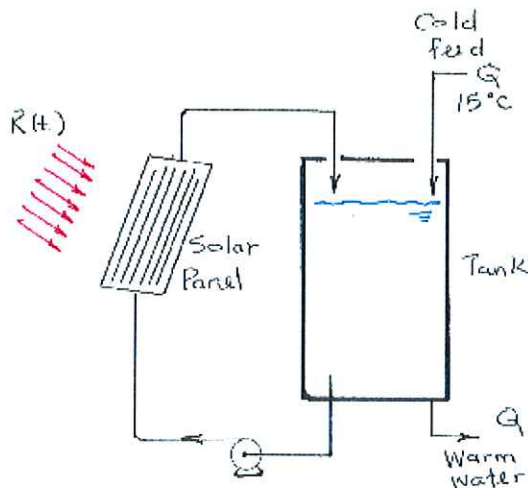
a) at what time of the day can the first drop of warm water be withdrawn? **Note:** *In setting up the problem, it is important to remember that the units must be the same on both sides of you an equation.*

b) What is the maximum total volume of water at  $T \geq 40^\circ\text{C}$  that can be withdrawn from the tank?



**Data:** Heat capacity for water = 4.179 kJ/kg; density of water = 1000 kg/m<sup>3</sup>. *Tables of mathematical functions and values are provided.*

Sketch Q.3



#### Question #4 (30 points)

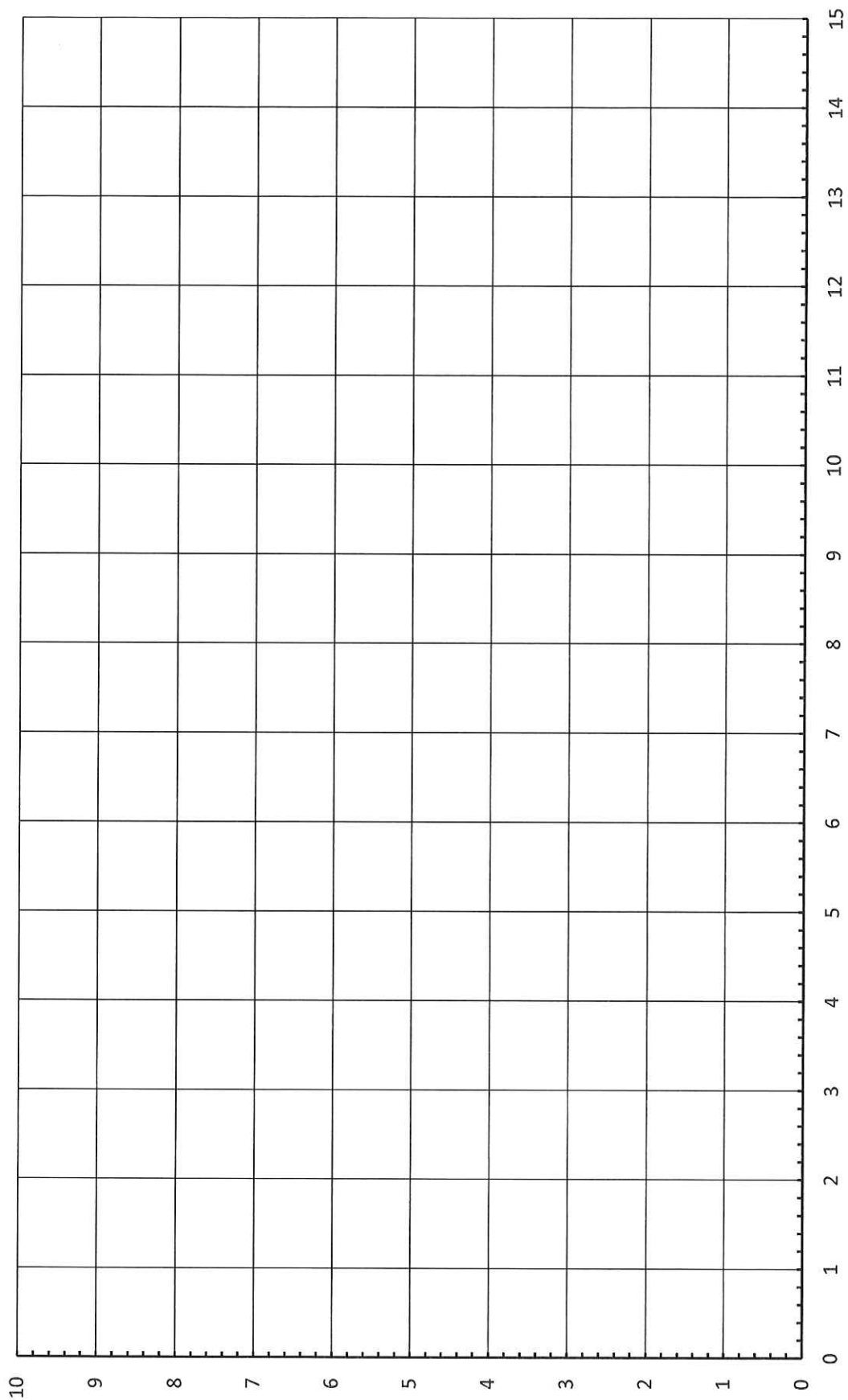
- a) (5 pts) A tire has just been installed on a rim and inflated to a pressure of 35 psig. The tire was then mounted on a car and the car was lowered onto the ground. The pressure was measured again. Is the pressure going to be less than, equal to or greater than 35 psig? Explain.
- b) (10 pts) Tailings ponds used by companies that extract metals (copper and gold) or bitumen from oilsands are large open surface ponds. The function of each pond exposed to the sun, wind and drier air is to allow water to evaporate, thereby concentrating the solution and forming sludge that settles to the bottom. Using the empirical relationship (equation 1.13), estimate the amount of water that will evaporate from a pond that is 80 by 120 m in 4 weeks if the average wind velocity is 20 km/hr, the humidity ratio at the surface of the pond is 0.014 kg H<sub>2</sub>O/kg dry air and 0.0081 kg H<sub>2</sub>O/kg dry air in the ambient. How much energy is required?
- c) (5 pts) Many engineering structures such as bridges, communication towers, cranes and others use cables to hold objects in place or lift them. Identify a dimensionless group that can be used to set the limit for the mass (m) of an object that can be lifted. You may neglect the mass of the cable that has a Young's modulus E and a diameter d.
- d) (10 pts) The Clausius-Clapeyron equation that relates vapor pressure of a liquid to temperature is

