GJ 12/12/12

AM

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena

Final Examination, Fall 2012

Time: 8.00 - 11.00 am **Friday, December 14, 2012**

Instructions: Attempt All Questions.
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Question #1 (30 points)

Flammable and toxic liquids are often collected into or stored in tanks that are not insulated and exposed to the elements – the sun, wind and precipitation. Examples of commercial products such handled are hydrocarbon liquids (crude oils, condensates and refinery products), ammonia and chlorine. Many of the storage tanks are not rated for high pressure, particularly if they are not equipped with vapor recovery systems. Such tanks would typically have a covered observation hatch (for visual inspection of content) and a vent. Tanks, such as petroleum storage vessels, are often located near residences and public spaces in rural areas or in open, unsecured locations. These may constitute hazards to people who socialized around the vessels in the case of explosions or accidental discharges of content due to leaks, grass fires, lightning, open flames from cigarette lighters and the sun's radiation. There have been several explosions of tanks for collecting oil and gas condensates, and these have led to fatalities of a number of young people who might have trespassed on unfenced facilities.

The current problem is on a tank for storing pure pentane, a diluent added to bitumen so that it can flow in pipes. The tank is a vertical cylinder sitting above ground on a ring mounted on supports. It may be assumed that the entire surface of the cylinder is exposed to the ambient. The tank has an inside diameter of 1 m and a height of 1.4 m. The tank is rated for a maximum of 6 atm. absolute pressure and it is completely sealed unless when material is to be pumped in or out of it. On a summer day when the temperature of the ambient, the tank and its content was 22°C, liquid was present in the tank to a level of 60 cm. A forest fire started and radiated heat from a distance on the tank such that the temperature of the tank and content rose. The ambient temperature near the tank remained unchanged at 22°C. With a gentle wind, the convective heat transfer coefficient around the tank was 86 W/m^2 K. The engineer on duty was asked to advise whether the tank was in danger of exploding - as the pressure in the tank rose in response to the temperature increase. The engineer, using a pyrometer, estimated that the heat rate from the fire on the tank was constant and was 5.5 kW. This heat is absorbed by the steel tank (mass 200 kg, C_p = 0.465 kJ/kg K), the liquid and the pentane vapor. Part of the heat would be used to vaporize some of the liquid as the tank content temperature is elevated. You may assume that each of the liquid and the vapor is always well-mixed, that only pure pentane vapor is in the tank above the liquid pentane, and that the vapor is an ideal gas. Also the tank and its content are always at the same temperature at any instant.

After how long of heating will the tank pressure exceed the rating limit? Show all your analysis, assumptions and derivations.

Data & Information:
Properties of pentane – density of liquid = 620 kg/m^3, Molar mass = 72.146 g/mole, boiling point = 36.07°C, latent heat of vaporization = 360 kJ/kg, C_p of liquid = 2.3 kJ/kg K, C_p of vapor = 1.8 kJ/kg K,

Temperature, °C	20	40	60	80	100	120	140
Vapor Pressure, Atm.	0.558	1.141	2.117	3.628	5.829	8.867	13.027

The Clasius-Clapeyron equation is:

$\ln (P/P_1) = (\Delta H_v/\mathbf{R})(1/T_1 - 1/T)$ where T is in **K** and **R** is the universal gas constant.

The ideal gas equation is: $PV = n\mathbf{R}T$ where n is in moles (mass/Molar mass)

Question #2 (25 points)

Underbalanced drilling is the process of drilling an oil well while air is injected at high pressure, instead of drilling mud, to oppose the pressure in the reservoir and avoid blowout. Often, the pressure of air at the bottom of the hole is less than the local reservoir pressure and oil or gas or both would be produced as the well traverses the layer containing hydrocarbon fluids. The produced fluids help transport cuttings to the surface where solids, oil, water and gas are separated. Underbalance operations have advantages that production occurs faster, the formation is not invaded and damaged by mud, bentonite is not partially lost and drilling rate is increased. Drilling is followed by insertion of casings to maintain the integrity of the walls and seal off water bearing layer below the overburden. Conductor casing is at the top, followed by surface, intermediate and production casings. The oil produced in one well is of current interest.

