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GJ Dec. 19

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena

Final Examination, Fall 2011

Time: 8.00 - 11.00 am **Wednesday, December 21, 2011**

Instructions: **Attempt All Questions.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (25 points)

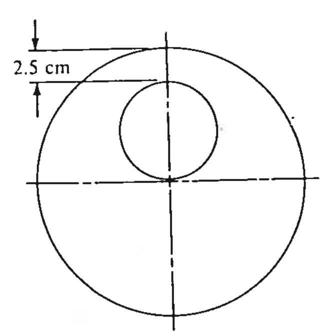
(a) (10 points)
The city of Calgary has many "plus 15" walkways or corridors connecting buildings and suspended above the streets. The corridors allow downtown employees to move about without having to wear heavy winter gears and avoid slipping on ice on sidewalks and on the main road. The +15 corridors are heated by forcing air flow between buildings or with space heaters. Your boss is interested in how much the heating is costing for a particular walkway.

The walkway is a square cross-section channel, made of a tough, transparent polymer material. The inside height of the channel is 2.6 m. The wall is 5 cm thick and the walkway is 25 m long. If the heat transfer coefficients inside and outside the walkway can be assumed large, and the temperatures of air inside and outside the walkway are 15°C and -35°C respectively, estimate the rate at which heat must be supplied into the walkway. The thermal conductivity of the wall material is 1.8 W/mK.

(b) (15 points)
Materials used for insulation degrade with time and either become compacted or shift position.

A long 5 cm outside diameter metal pipe is horizontal and a fluid at 180°C flows at a high rate through it in a crude oil refinery. The pipe is covered with insulation such that the overall outside diameter becomes 15 cm. The insulation is 85% magnesia wool with a thermal conductivity of 0.059 W/mk. Air outside the pipe is assume maintained at 5°C, the average temperature for southern Alberta, and the heat transfer coefficient is assumed large because of the high winds in the area. After 6 months, it is observed that the insulation has shifted around the pipe as per the diagram below.

Determine whether or not the insulation is performing better at 6 months than immediately after being installed. By how much has its effectiveness changed?



Problem #2 (25 points)

A solution of monoethanolamine is used to remove hydrogen sulphide from sour gas in a gas purification process. The liquid runs down on the inside surface of a 12 m tall, 1 m diameter wetted wall absorption column. A video camera is installed to monitor the inside of the tower. A patch of dirt film was observed to ride on the surface of the liquid layer downwards and it traveled the height of the tower in 0.5 seconds. Using the differential method,

a) estimate the thickness of the liquid layer on the tank wall. Derivations are not necessary.
b) What is the volume flow rate of the liquid solution down the column?

Data: Properties of the liquid - μ = 16.2 mPa s, ρ = 1022 kg/m^3

NOTE: Attempt one of #3 or #4 (not both).

Problem #3 (50 points)

The preparation of most packaged foods sold in stores involves chemical engineers. After packaging, the engineers must write instructions for how the consumer should handle the products, particularly heating prior to eating.

One such item is crushed fruits (such as apple) wrapped in a crepe for breakfast. The item is in the shape of a solid cylinder (4 cm diameter) that is heated from its frozen state at a temperature of -15°C to a temperature of 74°C along its centre-line in 15 minutes inside a convection oven set at 177°C.

Use the ***integral method*** to estimate the heat transfer coefficient (h, W/m^{2}K) around the cylinder suspended in the convection oven. Assume that the cylinder did not lose moisture, that the dimensions remained constant and the thermophysical properties are constant as given below. You may neglect latent heat and treat the problem as occuring in two stages - stage 1 up to when heat reaches the axis, and stage 2 thereafter. State all assumptions.