$$\frac{d \ln P_{vp}}{dT} = \phi = \frac{\Delta H_v}{RT^2}$$

If  $\Delta H_v = 308.5 \pm 0.6$  kJ/kg at a temperature of  $87.8 \pm 0.3$  °C for pentane<sup>1</sup>, what is the error (or measurement uncertainty) in  $\phi$  if the data are independent? The molar mass of pentane is 72.15 kg/kmol and the universal gas constant R is 8.314 kJ/kmol K.

<sup>1</sup> Kozicki, W. and Sage, B.H., Latent Heat of vaporization of n-pentane, J. Chemical & Engg Data, 8(3), 331 -3, 1960

Q2



$$\int e^{cx} dx = \frac{1}{c} e^{cx}$$

$$\int x e^{cx} dx = \frac{e^{cx}}{c^2} (cx - 1)$$

$$\int x^2 e^{cx} dx = \left( \frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3} \right) e^{cx}$$

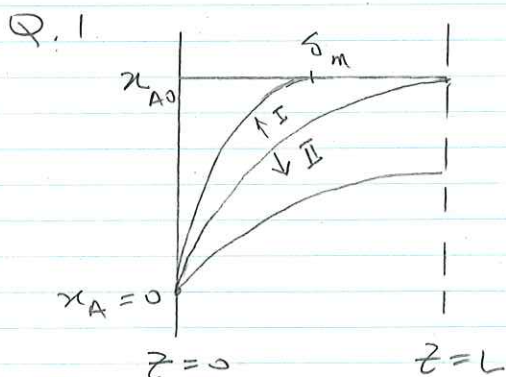
$$\int e^{-cx^2} dx = \sqrt{\frac{\pi}{4c}} \operatorname{erf}(\sqrt{c} x) \quad \{\text{erf is error function; } \operatorname{erf}(0) = 0, \operatorname{erf}(\infty) = 1, \operatorname{erf}(-x) = -\operatorname{erf}(x)\}$$

$$\int x e^{-cx^2} dx = -\frac{1}{2c} e^{-cx^2}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