Deep in the well, the outer diameter of the pipes to which the bit is attached (the string) is 10 cm. The inside diameter of the production casing is 16 cm. Heavy oil with a density of 920 kg/m^3 and a viscosity of 9.8 mPa s is in the annular space, flowing upwards. Oil was produced at rate of 13.25 m^3/hour.

a) Find an expression for the velocity profile in the annular space assumed filled only with oil.
b) What is the maximum velocity for the oil and at what radial distance is this observed?
c) If the production casing is 450 m long, what shear force acts on the casing?

Question #3 (30 points)

Paper is manufactured manually by a common method. Wood pulp or other fibres form a dilute suspension in water. The fibers are accumulated in random orientation on a screen mold or wire mesh until a thin coat forms. The coat is laid on a flat surface and allowed to dry. (Some of the water may be squeezed out by a variety of techniques to accelerate drying.)

In a process of interest, a 40 cm by 40 cm coat of fibres was laid on a flat, horizontal metal plate that is 40 cm wide by 60 cm long. Air at 20°C that has moisture at a mole fraction of 0.012 is blown over the surface in a direction normal to the width of the plate and parallel to it. The coat was at the back so that there was a 20 cm long section free of water at the leading edge. The free stream velocity was 0.5 m/s. The temperature of the plate was maintained at 20°C as water evaporated into the air and is carried off. The initial layer of free water on the plate was uniform at 0.5 mm thickness. As the water vaporized at a faster rate at the front end, water is drawn forward by capillary or wicking action (as for a paper towel) to keep all the paper hydrated until all the water is gone. The room pressure was 680 mm Hg.

a) Show how you would estimate the time required to remove all the free water from the coat? Use the **integral method** and show your derivations. Neglect the volume of the fibers.
b) (Bonus **5** pts only after doing part **a**) To evaporate the water faster, is it preferable to increase the temperature of the air and plate to 26°C or to increase the air free stream velocity by 50%?

Data:

Vapor pressure of water at 20°C = 17.5 mm Hg; at 26°C = 25.2 mm Hg; Density of air = 1.205 kg/m^3; Viscosity of air = 0.018 mPa. s; Diffusivity of water vapor in air = 2.16 (10^{-5}) m^2/s. Density of liquid water at 20°C = 998 kg/m^3, at 26°C = 996.8 kg/m^3. Molar mass of water = 18.016 g/mole. The universal gas constant is 8.314 J/mole K.

Question #4 (15 points)

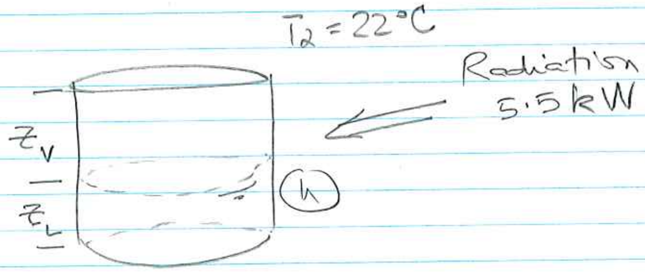
Samples of a sport drink were taken off an assembly line at random for testing. The specification is that each bottle contains less than 10 mg of impurities. The levels of impurities (in mg) from laboratory reports are as follows:

8.2, 8.7, 9.8, 10.1, 10.5, 8.9, 9.4, 8.3, 10.3, 16.1, 9.7, 10.9, 9.8, 8.1, 11.2, 10.3, 9.6, 9.9, 12, 11.5, 9.1, 11.7, 8.9, 9.6, 9.9

What are the average and best estimate of standard deviation?

ENCH 501 Final Exam Solution Dec. 14, 2012

Question #1



Heat radiated on tank is absorbed by the vessel and contents, and some is transferred into ambient air by convection.

Given at $t = 0$

$z_L = 0.6\,m$, $z_V = 0.8\,m$

Let the tank and content be the control volume.

Energy balance

$$\text{Input} + \underbrace{\text{Gen}}_{0} = \text{Output} + \text{Accum.}$$

$$(1) \quad q_{fire} = hA_t(T - T_a) + \frac{d}{dt}\Big[\underbrace{(m_t C_{p_T} + m_L C_{p_L} + m_V C_{p_V})(T - T_a)}_{\text{sensible heat content}}\Big] + \underbrace{\Delta H_V \frac{dm_V}{dt}}_{\text{latent heat}}$$

where subscripts t, L and V refer to the tank, the liquid and the vapor respectively.