Data: Properties of food item: k = 0.9 W/mK; C_p = 4.02 kJ/kg K; ρ = 1310 kg/m^3

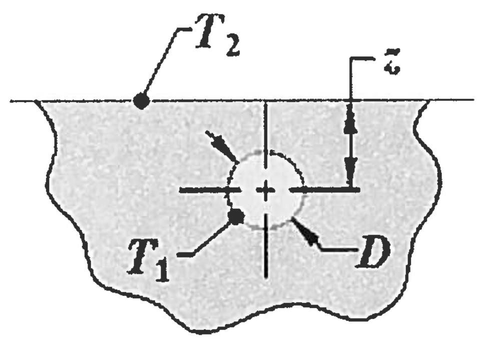
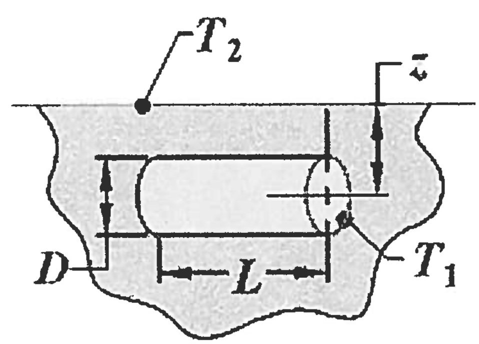
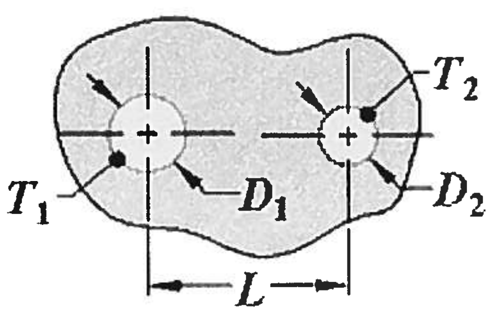
Problem #4 (50 points)

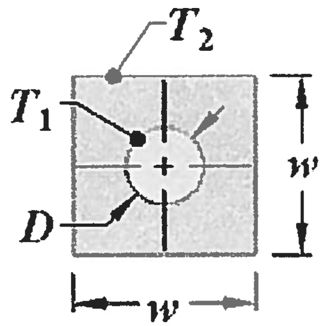
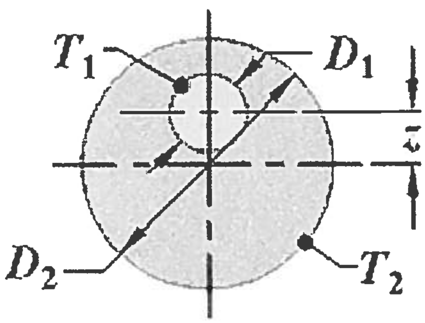
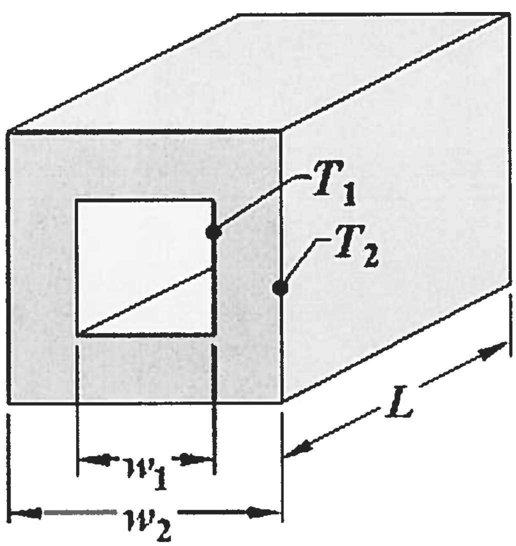
A rectangular tray contains turpentine (approx $C_{10}H_{16}$) that some painters use to clean their brushes. The tray is 30 cm wide and 56 cm long. The average depth of the tray is 2.5 cm. The tray is filled with turpentine to the brim (or edge) with fresh turpentine in a room at 20°C and the level is always maintained. Because it is winter that the painters are working inside a new building just constructed, they had a fan on to circulate heat from a space heat located in one corner of the room. The fan, however propelled air at a free stream velocity U_∞ = 0.3 m/s over the surface of the tray in a direction perpendicular to the width of the tray.

If the ambient pressure is 760 mmHg, the diffusivity of turpentine in air is 1.2 (10^{-5}) m^2/s, the concentration of turpentine in ambient air is considered negligible, and the temperature of both the gases and the liquid are constant at the room temperature, use the ***integral method*** to estimate the rate of loss of turpentine (in moles/second) from the tray. Assume that Raoult's law is obeyed at the liquid-vapor interface ($y_{AO} = P_{vp}/P_t$), and neglect the convective term for flux.

Data: Turpentine - At 20°C, Vapor Pressure, 5 mm Hg ; Air - ρ = 1.205 kg/m^3, kinematic viscosity = 1.511 (10^{-5}) m^2/s ; Universal Gas Constant = 0.08205 (m^3-atm)/(kmol-K) or 8.314 kJ/kmol K .