	Hundredths digit of x									
$x$	0	1	2	3	4	5	6	7	8	9
0.0	0.00000	0.01128	0.02256	0.03384	0.04511	0.05637	0.06762	0.07886	0.09008	0.10128
0.1	0.11246	0.12362	0.13476	0.14587	0.15695	0.16800	0.17901	0.18999	0.20094	0.21184
0.2	0.22270	0.23352	0.24430	0.25502	0.26570	0.27633	0.28690	0.29742	0.30788	0.31828
0.3	0.32863	0.33891	0.34913	0.35928	0.36936	0.37938	0.38933	0.39921	0.40901	0.41874
0.4	0.42839	0.43797	0.44747	0.45689	0.46623	0.47548	0.48466	0.49375	0.50275	0.51167
0.5	0.52050	0.52924	0.53790	0.54646	0.55494	0.56332	0.57162	0.57982	0.58792	0.59594
0.6	0.60386	0.61168	0.61941	0.62705	0.63459	0.64203	0.64938	0.65663	0.66378	0.67084
0.7	0.67780	0.68467	0.69143	0.69810	0.70468	0.71116	0.71754	0.72382	0.73001	0.73610
0.8	0.74210	0.74800	0.75381	0.75952	0.76514	0.77067	0.77610	0.78144	0.78669	0.79184
0.9	0.79691	0.80188	0.80677	0.81156	0.81627	0.82089	0.82542	0.82987	0.83423	0.83851
1.0	0.84270	0.84681	0.85084	0.85478	0.85865	0.86244	0.86614	0.86977	0.87333	0.87680
1.1	0.88021	0.88353	0.88679	0.88997	0.89308	0.89612	0.89910	0.90200	0.90484	0.90761
1.2	0.91031	0.91296	0.91553	0.91805	0.92051	0.92290	0.92524	0.92751	0.92973	0.93190
1.3	0.93401	0.93606	0.93807	0.94002	0.94191	0.94376	0.94556	0.94731	0.94902	0.95067
1.4	0.95229	0.95385	0.95538	0.95686	0.95830	0.95970	0.96105	0.96237	0.96365	0.96490
1.5	0.96611	0.96728	0.96841	0.96952	0.97059	0.97162	0.97263	0.97360	0.97455	0.97546
1.6	0.97635	0.97721	0.97804	0.97884	0.97962	0.98038	0.98110	0.98181	0.98249	0.98315
1.7	0.98379	0.98441	0.98500	0.98558	0.98613	0.98667	0.98719	0.98769	0.98817	0.98864
1.8	0.98909	0.98952	0.98994	0.99035	0.99074	0.99111	0.99147	0.99182	0.99216	0.99248
1.9	0.99279	0.99309	0.99338	0.99366	0.99392	0.99418	0.99443	0.99466	0.99489	0.99511
2.0	0.99532	0.99552	0.99572	0.99591	0.99609	0.99626	0.99642	0.99658	0.99673	0.99688
2.1	0.99702	0.99715	0.99728	0.99741	0.99753	0.99764	0.99775	0.99785	0.99795	0.99805
2.2	0.99814	0.99822	0.99831	0.99839	0.99846	0.99854	0.99861	0.99867	0.99874	0.99880
2.3	0.99886	0.99891	0.99897	0.99902	0.99906	0.99911	0.99915	0.99920	0.99924	0.99928
2.4	0.99931	0.99935	0.99938	0.99941	0.99944	0.99947	0.99950	0.99952	0.99955	0.99957
2.5	0.99959	0.99961	0.99963	0.99965	0.99967	0.99969	0.99971	0.99972	0.99974	0.99975
2.6	0.99976	0.99978	0.99979	0.99980	0.99981	0.99982	0.99983	0.99984	0.99985	0.99986
2.7	0.99987	0.99987	0.99988	0.99989	0.99989	0.99990	0.99991	0.99991	0.99992	0.99992
2.8	0.99992	0.99993	0.99993	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	0.99996
2.9	0.99996	0.99996	0.99996	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998
3.0	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99998	0.99999	0.99999	0.99999
3.1	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999	0.99999
3.2	0.99999	0.99999	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000





The problem is treated in 2 stages. For stage I, the system is a semi-infinite domain.