- The masses of liquid (m_L) and vapor (m_V) may vary with time but the total is unchanged at T.
- For the vapor phase, m_V is related to both T and P by the ideal gas law

$$(2) \quad m_V = \frac{MPV_V}{RT}\,; \quad V_V = \text{volume of vapor.} \quad M = \text{molar mass pentane}$$

The total volume inside the tank also remains constant, $\therefore$ $V_V = V_{total} - V_L$

and $V_L = \frac{1}{\rho_L}(\Gamma - m_V)$

In turn, the vapor pressure P is related to the absolute temperature of tank content T by the Clausius - Clapeyron eq.

(3) $\frac{d \ln P}{dT} = \frac{\Delta H_V}{RT^2}$ or $\ln\left(\frac{P}{P_1}\right) = \frac{\Delta H_V}{R}\left[\frac{1}{T_1} - \frac{1}{T}\right]$

These are 3 equations for 3 unknowns — P, T, m_V. Using the relationships for V_L and V_V, eq. (2) may be expressed as

(4) $$m_V = \frac{V_{total} - \Gamma/\rho_L}{\left(\frac{R}{M}\right)\frac{T}{P} - \frac{1}{\rho_L}}$$

Equation (1) may now be solved with eqs (3) and (4). The equations look complicated, and in similar cases, assumptions are made to simplify the problem.

II Possible assumptions

ⓐ System reaches a steady state and heat input by radiation = heat output by convection. The tank and content are heated up but the final pressure is lower than the tank rating.

(b) neglect the convective heat loss term and solve the unsteady problem – by MATLAB or through an exploratory approach.

With assumption (a)

(5) $$q_{fire} = hA(T - T_a)$$

$$5500 = 85(2)\pi\left(0.5^2 + 0.5(1.4)\right)(T - 22)$$

$$T = 32.84\,^\circ C$$

Apply the Clasius – Clapeyron equation

$$\ln\left(\frac{P}{0.558}\right) = \frac{360(72.146)}{8.314}\left(\frac{1}{293.15} - \frac{1}{305.99}\right)$$

$$P = 0.8727 \text{ atm} \quad (\ll 6\text{atm})$$

Hence, although the tank and content will see a temperature rise of almost $11\,^\circ C$, the final pressure will be below the tank limit. The tank will not explode! $\longrightarrow$ Ans.

Making assumption (b), the equation to be solved is

(6) $$q_{fire} = \frac{d}{dt}\left[\left(m_t C_{p_T} + m_L C_{p_L} + m_v C_{p_v}\right)(T - T_a)\right] + \Delta H_v \frac{dm_v}{dt}$$

where $m_L = \Gamma - m_v$

and $$m_v = \frac{V_{total} - \Gamma/\rho_L}{\frac{RT}{MP} - \frac{1}{\rho_L}} \qquad (eq.\ 4)$$

For the problem, at $t = 0$, $T = 22°C$

□ mass of liquid $= \pi(0.5)^2(0.6)(620) = 292.168$ kg.

Pressure of vapor is estimated from data:

$$\ln \frac{P}{0.558} = \frac{360(72.146)}{8.314}\left\{\frac{1}{293.15} - \frac{1}{295.15}\right\}$$

$$P = 0.558(1.0749) = 0.5998 \text{ atm}$$

□ mass of vapor in tank at $22°C$ (from ideal gas eq.)

$$m_v = \frac{0.5998(\pi)(0.5)^2(0.8)(72.146)}{0.082056(295.15)}$$

$$= 1.1226 \text{ kg}$$

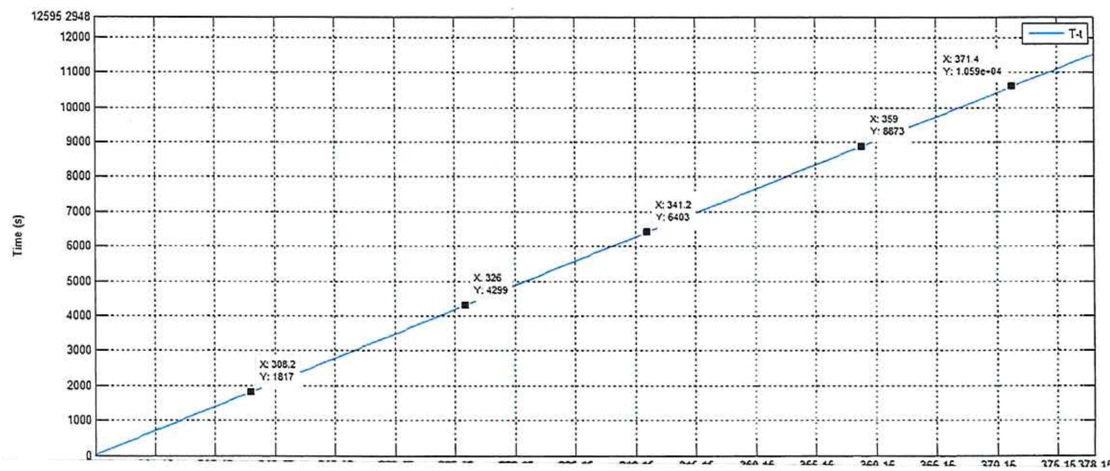
$$\therefore \Gamma = m_v + m_L = 293.29 \text{ kg}$$