Conduction Shape Factors

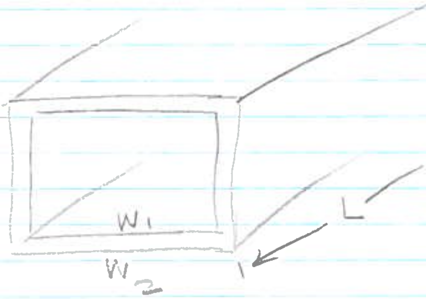
System	Schematic	Restrictions	Shape Factor
Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L >> D$ $L >> D$ $z > 3D/2$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
Conduction between two cylinders of length L in infinite medium		$L >> D_1, D_2$ $L >> w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$

Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L >> w$	$\dfrac{2\pi L}{\ln(1.08w/D)}$
Eccentric circular cylinder of length L in a cylinder of equal length		$D_2 > D_1$ $L >> D_2$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D_2^2 + D_1^2 - 4z^2}{2D_1D_2}\right)}$
Square channel of length L		$w_2/w_1 < 1.4$ $w_2/w_1 > 1.4$ $L >> w_2$	$\dfrac{2\pi L}{0.785 \ln(w_2/w_1)}$ $\dfrac{2\pi L}{0.930 \ln(w_2/w_1) - 0.050}$

ENCH 501 Final Exam Solution Dec. 21, 2011

#1

(a)



$W_1 = 2.6$ m

$W_2 = 2.7$ m

$$\frac{W_2}{W_1} = \frac{2.7}{2.6} = 1.039 < 1.4$$

From table supplied,

$$S = \frac{2\pi L}{0.785 \ln(W_2/W_1)} = \frac{2\pi(25)}{0.785 \ln(1.039)} \text{ m}$$

Using shape factor method
S

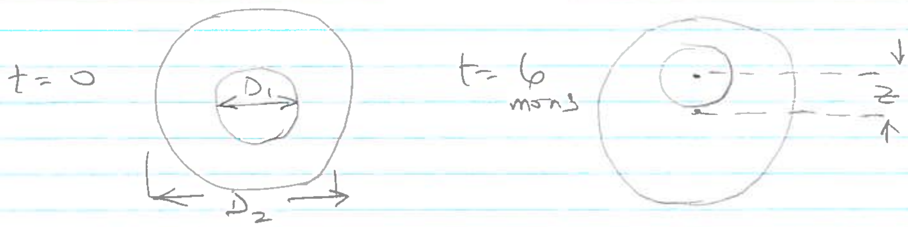
$$Q = kS\Delta T.$$

$$= 1.8(5{,}302.059)(15-(-35)) \quad \text{W}$$

Rate of heat supply $= 477.1853$ kW

$\longrightarrow$

(b)



$$S \qquad \frac{2\pi L}{\ln \frac{D_1}{D_2}} \qquad\qquad \frac{2\pi L}{\cosh^{-1}\left(\frac{D_2^2 + D_1^2 - 4z^2}{2D_1 D_2}\right)}$$

From Notes

From table supplied.

For the same length of pipe, the same k and the same ΔT, the relative heat rates is simply the ratio of the values for S

$$\therefore \quad \frac{q_{t=0}}{q_{t=6\,mons}} = \frac{\cosh^{-1}\left(\dfrac{D_2^2 + D_1^2 - 4z^2}{2 D_1 D_2}\right)}{\ln\left(D_1/D_2\right)}$$

$D_1 = 5\,cm$; $D_2 = 15\,cm$ and

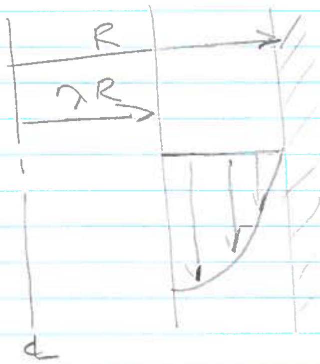
$z = 2.5\,cm$

$$\frac{q_0}{q_6} = \frac{\cosh^{-1}(3/2)}{\ln(3)} = \frac{0.9624}{1.0986}$$

$$= 0.876$$

This means less heat is lost from the system at $t = 0$ compared to at $t = 6$ months when 14.15 % more heat is lost.

2



The formulation and derivations for this problem are in the Notes: – eq. 6.17 (p 137). The velocity profile is given as:

$$u = \frac{\rho g R^2}{4\mu}\left\{1 - \frac{r^2}{R^2} + 2\lambda^2 \ln \frac{r}{R}\right\}$$

At the free surface, $r = \lambda R$, $u = 24$ m/s.