$$VOC = A$$

medium = B

The flux of A, from

$$N_A = -c D_{AB} \frac{dx_A}{dz} + x_A (N_A + N_B), \text{ with } N_B = 0$$

becomes

$$N_A = - \frac{c D_{AB}}{1 - x_A} \frac{dx_A}{dz}$$

material balance on A in the region  $0 \leq z \leq L$  is

$$\text{Input} + \text{Generation} = \text{Output} + \text{Accum.}$$

$\downarrow$   $\downarrow$   $\downarrow$   
 $z=0$   $z$

$$0 = -N_A \Big|_{z=0} + \frac{d}{dt} \left[ \int_0^L c x_A dz \right] \quad \begin{array}{l} \text{per} \\ \text{unit} \\ \text{x-section} \\ \text{area} \end{array}$$

(w/ the -ve z-direction)

or

$$\left( - \frac{c D_{AB}}{1 - x_A} \frac{dx_A}{dz} \right) \Big|_{z=0} = \frac{d}{dt} \left[ \int_0^L c x_A dz \right]$$

or

$$\textcircled{1} \quad - D_{AB} \frac{dx_A}{dz} \Big|_{z=0} = \frac{d}{dt} \left[ \int_0^L x_A dz \right] \quad \begin{array}{l} \text{integral} \\ \text{equation} \end{array}$$

Since  $c = \text{const}$  and  $x_A|_{z=L} = 0$

The conditions for stage I are

$$z=0 \quad x_A=0 \quad (i)$$

$$z=\delta_m \quad x_A = x_{A0} \quad (ii)$$

$$z=\delta_m \quad \frac{dx_A}{dz} = 0 \quad (iii)$$

Assume a concentration profile with 3 constants -

$$x_A = a + bz + cz^2 \Rightarrow \frac{dx_A}{dz} = b + 2cz$$

Apply b.c.s

$$(2) \quad \frac{x_A}{x_{A0}} = 2 \frac{z}{\delta_m} - \frac{z^2}{\delta_m^2} \quad \text{or} \quad 1 - \frac{x_A}{x_{A0}} = \left(1 - \frac{z}{\delta_m}\right)^2$$

Substitute (2) into integral equation (1)

$$- \frac{D_{AB}}{\delta_m} \frac{2x_{A0}}{\delta_m} = \frac{d}{dt} \left[ \int_0^{\delta_m} x_A dz + x_{A0}(L - \delta_m) \right]$$

$$= \frac{d}{dt} \left[ \int_0^{\delta_m} x_{A0} \left( 2 \frac{z}{\delta_m} - \frac{z^2}{\delta_m^2} \right) dz - x_{A0}(L - \delta_m) \right]$$

$$- \frac{2 D_{AB}}{\delta_m} = \frac{d}{dt} \left[ \int_0^{\delta_m} \left( 2 \frac{z}{\delta_m} - \frac{z^2}{\delta_m^2} \right) dz \right] - \frac{d\delta_m}{dt}$$

$$= \frac{d}{dt} \left[ \delta_m \int_0^1 (2\eta - \eta^2) d\eta \right] - \frac{d\delta_m}{dt}$$

$$= \frac{d}{dt} \left[ \delta_m \left( 1 - \frac{1}{3} - 1 \right) \right]$$

$$6 D_{AB} = \delta_m \frac{d\delta_m}{dt} \quad \text{or} \quad 12 D_{AB} = \frac{d\delta_m^2}{dt}$$

subject to the condition  $t=0, \delta_m=0$

Solve

$$\delta_m = \sqrt{12 D_{AB} t}$$

At the end of stage I,  $\delta_m = L$

$$\text{Hence } t_I = L^2 / 12 D_{AB}$$

For stage II,

conditions are  $z=0 \quad x_A=0$

$$z=L \quad \frac{dx_A}{dz} = 0$$

and at  $t=0$

$$\frac{x_A}{x_{A0}} = 2\left(\frac{z}{L}\right) - \frac{z^2}{L^2}$$

Assume the profile

$$(3) \quad \frac{x_A}{x_{A0}} = \left(2\left(\frac{z}{L}\right) - \frac{z^2}{L^2}\right) P(t) \quad \text{with } P(0) = 1$$

$$\frac{dx_A}{dz} = \left(\frac{2}{L} - \frac{2z}{L^2}\right) P(t)$$

The conditions are satisfied.