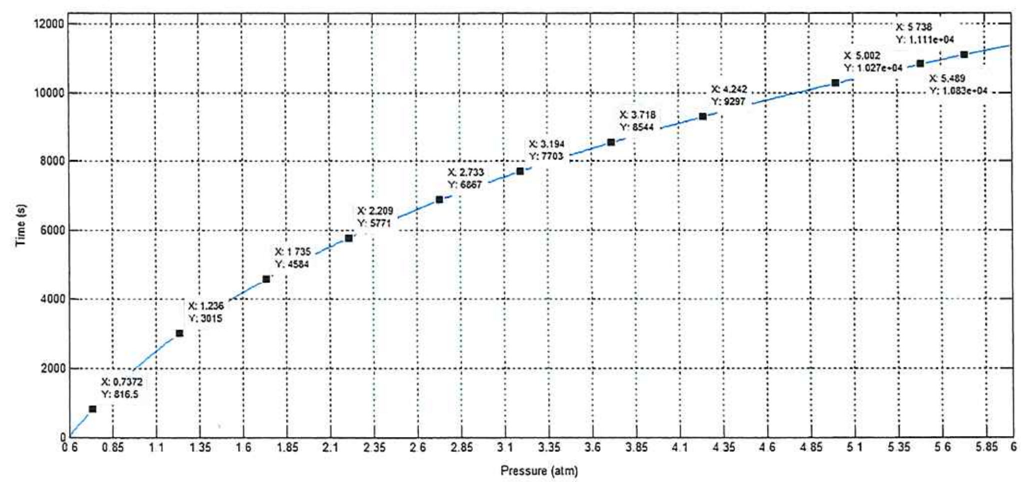
Equation (6) is hence (keeping all units consistent)

$$5500 = \frac{d}{dt}\left\{\left[200(465) + (293.29 - m_v)(2300)\right.\right.$$

$$(7) \qquad \left.\left. + m_v(1800)\right](T - 295.15)\right\} + 360(10^3)\frac{dm_v}{dt}$$

Time (s)	mv (kg)	P (atm)	Temp (K)
0	1.1228	0.6	295.15
500		0.68	
1000		0.775	302.4
1500		0.8744	
2000		0.98	309
2500		1.099	
3000		1.23	316.7
3500		1.373	
4000		1.535	323
4500		1.7	
5000		1.897	331
5500		2.097	
6000		2.309	338.4
6500		2.545	
7000		2.807	345
7500		3.082	
8000		3.369	352.6
8500		3.693	
9000		4.03	360
9500		4.391	
10000		4.79	367.1
10500		5.2	
11000		5.625	374.4
11500	8.9439	6	378
13200			393.15





with

$$(8) \qquad m_v = \frac{1.0995 - \dfrac{293.29}{620}}{\dfrac{(0.082056)\,T}{(72.146)\,P} - \dfrac{1}{620}}$$

and

$$(9) \qquad \ln \frac{P}{0.558} = \frac{360\,(72.146)}{8.314}\left\{\frac{1}{293.15} - \frac{1}{T}\right\}$$

Equations (7), (8) and (9) are solved using MATLAB. The results are presented in the tables and plots attached.

The pressure limit for the tank will be reached at $t \simeq 11{,}500$ s or 3.194 hrs. and the tank temperature would be $\sim 104.85°C$ $\Longrightarrow$

This is the solution if there was no convective heat loss!

* The engineer reports that, with the wind, there would be no explosion! If the wind stops, the tank will explode in ~3 hrs.

The foregoing may also be solved approximately without the use of MATLAB as follows.

- Extend the assumptions
  - ↳ no convective heat loss
  - ↳ neglect heat for phase change
  - ↳ neglect sensible heat for vapor.