$$\therefore \quad 24 = \frac{(1022)(9.81)(0.5)^2}{4(16.2)(10^{-3})}\left\{1 - \lambda^2 + 2\lambda^2 \ln \lambda\right\}$$

where $0 \le \lambda < 1$

or $$6.2048\,(10^{-4}) = 1 - \lambda^2(1 - 2\ln\lambda)$$

By trial + error, $\lambda \to 1$ means $\ln\lambda \to 0$

$\lambda \simeq 0.9996897$ for 1st try

λ	0.9996897	0.99	0.985	0.98
RHS	$2.183\,(10^{-7})$	$1.9933\,(10^{-4})$	$4.4774\,(10^{-4})$	$7.946\,(10^{-4})$
	0.983	0.9825	0.9823	
	$5.7471\,(10^{-4})$	$6.089\,(10^{-4})$	$6.2287\,(10^{-4})$ close to LHS	

so $\lambda \simeq 0.9823$

The layer thickness, $\delta = (1-\lambda)R$

$= (1 - 0.9823)(0.5)\,\text{m} = 0.00885\,\text{m}$

or 8.85 mm

From eq. 6.19 Notes, the volume rate is given by

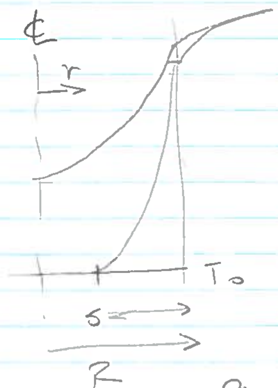
$$Q = \frac{\pi R^4 \rho g}{8\mu}\left\{1 - 4\lambda^2 + \lambda^4(3 - 4\ln\lambda)\right\}$$

with $\lambda = 0.9823$

$$= \frac{\pi (0.5)^4 (1022)(9.81)}{8(16.2)(10^{-3})}(2.90525)(10^{-5})$$

$$= 0.4413 \quad m^3/s$$

Q #3



Energy balance eq

Input + Gen = output + Accum.
(Gen = 0, output = 0)

$$(2\pi R L) q_R = \frac{d}{dt}\left[\int_0^R (2\pi r\,dr\,L)\rho c_p (T - T_0)\right]$$

$$q_R = k \left.\frac{dT}{dr}\right|_R = h\left(T_\infty - T_s(t)\right)$$

Integral energy eq is

$$\frac{R\,h}{\rho c_p}(T_\infty - T_s) = \frac{d}{dt}\left[\int_0^R r (T - T_0)\,dr\right] \qquad (1)$$

or $$\frac{h}{\rho c_p}(T_\infty - T_s) = \frac{d}{dt}\left[R \int_0^1 x (T - T_0)\,dx\right] \quad ; \; x = r/R$$

Divide problem into 2 stages — before and after heat reaches the centre line (as in sketch above)

Stage 1 :

δ is distance of heat penetration into the cylinder

$r = R - \delta$	$T = T_0$	b.c. 1
$r = R - \delta$	$dT/dr = 0$	b.c. 2
$r = R$	$k \left.\frac{dT}{dr}\right|_R = h(T_\infty - T_s)$	b.c. 3

Assume a temperature profile

$$\frac{T - T_0}{T_s - T_0} = (1 - \eta)^2 \quad ; \quad \eta = \frac{R - r}{\delta}$$

where both T_s + δ are functions of time.

(This is the profile for $h \to \infty$, $T_\infty = T_s$ case

for this stage with the assumed temp profile given as $T = a + b(R-r) + c(R-r)^2$

From b.c. 3 (not yet used)

$$k \left.\frac{dT}{dr}\right|_R = h(T_\alpha - T_s) = \frac{2}{\delta}(T_s - T_0)k.$$

$$\text{or} \quad \frac{h\delta}{2k} = \frac{(T_s - T_0)}{(T_\alpha - T_s)}$$

Divide the integral equation by a constant, $T_\alpha - T_0$

$$\frac{h}{\rho c_p}\left(\frac{T_\alpha - T_s}{T_\alpha - T_0}\right) = \frac{d}{dt}\left[R\int_0^1 \left(\frac{T - T_0}{T_\alpha - T_0}\right) x\, dx\right]$$

$$\frac{T_\alpha - T_s}{T_\alpha - T_0} = \frac{1}{1 + \frac{T_s - T_0}{T_\alpha - T_s}} = \frac{1}{1 + h\delta/2k}$$

$$\frac{T - T_0}{T_\alpha - T_0} = \frac{T - T_0}{T_s - T_0}\,\frac{T_s - T_0}{T_\alpha - T_0} = \frac{T_s - T_0}{T_\alpha - T_0}(1-\eta)^2$$

$$= \frac{h\delta/2k}{1 + h\delta/2k}(1-\eta)^2 \quad ; \quad \eta = \frac{R}{\delta}(1-x)$$

$$\frac{h}{R\rho c_p}\,\frac{2k}{2k + h\delta} = \frac{d}{dt}\left[\frac{h\delta}{2k + h\delta}\int_0^1 \left(1 - \frac{R}{\delta} + \frac{R}{\delta}x\right)^2 x\, dx\right]$$