Substitute (3) into the integral equation (1)

$$- D_{AB} x_{A0} \frac{2}{L} P = \frac{d}{dt} \left[ \int_0^L x_{A0} \left(2\frac{z}{L} - \frac{z^2}{L^2}\right) P dz \right]$$



$$-2 \frac{D_{AB}}{L} \bar{r} = \frac{d}{dt} \left[ \bar{r} L \int_0^1 (2z - z^2) dz \right] ; \frac{1}{2} = \frac{z}{L}$$

$$= \frac{d}{dt} \left[ \bar{r} L \left(1 - \frac{1}{3}\right) \right]$$

$$- \frac{3 D_{AB}}{L^2} = \frac{1}{\bar{r}} \frac{d\bar{r}}{dt} = \frac{d \ln \bar{r}}{dt}$$

Integrate

$$\ln \bar{r} = - \frac{3 D_{AB} t}{L^2} + C$$

But  $\bar{r}(0) = 1 \quad \therefore C = 0$

$$\bar{r} = e^{- \frac{3 D_{AB} t}{L^2}}$$

For stage II

$$\therefore \frac{x_A}{x_{A0}} = \left( 2 \left( \frac{z}{L} \right) - \frac{z^2}{L^2} \right) e^{- \frac{3 D_{AB} t}{L^2}}$$

When the concentration at the mid-plane ( $z=L$ ) is half the original value,  $x_A/x_{A0} = \frac{1}{2}$

$$\therefore \frac{1}{2} = e^{- \frac{3 D_{AB} t}{L^2}} \quad \text{or} \quad \frac{3 D_{AB} t}{L^2} = \ln 2$$

$$t_{II} = \frac{\ln 2 L^2}{3 D_{AB}}$$

Hence total time

$$t_{\text{total}} = t_I + t_{II} = \frac{L^2}{3 D_{AB}} \left( \frac{1}{4} + \ln 2 \right)$$

But  $t_{\text{total}} = 24(3600) \text{ s}$ ,  $L = 5 \text{ cm or } 5(10^{-2}) \text{ m}$

Hence

$$24(3600) = \frac{25(10^{-4})}{3 D_{AB}} (0.25 + 0.6931)$$

$$D_{AB} = 9.0967(10^{-9}) \text{ m}^2/\text{s}$$

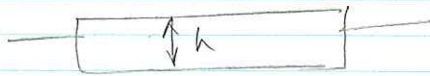
(b) At  $t = 24 \text{ hours}$ , the ratio of VOE remaining to present originally is

$$\phi = \frac{\int_0^L x_A|_{t=24\text{hrs}} dx}{\int_0^L x_{A0} dx} = \frac{\int_0^L x_{A0} \left(2\frac{z}{L} - \frac{z^2}{L^2}\right) \frac{1}{2} dz}{\int_0^L x_{A0} dz}$$

$$= \frac{1}{2} \int_0^1 (2\xi - \xi^2) d\xi \quad ; \quad \xi = \frac{z}{L}$$

$$\phi = \frac{1}{3}$$

Q. 2



$$(a) \text{ Volume } V = \pi R^2 h = \pi (148.4)^2 (4.2)(10^{-2})$$

$$= 2905.8 \text{ m}^3$$

$$(b) \text{ At } \ln t = 0, \quad t = 1 \text{ s} \quad \ln R = 3 \quad \therefore R = 20.086 \text{ m}$$

$$\text{But } \pi (20.086)^2 h = 2905.8 \Rightarrow h = 2.293 \text{ m}$$

$$(c) \text{ The transitions are, from plot of } \ln R \text{ vs } \ln t, \text{ at}$$

$$\ln t = 7.0 \quad \text{or } t = 109.646 \text{ s} \approx 18 \text{ mins } 17 \text{ s}$$

$$\ln t = 11.42 \quad \text{or } t = 91,126.14 \text{ s} = 25 \text{ hrs, } 18 \text{ mins and } 46 \text{ s}$$