That is, heat from fire simply heats up the vessel and liquid pentane.

First check the data provided.

Estimate the temperature at which the tank pressure will be 6 atm. – use given $\Delta H_v = 360$ kJ/kg

$$\ln\left(\frac{6}{5.829}\right) = \frac{360(72.146)}{8.314}\left[\frac{1}{373.15} - \frac{1}{T}\right]$$

$T = 374.44$ K or $101.293\ °C$. (Eq. 9 will give a higher T)

Estimate ΔH_v w/ same range of data provided

$$\ln\left(\frac{8.867}{5.829}\right) = \frac{\Delta H_v(72.146)}{8.314}\left[\frac{1}{373.15} - \frac{1}{393.15}\right]$$

$\Delta H_v = 354.6$ kJ/kg

This close to the value provided and T is not much changed if the latter ΔH_v is used.

Now the energy balance if vessel and liquid are heated from 22°C to 101.3 °C.

Total heat gained, Q, is

$$Q = \{292.18(2.3) + 200(0.465)\}(101.3 - 22)$$
$$= 60{,}663.443 \text{ kJ}$$

With heat input rate = 5.5 kW, the time required = 11,029.7 s or 3.0638 hrs.

Given that $T = 101.3°C$ and $P = 6$ atm, if volume of vapor is unchanged from the value at 22 °C, then

$$m_v\Big|_{101.3°C} = \frac{6(\pi)(0.5)^2(0.8)(72.146)}{0.082056(374.4)}$$

$$= 8.85 \text{ kg}$$

The volume of liquid evaporated $= \dfrac{8.85 - 1.1224}{620}$

$= 0.0125 \text{ m}^3$

This is 2.64% of the original volume of liquid.
The heat required to evaporate this amount of liquid pentane is

$$\{7.7274(360) + (8.85)(1.8)(101.3-22)\} = 4{,}047.76 \text{ kJ}$$

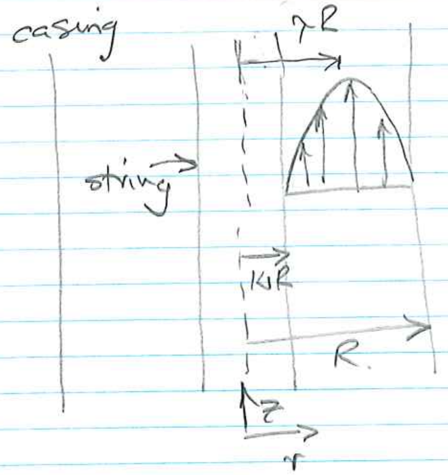
This is 6.67% of heat gained by liquid and tank (without evaporation)

The time required to effect the phase change is 736 s or 0.204 hr.

Adding this to previous value gives 3.272 hrs to reach 6 atm — when there is no convective heat loss

A better answer can be obtained by iteration. The answer is close to the MATLAB solution.

Question #2



Is flow laminar?

$$Q = 13.25 \ m^3/hr \quad \text{or}$$

$$= 3.681 (10^{-3}) \ m^3/s$$

X-sectional area

$$A = \pi (R^2 - \kappa^2 R^2)$$

$$R = 8 cm \qquad \kappa R = 5 cm, \quad \kappa = 5/8$$

$$A = \pi (0.08)^2 (1 - (5/8)^2) \ m^2$$

$$= 0.01227 \ m^2$$

$$\therefore \bar{u} \doteq 0.3 \ m/s$$

and $\quad Re = \dfrac{D_h \bar{u} \rho}{\mu} \quad ; \quad D_h = \dfrac{4 \pi R^2 (1 - \kappa^2)}{2 \pi R (1 + \kappa)}$

Hydraulic Diam.

$$= 2R(1 - \kappa)$$

$$Re = \frac{2(0.08)(1 - 5/8)(0.3)(920)}{9.8(10^{-3})}$$

$$= 1,996.3 \qquad < 2100 \qquad \therefore \text{flow laminar}$$

(a)

This problem has been solved in the Notes:- Equation 6.36

$$u = \frac{\alpha R^2}{4 \mu L} \left\{ 1 - \frac{r^2}{R^2} + \frac{1 - \kappa^2}{\ln(1/\kappa)} \ln\left(\frac{r}{R}\right) \right\}$$

where $\quad \dfrac{\alpha}{L} = - \left[\dfrac{dP}{dz} + \rho g \sin \beta \right] \quad ; \quad \beta = 90^\circ$ for this problem