Let $a = 1 - R/\delta$ and $b = R/\delta$

$$\int_0^1 (a + bx)^2 x\, dx = \frac{a^2}{2} + \frac{2}{3}ab + \frac{b^2}{4}$$

The integral equation yeilds a non-linear, ordinary differential equation

$$\frac{h}{R\rho c_p}\left(\frac{2k}{2k+h\delta}\right) = \frac{d}{dt}\left[\frac{h\delta}{2k+h\delta}\left(\frac{(1-R/\delta)^2}{2} + \frac{2}{3}\left(1-\frac{R}{\delta}\right)\frac{R}{\delta} + \frac{1}{4}\frac{R^2}{\delta^2}\right)\right]$$

$$\frac{h}{R\rho c_p}\left(\frac{2k}{2k+h\delta}\right) = \frac{d}{dt}\left[\frac{h\delta}{2k+h\delta}\left(\frac{1}{2} - \frac{1}{3}\frac{R}{\delta} + \frac{1}{12}\frac{R^2}{\delta^2}\right)\right]$$

with the condition: $\delta = 0$ at $t = 0$

$$\frac{12\,h}{R\rho c_p}\left(\frac{2k}{2k+h\delta}\right) = \frac{d}{dt}\left[\frac{h\delta}{2k+h\delta}\left(6 - 4\frac{R}{\delta} + \frac{R^2}{\delta^2}\right)\right]$$

$$= \frac{d}{dt}\left(\frac{6h\delta}{2k+h\delta}\right) - \frac{d}{dt}\left[\frac{4hR}{2k+h\delta}\right] + \frac{d}{dt}\left[\frac{hR^2}{\delta(2k+h\delta)}\right]$$

$$= \left[\frac{12k}{(2k+h\delta)^2} - \frac{4hR}{(2k+h\delta)^2} - \frac{h^2R^2(2k+2h\delta)}{(h\delta)^2(2k+h\delta)^2}\right]\frac{d(h\delta)}{dt}$$

$$\frac{24\alpha}{R} = \left[\frac{12k}{2k+h\delta} - \frac{4hR}{2k+h\delta} - \frac{h^2R^3(2k+2h\delta)}{(h\delta)^2(2k+h\delta)}\right]\frac{d\delta}{dt}$$

$$24\alpha\int_0^t dt = 12kR\int_0^\delta \frac{d\delta}{2k+h\delta} - 4hR^2\int_0^\delta \frac{d\delta}{2k+h\delta}$$

$$-2h^2R^3\int_0^\delta \frac{(k+h\delta)}{(h\delta)^2(2k+h\delta)}\,d\delta$$

$$24\alpha t = \frac{1}{h}\left(12kR - 4hR^2\right)\ln\left(\frac{2k+h\delta}{2k}\right) -$$

$$2hR^3\int_0^{h\delta}\frac{(k+h\delta)}{(h\delta)^2(2k+h\delta)}\,d(h\delta)$$

$$= \left\{12\frac{kR}{h} - 4R^2\right\}\ln\left(\frac{2k+h\delta}{2k}\right) -$$

$$4hR^3\left\{-\frac{1}{2h\delta} - \frac{1}{4k}\ln\left(\frac{h\delta+2k}{h\delta}\right)\right\}_0^{h\delta}$$

$$6\alpha t = \left(\frac{3kR}{h} - R^2\right)\ln\left(\frac{2k+h\delta}{2k}\right)$$

$$+ hR^3\left\{\frac{1}{2h\delta} + \frac{1}{4k}\ln\left(\frac{h\delta+2k}{h\delta}\right) - \frac{1}{k}\right\}$$

Note: This last result requires using limits as $\delta \to 0$,

$$\ln x \simeq 2\left(\frac{x-1}{x+1}\right) \quad \text{for } x > 0$$

and the application of L'Hopital's rule

The expression relates δ, t and h

Time is required when $\delta = R$ at the end of stage 1.

Subst. $r = R$ to obtain

$$\varphi\alpha t = \left(\frac{3kR}{h} - R^2\right)\ln\left(\frac{2k+hR}{2k}\right) +$$

$$hR^3\left\{\frac{1}{2hR} + \frac{1}{4k}\ln\left(\frac{2k+hR}{hR}\right) - \frac{1}{k}\right\}$$

A guess of h would allow t to be estimated at the end of stage 1.