$$(d) \pi (13359.7)^2 h = 2905.8 \text{ m}^3$$

$$h = 5.182 (10^{-4}) \text{ m} \approx 5.2 \mu\text{m}$$

$$(e) \text{ Given } R = \beta (g \Delta V)^{1/4} t^{1/2}$$

$$\Delta = 1 - \rho_0 / \rho_w = 1 - 989 / 1024 = 0.036$$

$$g = 9.81 \text{ m/s}^2$$

$$V = 2905.8 \text{ m}^3$$

$$\text{when } t = 54.6 \text{ s, } R = 148.4 \text{ m}$$

$$148.4 = \beta \left[ (9.81)(0.036)(2905.8) \right]^{1/4} (54.6)^{1/2}$$

$$\beta = 3.5484$$

$$\text{At } t = 403.4 \text{ s, } \ln t = 6 \quad \text{From plot } \ln R = 6$$

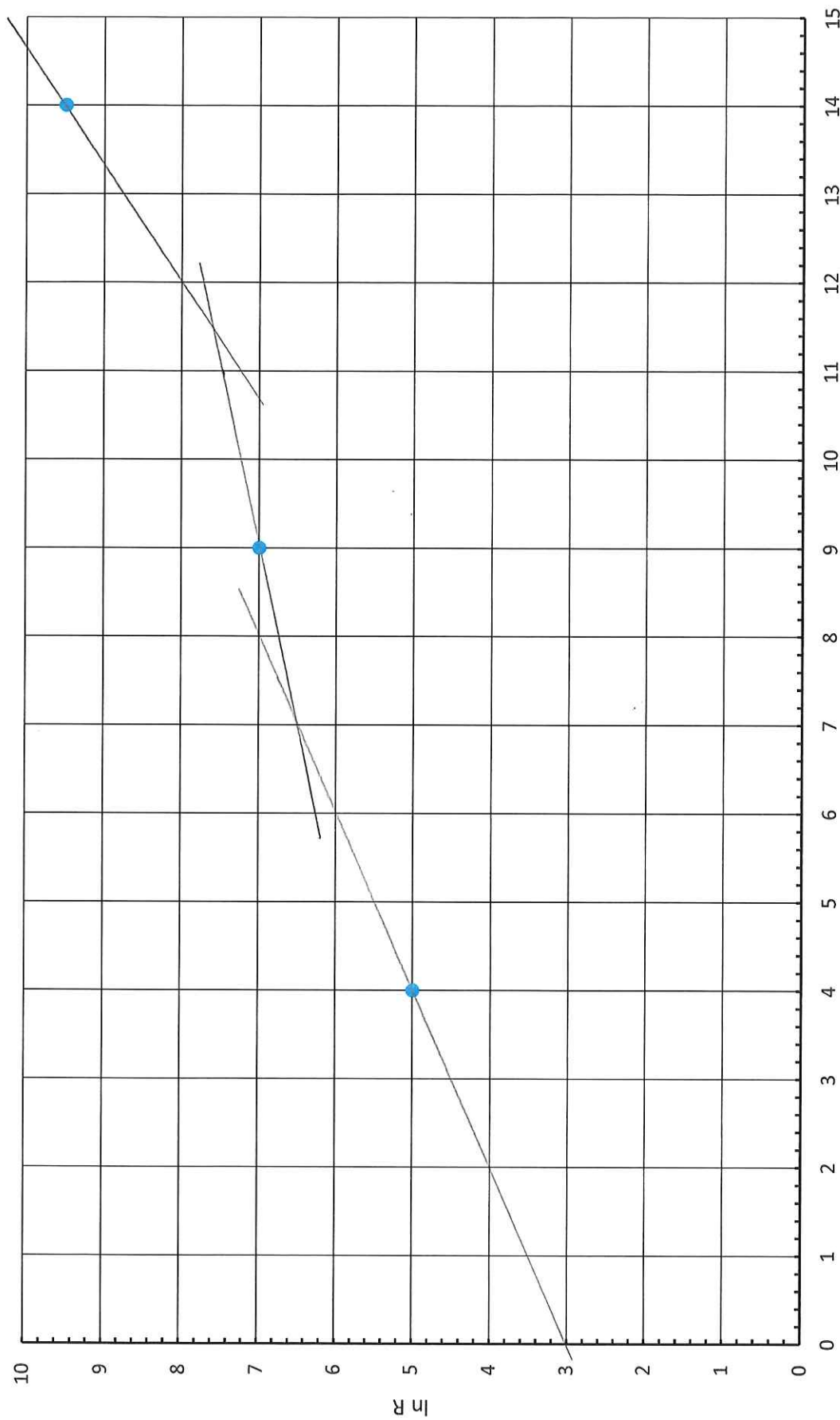
$$\therefore R_{\text{exp}} = 403.4 \text{ m}$$

$$\text{From eq. } R = 3.5484 (9.81 \times 0.036 \times 2905.8)^{1/4} (403.4)^{1/2}$$