$\longrightarrow$

(b) The maximum velocity is given by eq. 6.37

$$u_{max} = \frac{\alpha R^2}{4\mu L}\left[1 - \left(\frac{1-\kappa^2}{2\ln(\frac{1}{\kappa})}\right)\left(1 - \ln\left[\frac{1-\kappa^2}{2\ln(\frac{1}{\kappa})}\right]\right)\right]$$

and the maximum is at a radial distance λR

where $$\lambda = \left[\frac{1-\kappa^2}{2\ln(1/\kappa)}\right]^{1/2}$$

Both of these values can be calculated.

From eq. 6.38

av. vel $$\bar{u} = \frac{\alpha R^2}{8\mu L}\left[\frac{1-\kappa^4}{1-\kappa^2} - \frac{1-\kappa^2}{\ln(1/\kappa)}\right],$$

$$\varepsilon = \frac{\alpha R^2}{4\mu L}$$ can be calculated.

$$\bar{u} = \frac{\varepsilon}{2}\left\{0.0941\right\} = 0.3 \text{ m/s.}$$

$$\therefore \quad \varepsilon = 6.3767$$

$$\lambda = 0.8051$$

$$u_{max} = \varepsilon\left\{1 - \lambda^2\left(1 - \ln\lambda^2\right)\right\}$$

$$= 6.3767\left\{1 - 0.6483\left(1 - \ln 0.6483\right)\right\}$$

$$= 0.451 \text{ m/s.}$$

(c) From eq. 6.33

wall shear $\tau\Big|_{r=R} = \frac{\chi R}{2L}\left[1 - \lambda^2\right]$

and $\frac{\chi R}{2L} = \frac{2\mu\varepsilon}{R}$

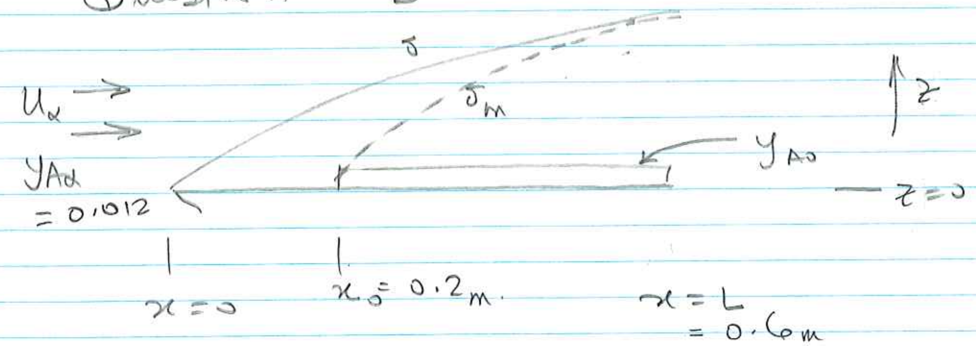
$\therefore$ Shear force on casing

$$F = (2\pi R L)\,\tau\Big|_{r=R}$$

$$= 2\pi(\cancel{0.08})(450)\,\frac{2(9.8)(10^{-3})\,6.3747}{\cancel{0.08}} \times (1-0.6483)$$

$$= 124.2845 \quad N \quad \longrightarrow$$

Question #3

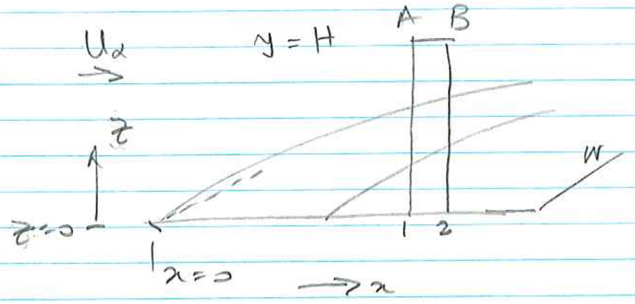


From Raoult's Law

$$y_{A0} = \frac{P_{vp}^{A}}{P_t} = \frac{17.5}{680} = 0.025735$$

Consider a control volume 1AB2.

$\left\{ \begin{array}{l} A \equiv \text{water} \\ B \equiv \text{air} \end{array} \right.$



from notes:

Mass through AB is given by

$$-\frac{d}{dx}\left[\int_0^H W u \rho \, dz\right] dx$$

where W is plate width.