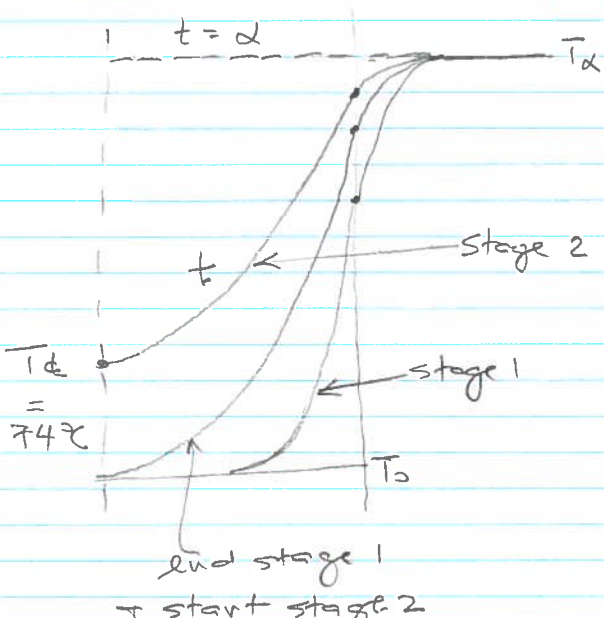
For stage 2, the conditions are:

b.c. 1 $\quad r = 0 \quad \frac{dT}{dr} = 0$ (symmetry)

b.c. 2 $\quad r = R \quad k\frac{dT}{dr} = h(T_\alpha - T_s)$ (as for stage 1)

all R, $t \to \alpha \quad T = T_\alpha$ (steady state temp.)

The temperature profile is anticipated as per diagram below.



Assume a profile

$$T = T_{\mathbb{C}}\left(1 + a\,r^2/R^2\right)$$

where both $T_{\mathbb{C}}$ (centre line temp) and a are functions of time.

Apply b.cs 1 & 2 above

$$T = T_{\mathbb{C}} + \beta\left(T_\alpha - T_{\mathbb{C}}\right) r^2/R^2$$

where $\frac{1}{\beta} = \left(\frac{2}{hR} + 1\right)$ and $T_{¢}(t)$

Substitute for T in the integral equation (1)

$$\frac{R}{\rho C_p} h\left(T_\alpha - T\big|_{e}\right) = \frac{d}{dt}\left[\int_0^R \left(T_{¢} - T_0 + \beta\left(T_\alpha - T_{¢}\right) r^2/R^2\right) r\,dr\right]$$

$$= \frac{d}{dt}\left[\frac{R^2}{4}\left((2-\beta)T_{¢} - 2T_0 + \beta T_\alpha\right)\right]$$

$$\frac{4h}{R\rho C_p (2-\beta)}\left(T_\alpha - (1-\beta)T_{¢} - \beta T_\alpha\right) = \frac{d}{dt} T_{¢}$$

$$\frac{4h(1-\beta)}{R\rho C_p (2-\beta)}\left(T_\alpha - T_{¢}\right) = \frac{dT_{¢}}{dt}$$

Subject to $t = 0$ $T_{¢} = T_0$ for stage 2.

$$\Gamma\, dt = \frac{dT_{¢}}{T_\alpha - T_{¢}} \quad ; \quad \Gamma = \frac{4h(1-\beta)}{R\rho C_p (2-\beta)}$$

$$\ln\left(T_\alpha - T_{¢}\right)\Big|_{T_0}^{T_{¢}} = -\Gamma t$$

or $$\frac{T_\alpha - T_{¢}}{T_\alpha - T_0} = e^{-\Gamma t}$$

$$T_{¢} = T_\alpha - \left(T_\alpha - T_0\right) e^{-\Gamma t}$$

This is substituted into the profile

$$T = \left[T_\alpha - (T_\alpha - T_o)e^{-\Gamma t}\right] + \beta (T_\alpha - T_o)e^{-\Gamma t} \, r^2/R^2$$

For the problem, we are given the following information:

Total time, both stages = 15 min or 900 s

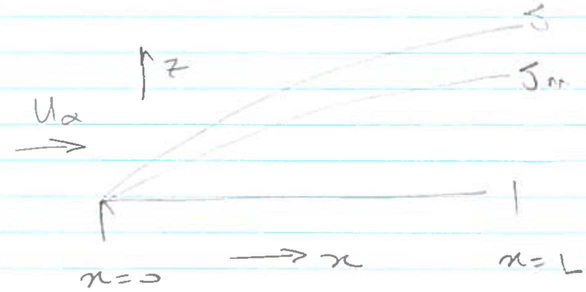
$T\big|_{r=0} = 74°C$; $T_\alpha = 177°C$; $T_o = -15°C$

Since h is in both β and Γ for stage 2, and h has to be estimated for stage 1 to get a time of transition, trial-and-error is involved.