$$= 403.37 \text{ m} \quad \text{V. Good} \rightarrow$$

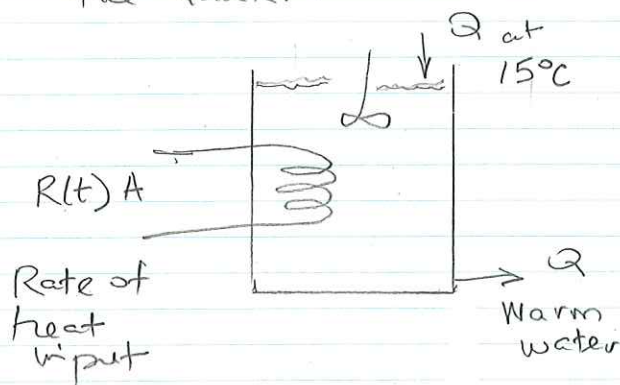


# Q2



$\ln t$	$\ln R$
54.65	4
81035	9
1.202(10 <sup>6</sup> )	14
	148.4 m
	1096.6 m
	13,359.7 m
	9.5

#3 The circuit for the solar collector simply transfers heat into the tank. Hence the problem is adding heat at a rate of  $R(t) \cdot A$  to the tank.



The problem has two stages. In stage I, no water is fed into or removed from the tank. In stage II, the tank temperature is maintained at  $40^\circ\text{C}$ .

Heat added is removed by flow stream.

Stage I

Energy balance on water in the tank.

$$\underset{\text{in}}{\text{Input}} + \underset{\text{in}}{\text{Generation}} = \underset{\text{out}}{\text{Output}} + \underset{\text{out}}{\text{Accum}}$$

$$R(t) A = \frac{d}{dt} [m c_p (T - 10)]$$

$$\beta \exp(-\alpha (t-12)^2) = m c_p \frac{dT}{dt} \quad (1)$$

where

$$\beta = (1060)(81) \text{ W}, \quad \alpha = 0.136261 \text{ hr}^{-2}$$

$$m = V_{\text{water in tank}} \cdot \rho = 454.25 \text{ kg}$$

$$c_p = 4179 \text{ J/kg}$$

Re-arrange ①

$$\frac{m C_p}{\beta} dT = e^{-\alpha(t-12)^2} dt (3600) s$$

The objective is to find time  $t$  at which the water in the tank reaches  $40^\circ\text{C}$  from the initial temperature of  $10^\circ\text{C}$

$$\frac{m C_p}{(3600)\beta} \int_{10}^{40} dT = \int_0^t e^{-\alpha(t-12)^2} dt$$

Let  $x = t - 12$ , then

$$\int_{-12}^{t-12} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{4\alpha}} \operatorname{erf}(\sqrt{\alpha} \cdot x) \Big|_{-12}^{t-12}$$

where  $\operatorname{erf}$  is the error function

Hence

$$\begin{aligned} \sqrt{\frac{4\alpha}{\pi}} \frac{m C_p (30)}{\beta 3600} &= \operatorname{erf}(\sqrt{\alpha} (t-12)) - \operatorname{erf}(\sqrt{\alpha} (-12)) \\ &= \operatorname{erf}(\sqrt{\alpha} (t-12)) + \operatorname{erf}(\sqrt{\alpha} 12) \end{aligned}$$

$$\begin{aligned} \operatorname{erf}(\sqrt{\alpha} (t-12)) &= \frac{454.25 (4179) (30)}{(3600) 1060 (81)} \sqrt{\frac{4(0.136261)}{\pi}} - \\ &\quad \operatorname{erf}(4.4297) \end{aligned}$$

$$= 0.07674 - 1 = -0.9233$$

$$-\operatorname{erf}(\sqrt{\alpha} (t-12)) = \operatorname{erf}(-\sqrt{\alpha} (t-12)) = 0.9233$$



From error function table,

$$-\sqrt{0.136261} (t - 12) = 1.252$$

$$t = 8.608 \text{ hrs or } 8 \text{ hrs } 36 \text{ min } 30 \text{ s}$$

(b)

For the maximum total volume of warm water, the tank water has to be at a temperature of  $40^\circ\text{C}$  always. That is the feed rate of  $15^\circ\text{C}$  water (equal the outflow) has to be appropriate to remove the heat added by the solar collector system. If the feed rate is less than optimum at any time, the tank water temperature will rise above  $40^\circ\text{C}$  and the outflow stream will be warmer than  $40^\circ\text{C}$ . This means less total volume. A faster than optimum feed rate gives outflow colder than  $40^\circ\text{C}$  for any period. Thus, at any instant,

$$\underset{\text{m}^3/\text{s}}{Q(\tau) \rho C_p (40 - 15)} = 1060(81) \exp(-\alpha(t-12)^2) W$$