From momentum balance on the control volume, where U_α (and therefore Pressure) is constant,

Eq. 5.16 $$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{z}{\delta}\right) - \frac{1}{2}\left(\frac{z}{\delta}\right)^3 \quad \text{where } \delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

For mass transfer, molar basis

$$N_A = -c\,\mathcal{D}_{AB}\frac{dy_A}{dz} + y_A(N_A + N_B)$$

Note that $y_A \le 0.026$

at $z = 0$ (the wall), $N_B = 0$; Material balance on A per unit width

(12) Input $\therefore$ $$N_A\Big|_{z=0} = \left(-\frac{c\,\mathcal{D}_{AB}}{1-y_A}\frac{dy_A}{dz}\right)_{z=0} \cdot dx$$

(A1) input $$\int_0^H c\, y_A u \, dy$$

(BZ) output $\displaystyle \int_0^H c y_A u\, dy + \frac{d}{dx}\left[\int_0^H c y_A u\, dz\right] dx$

(AB) output volume displaced × conc.

$$- \frac{d}{dx}\left[\int_0^H u\, dz\right] dx \times c y_{A\infty}$$

$$\therefore \left(\frac{-c D_{AB}}{1-y_A}\frac{dy_A}{dz}\right)\Bigg|_{z=0} dx = \frac{d}{dx}\left[\int_0^H c y_A u\, dz\right] dx$$

$$- \frac{d}{dx}\left[\int_0^H c y_{A\infty} u\, dz\right] dx$$

Since $c = P/RT = \text{const}$

$$\left(-\frac{D_{AB}}{1-y_A}\frac{dy_A}{dz}\right)_{z=0} = \frac{d}{dx}\left[\int_0^H (y_A - y_{A\infty}) u\, dz\right]$$

This is the integral material balance equation

Conditions:

$z = 0 \quad y_A = y_{A0} \quad \theta = \theta_0 = y_{A0} - y_{A\infty}$

$z = \delta_m \quad y_A = y_{A\infty} \quad \theta = 0$

$z = \delta_m \quad \dfrac{dy_A}{dz} = 0 \quad \dfrac{d\theta}{dz} = 0$

With $\theta = y_A - y_{A\infty}$

Let $\theta = a + bz + cz^2$

apply conditions

$$\theta/\theta_0 = \left(1 - \frac{z}{\delta_m}\right)^2 = \frac{y_A - y_{A\infty}}{y_{A0} - y_{A\infty}}$$

Substitute into integral material balance eq.

$$+\frac{2D_{AB}}{1-y_{A0}}\frac{(y_{A0}-y_{A\alpha})}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m}(y_A-y_{A\alpha})\,u\,dz\right]$$

where the upper limit of integral H has been replaced with δ_m since $y_A - y_{A\alpha} = 0$ for $z \geq \delta_m$

Subst. profile

$$\frac{2D_{AB}}{1-y_{A0}}\frac{1}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m} U_\alpha\left(1-\frac{z}{\delta_m}\right)^2\left(\frac{3}{2}\frac{z}{\delta}-\frac{1}{2}\left(\frac{z}{\delta}\right)^3\right)dz\right]$$

Let $\delta_m = \xi\delta$ and $\eta = z/\delta$

Then

$$\frac{2D_{AB}}{1-y_{A0}}\frac{1}{\xi\delta} = \frac{d}{dx}\left[U_\alpha\delta\int_0^{\xi}\left(1-\frac{\eta}{\xi}\right)^2\left(\frac{3}{2}\eta-\frac{1}{2}\eta^3\right)d\eta\right]$$

$$= \frac{d}{dx}\left[U_\alpha\delta\left(\frac{\xi^2}{8}-\cancel{\frac{1}{120}\xi^4}\right)\right]$$

neglect if $\xi < 1$

$$\underset{\text{const.}}{\beta} = \frac{16D_{AB}}{1-y_{A0}}\frac{1}{U_\alpha} = \xi\delta\frac{d}{dx}\left[\delta\xi^2\right]$$

$$\beta = 2\xi^2\delta^2\frac{d\xi}{dx} + \xi^3\delta\frac{d\delta}{dx}$$

But from Notes $\delta\dfrac{d\delta}{dx} = \dfrac{140}{13}\dfrac{\nu}{U_\alpha}$ and $\delta^2 = \dfrac{280}{13}\dfrac{\nu x}{U_\alpha}$