Notes: (a) Many intermediate steps were skipped.

(b) For the examination, how to set up the problem was most important. The final answer is worth only 10% for the problem, i.e. 5 of 50 points.

4



Schmidt #, $S_c = \nu / D_{AB}$

$$= \frac{1.511(10^{-5})}{1.2(10^{-5})} = 1.2592$$

$\therefore \delta_m < \delta$

Is the b.l. laminar?

$$Re_x\Big|_L = \frac{U_\infty L \rho}{\mu} = \frac{(0.3)(0.56)}{1.511(10^{-5})} = 1.1118(10^4)$$

This is $< 5(10^5)$. $\therefore$ b.l. is laminar.

The concentration at the liquid-gas interface, gas side, is given by Raoult's law.

$$y_{A0} = \frac{P_{vp}}{P_t} = \frac{5}{760} = 0.0066 \; (<< 1)$$

The flux of turpentine (A) in air (B) is

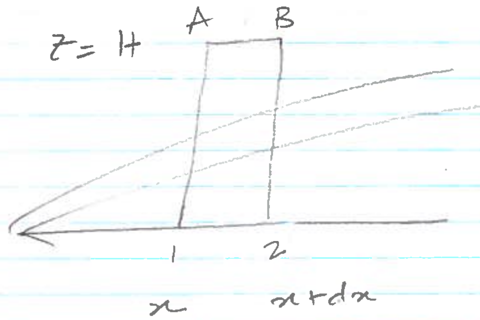
$$N_A = -c\mathcal{D}_{AB}\frac{dy_A}{dz} + y_A(N_A + N_B)$$

in the z-direction

Since $y_A \le y_{A0} << 1$, neglect the convective term

$$\therefore \quad N_A = -c\,D_{AB}\frac{dy_A}{dz} \quad \text{for the problem}$$

By the integral method, consider the control volume as shown below



Material balance on component A

In A1 $\int_0^H c y_A u W dz$

out B2 $\int_0^H c y_A u W dz + \frac{d}{dx}\left[\int_0^H c y_A u W dz\right] dx$

out AB $-\frac{d}{dx}\left[\int_0^H u W dz\right] dx \,(y_{A\infty} c) = 0$

in 12 $N_A\Big|_{z=0} (W dx)$

The balance equation is

$$N_A\Big|_{z=0} (dx\, W) = \frac{d}{dx}\left[\int_0^H c y_A u W dz\right] dx$$

or

$$N_A\Big|_{z=0} = -c\mathcal{D}_{AB} \frac{dy_A}{dz}\Big|_{z=0} = \frac{d}{dx}\left[\int_0^{\delta_m} c y_A u dz\right]$$

The b.c. are

$z = 0 \quad y_A = y_{A0}$, a const.

$z = \delta_m \quad y_A = 0$

$z = \delta_m \quad \frac{dy_A}{dz} = 0$

Choose an equation

$$y_A = a + bz + cz^2$$

Apply b.c.

$$\frac{y_A}{y_{A0}} = \left(1 - z/\delta_m\right)^2$$

and

$$\frac{dy_A}{dz} = -\frac{2y_{A0}}{\delta_m}\left(1 - \frac{z}{\delta_m}\right)$$

Substitute into the integral eq.

$$-c\mathcal{D}_{AB}\left(-\frac{2y_{A0}}{\delta_m}\right) = \frac{d}{dx}\left[\int_0^{\delta_m} c\,y_{A0}\left(1 - \frac{z}{\delta_m}\right)^2 u\,dz\right]$$

Given in Notes

$$\frac{u}{U_\infty} = \frac{3}{2}\left(z/\delta\right) - \frac{1}{2}\left(z/\delta\right)^3 \quad \text{for vel profile}$$

$$2\frac{\mathcal{D}_{AB}}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m} U_\infty\left(1 - \frac{2z}{\delta_m} + \frac{z^2}{\delta_m^2}\right)\left(\frac{3}{2}\frac{z}{\delta} - \frac{1}{2}\left(\frac{z}{\delta}\right)^3\right)dz\right]$$

Let $\xi = \delta_m/\delta$ and $\eta = z/\delta$

$$\frac{2\mathcal{D}_{AB}}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m}\left(1 - \frac{2\eta}{\xi} + \frac{\eta^2}{\xi^2}\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\delta\frac{dz}{\delta}\right]$$

$$= \frac{d}{dx}\left[\delta\int_0^{\xi}\left(1 - \frac{2\eta}{\xi} + \frac{\eta^2}{\xi^2}\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)d\eta\right]$$