Total volume, up to sunset at  $\sim 19$  hours, is

$$V \approx \int_{8.608}^{19} Q(\tau) dt (3600) \quad ; \quad t \text{ in hours}$$

$$= \frac{1060(81)(3600)}{1000(4179)(25)} \int_{8.608}^{19} \exp(-\alpha(t-12)^2) dt \quad \text{m}^3$$

$$V = 2.9586 \int_{-3.392}^7 \exp(-\alpha x^2) dx \quad ; \quad x = t - 12$$

$$= 2.9586 \cdot \sqrt{\frac{\pi}{4\alpha}} \left[ \operatorname{erf}(\sqrt{\alpha} 7) - \operatorname{erf}(\sqrt{\alpha} (-3.392)) \right]$$

where  $\alpha = 0.136261$

$$V = 7.103 \left\{ \operatorname{erf}(2.5839) + \operatorname{erf}(1.2521) \right\}$$

$$= 7.103 (0.99974 + 0.9234)$$

Total Volume =  $13.66 \text{ m}^3$  or  $\sim 3608.6 \text{ gals.}$



Q. 4

- (a) After the car is lowered, the tire pressure remains the same at 35 psig. The volume of gas is not changed even if the tire deforms on carrying the car weight.

- (b) Equation 1.13 in Notes is

$$\left(\frac{\text{kg}}{\text{s}}\right) \dot{g}_h = \Theta A (x_s - x)$$

where  $\Theta = (25 + 19V)$  where  $V = 5.56 \text{ m/s}$   
or  $20 \text{ km/hr}$ .

$$A = (80 \times 120) = 9,600 \text{ m}^2$$

$$x_s = 0.014 \quad \text{and} \quad x = 0.0081$$

$$\dot{g}_h = (25 + 19(5.555)) 9600 (0.014 - 0.0081)$$

$$\text{kg/hr} = 7,394.6 \quad \text{kg/hr.}$$

Over 4 weeks, total evaporated is

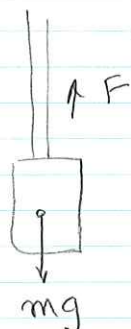
$$G = 4(7)(24)(7,394.6) \quad \text{kg} \\ = 4.969(10^6) \text{ kg} \rightarrow$$

The amount of energy required is

$$Q = G \cdot \Delta H_v = G(2270) \text{ kJ} \\ = 11.28(10^9) \text{ kJ} \rightarrow$$



(c)



$$F = \sigma A = E \epsilon A$$

if cable extension is elastic

The condition is

$$F \epsilon_{\max} A < mg$$

or

$$\frac{mg}{F \epsilon_{\max} A} > 1 \quad \text{where } A = \frac{\pi d^2}{4}$$

(d)

$$\Delta H_v = 308.5 \pm 0.6 \text{ kJ/kmol} ; \text{ Convert units.}$$

$$= 22258.275 \pm 43.29 \text{ kJ/kmol}$$

$$\text{temperature} = 87.8 \pm 0.3^\circ\text{C}$$

$$T = 360.9 \pm 0.3 \text{ K.}$$

With data independent (from eq. 3.17, Notes)

$$\Delta \phi = \sqrt{\left( \frac{\partial \phi}{\partial \Delta H_v} \right)^2 [\Delta(\Delta H_v)]^2 + \left( \frac{\partial \phi}{\partial T} \right)^2 (\Delta T)^2}$$

$$\frac{\partial \phi}{\partial \Delta H_v} = \frac{1}{RT^2} = \frac{1}{(8.314)(360.9)^2} = 9.23456(10^{-7})$$

$$\begin{aligned} \frac{\partial \phi}{\partial T} &= \frac{\Delta H_v}{R} (-2) \frac{1}{T^3} = -\frac{2(22258.275)}{8.314 (360.9)^3} \\ &= -1.13907(10^{-4}) \end{aligned}$$

$$\Delta \phi = \sqrt{\left\{ 1.599811(10^{-9}) + 1.16774(10^{-9}) \right\}}$$

$$= 5.2591(10^{-5})$$

$$\phi = \frac{22,258.275}{8.314(360.9)^2} = 2.05545(10^{-2})$$

$$\phi_r = \frac{\Delta \phi}{\phi} = 2.5558(10^{-3})$$

→