$$\beta = \frac{560}{13}\frac{\nu}{U_\alpha}x\,\xi^2\frac{d\xi}{dx} + \xi^3\left(\frac{140}{13}\frac{\nu}{U_\alpha}\right)$$

$$\beta = \frac{560}{39}\frac{\nu}{U_\alpha} x \frac{d\xi^3}{dx} + \xi^3\left(\frac{140}{13}\frac{\nu}{U_\alpha}\right)$$

$$\frac{16}{1-y_{A0}}\frac{D_{AB}}{\nu}\frac{\cancel{U_\alpha}}{\cancel{U_\alpha}}\frac{13}{140} = \frac{4}{3} x \frac{d\xi^3}{dx} + \xi^3$$

$$\frac{52}{35}\frac{1}{1-y_{A0}}\left(\frac{D_{AB}}{\nu}\right) = \frac{4}{3} x \frac{d\xi^3}{dx} + \xi^3$$

Integrate

$$\xi^3 = C x^{-3/4} + \frac{52}{35}\frac{1}{1-y_{A0}}\left(\frac{D_{AB}}{\nu}\right)$$

The constant of integration is determined from $x = x_0$, $\xi = 0$

$$\therefore \quad \xi = \left[\frac{52}{35}\frac{1}{1-y_{A0}}\cdot\frac{D_{AB}}{\nu}\right]^{\frac{1}{3}}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{\frac{1}{3}}$$

Evaporation rate

$$Q = \int_{x_0}^{L} W\, N_A\big|_{z=0}\, dx$$

$$= W\left(\frac{-c\,D_{AB}}{1-y_{A0}}\right)\int_{x_0}^{L} \frac{dy_A}{dz}\bigg|_{z=0} dx$$

But

$$\frac{dy_A}{dz}\bigg|_{z=0} = -\frac{2\,(y_{A0}-y_{A\alpha})}{\delta_m}$$

$$Q = \frac{2WcD_{AB}(y_{A0}-y_{A\delta})}{(1-y_{A0})} \int_{x_0}^{L} \frac{dx}{\delta_m}$$

where $\delta_m = 4.64\sqrt{\frac{\nu x}{U_\alpha}}\left(\lambda^{\frac{1}{3}}\right)\left(1-\left(\frac{x_0}{x}\right)^{3/4}\right)^{1/3}$

and $\lambda = \frac{52}{35}\frac{1}{1-y_{A0}}\frac{D_{AB}}{\nu}$

The amount of water initially present is

$$\phi = \frac{(h \times \text{area})\rho_L}{M_w} = \frac{0.5(10^{-3})(40)^2(10^{-4})\,998}{18.016} \text{ kmols}$$

Time for evaporation $= \frac{\phi}{Q}$

The integral for Q may have to be determined numerically – as δ_m is a complicated function of x.

Question #4

On inspection of the data, most values are between 8 and 12. There, however, is a value at 16.1. This is treated as an outlier and removed.

	x (mm/s)	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
1	8.2	1.65	2.7225
2	8.7	1.15	1.3225
3	9.8	0.05	0.0025
4	10.1	0.25	0.0625
5	10.5	0.65	0.4225
6	8.9	0.95	0.9025
7	9.4	0.45	0.2025
8	8.3	1.55	2.4025
9	10.3	0.45	0.2025
10	9.7	0.15	0.0225
11	10.9	1.05	1.1025
12	9.8	0.05	0.0025
13	8.1	1.75	3.0625
14	11.2	1.35	1.8225
15	10.3	0.45	0.2025
16	9.6	0.25	0.0625
17	9.9	0.05	0.0025
18	12	2.15	4.6225
19	11.5	1.65	2.7225
20	9.1	0.75	0.5625
21	11.7	1.85	3.4225
22	8.9	0.95	0.9025
23	9.6	0.25	0.0625
24	9.9	0.05	0.0025

$\Sigma \dfrac{236.4}{24}$

$\bar{x} = 9.85$

$\dfrac{\Sigma \lvert x_i - \bar{x} \rvert}{N}$ Average Deviation

$\sqrt{\dfrac{(x_i - \bar{x})^2}{N-1}}$ Best Estimate standard Deviation

$\dfrac{19.9}{24} = 0.829$

$\sqrt{\dfrac{26.82}{23}} = 1.08$

$x = 9.85 \pm 0.829$ for 58% of data

$x = 9.85 \pm 1.08$ for 68% of data $\longrightarrow$.