Solve to obtain

$$\frac{2 D_{AB}}{\delta \xi} = \frac{d}{dx}\left[\delta U_\infty \left\{ \frac{1}{8}\xi^2 - \frac{1}{120}\xi^4 \right\} \right]$$

neglect for $\xi < 1$

$$\frac{16 D_{AB}}{U_\infty \delta \xi} = \frac{d}{dx}\left[\delta \xi^2 \right]$$

or

$$\frac{16 D_{AB}}{U_\infty} = \delta^2 \xi \frac{d\xi^2}{dx} + \xi^3 \delta \frac{d\delta}{dx}.$$

From Notes: eq 5.18 and 5.19

$$\delta \frac{d\delta}{dx} = \frac{140}{13}\left(\frac{\nu}{U_\infty}\right) \; ; \; \delta^2 = \frac{280}{13}\frac{\nu x}{U_\infty}$$

Substitute

$$\frac{16 D_{AB}}{U_\infty} = \frac{280}{13}\frac{\nu x}{U_\infty} 2\xi^2 \frac{d\xi}{dx} + \xi^3 \left(\frac{140}{13}\frac{\nu}{U_\infty} \right)$$

This gives

$$\frac{52}{35}\frac{D_{AB}}{\nu} = 4x\xi^2 \frac{d\xi}{dx} + \xi^3 = \beta$$

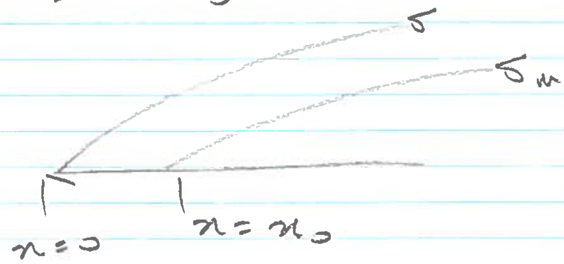
solve 1st order o.d.e.

$$\xi^3 = C x^{-3/4} + \beta$$

Now to apply b.c. to obtain constant

For the problem, $\xi = \delta_m / \delta$ is indeterminate at $x=0$

$\therefore$ change system — no flux at leading edge.



$\therefore \xi = 0$ at $x = x_0$

Apply

$$\xi = \frac{\delta_m}{\delta} = \beta^{\frac{1}{3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{\frac{1}{3}}$$

When $x_0 \longrightarrow 0$ $\qquad \xi = \beta^{\frac{1}{3}}$

$$\therefore \quad y_A = y_{A0}\left(1 - \frac{z}{\delta \beta^{\frac{1}{3}}}\right)^2 \; ; \quad \delta = 4.64\sqrt{\frac{\nu x}{U_\infty}}$$

From Notes

With the concentration profile known, the flux can be estimated.

$$N_A\Big|_{z=0} = -c\,D_{AB}\frac{dy_A}{dz}\Big|_{z=0} = \left(-c\,D_{AB}\right)\left(-\frac{2y_{A0}}{\delta_m}\right)$$

$$= \frac{2\,c\,y_{A0}\,D_{AB}}{\delta_m}$$

The rate of loss of turpentine is given by

$$\dot{Q} = \int_0^L N_A\Big|_{z=0} W\,dx \qquad \text{moles}/\text{s}$$

$$= \int_0^L 2 c y_{A_0} D_{AB} W \left(\frac{1}{\beta^{1/3} \varepsilon} \right) \frac{dx}{x^{1/2}} \quad ; \quad \varepsilon = 4.64 \sqrt{\frac{\nu}{U_\alpha}}$$

$$= \frac{2 c y_{A_0} D_{AB} W}{\beta^{1/3} \varepsilon} \cdot 2 x^{1/2} \Big|_0^L$$

Given $L = 0.56\,m$, $W = 0.3\,m$, $y_{A_0} = 0.0066$

$$c = \frac{P}{RT} = \frac{1}{0.08205\,(273.15+20)} \ \text{kmols}/m^3$$

$$= 0.041575 \ \text{kmols}/m^3$$

$$\varepsilon = 4.64 \sqrt{\frac{1.511(10^{-5})}{0.3}} = 0.03293$$

$$\beta^{1/3} = \left(\frac{52}{35} \; \frac{1}{1.2592} \right)^{1/3} = 1.05668$$

$\therefore$ Rate of loss, $\dot{Q} =$

$$\frac{2(0.041575)(0.0066)(1.2)(10^{-5})(0.3) \cdot 2(0.56)^{1/2}}{1.05668(0.03293)}$$

$$= 8.4976\,(10^{-8}) \ \text{kmols}/s$$

or $8.5\,(10^{-5})$ moles/s