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ENCH 501: Transport Phenomena

Final Examination, Fall 2010

Time: Noon - 3.00 pm **Monday, December 13, 2010**

Instructions: **Attempt All Questions.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (25 points)

New optical and electronic gadgets such as cameras, tablets and laptops are often packaged with packets of absorbents or desiccants. This is necessary to prevent condensation on lenses, mirrors and circuits, especially if the items are transported in airplanes in unheated cargo spaces with high humidity. Films of water cause damage. One such desiccants is Barium Chlorate ($Ba[ClO_3]_2$) that is used in pyrotechnics and as a mordant in dyeing.

Anhydrous (water-free) crystals of the salt (ρ = 3179 kg/m^3) are contained in a jar, 4cm i.d and 5 cm deep. The jar is filled with the crystals and the void fraction of the volume is 38%. The jar is in a refrigerator at 0°C and a fine wire gauze covers the mouth so that there are no convection currents in the porous medium. The gauze did not hinder the diffusion of water vapor into the jar. The vapor pressure of the water in the fridge was 3.5 mm Hg. The ambient pressure is 760 mm Hg. A fan in the fridge circulated air and the air flowed over the mouth of the jar.
Barium chlorate is saturated when it contains 0.17 g water per g desiccant. The residual water, given in mg H_2O per litre of dry air, is 0.8. This is concentration of air in equilibrium with the saturated desiccant. Assume that the top layer of the bed is rapidly saturated with moisture and that a sharp (but moving) boundary separates the region with moisture from the zone with just dry desiccant.

Estimate the time for the moisture to reach the bottom of the jar. Use the differential method and show your steps.

Data: Diffusivity of water vapor in air at 0°C = 2.2(10^{-7}) m^2/s

Problem #2 (25 points)

A Newtonian liquid flows upwards in the annular space between two concentric vertical steel pipes. The inner pipe is 4 in schedule 40s and the outer pipe is 8 in schedule 80. The volume flow rate is 539.12 m^3/day and the pipes are 2 km long. (The problem is based on production from a deep well.)

a) Estimate the pressure drop across the length of the pipes. Show important steps of your analysis.

b) What is the maximum velocity in the annular space?

Data: Properties of the liquid - μ = 18 mPa s, ρ = 1450 kg/m^3

Problem #3 (25 points)

The concrete slab for runways for large aircrafts, like the Boeing 747 and Airbus A380, are very thick (say 3 m thick). On a hot summer day, the slab attained a uniform temperature of 60°C. Hot runways and the friction between tires and the pavement on contact can cause tire blowout and the pilot to lose control. In anticipation of a plane landing, the airport supervisor instructed that jets of water at 7°C be sprayed continuously over the surface. The heat transfer coefficient between the concrete and the water is estimated to be 85 W/m^{2}K. Show your analysis and use the **integral** method to
a) estimate the surface temperature of the slab after 10 minutes of spraying.
b) How long will it take for the temperature of the surface to drop to 15°C?

Data: Properties of concrete:

k = 1.21 W/mK; Cp = 0.88 kJ/kg K; ρ = 2310 kg/m^3

Problem #4 (25 points)

Pure liquid ethanol is charged into a flat-bottomed, uniform-cross section tube to a depth of 4 cm. The overall tube length is 10 cm and both the tempeatures of the tube (and content) and the ambient are maintained at 25°C. Dry (ethanol-free) air is blown over the mouth of the tube, with care taken that there are no convection currents inside the tube.

a) Estimate the time for the liquid level to drop by 1 cm. Show your method.
b) After how long will all the liquid have evaporated?

Data: Ethanol

Temperature, °C	19	34.9	63.5	78.4
Vapor Pressure, mm Hg	40	100	400	760

Liquid density = 785.2 kg/m^3; Molar Mass = 46.07 kg/kmol; Diffusivity in air = 1.45 (10^{-5}) m^2/s; Ambient or Room Pressure = 640 mm Hg; and
the Universal Gas Constant = 0.08205 (m^3-atm)/(kmol-K)

Nominal Pipe Size (NPS) 4 to 9 inches

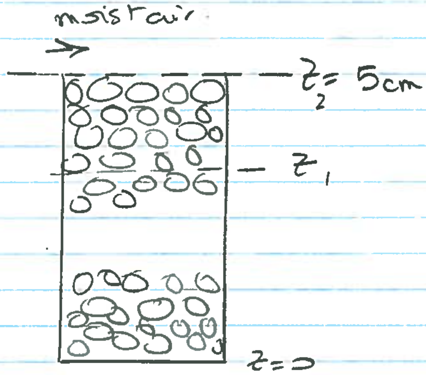
Wall thickness

NPS, in	DN mm	OD [in (mm)]	SCH 5	SCH 10s/10	SCH 40s/40 /STD	SCH 80s/80 /XS	SCH 120	SCH 160	XXS
4	100	4.500 (114.30)	0.083 (2.108)	0.120 (3.048)	0.237 (6.020)	0.337 (8.560)	0.437 (11.100)	0.531 (13.487)	0.674 (17.120)
4½	115	5.000 (127.00)	—	—	0.247 (6.274)	0.355 (9.017)	—	—	0.710 (18.034)
5	125	5.563 (141.30)	0.109 (2.769)	0.134 (3.404)	0.258 (6.553)	0.375 (9.525)	0.500 (12.700)	0.625 (15.875)	0.750 (19.050)
6	150	6.625 (168.27)	0.109 (2.769)	0.134 (3.404)	0.280 (7.112)	0.432 (10.973)	0.562 (14.275)	0.718 (18.237)	0.864 (21.946)
7	—	7.625 (193.68)	—	—	0.301 (7.645)	0.500 (12.700)	—	—	0.875 (22.225)
8	200	8.625 (219.08)	0.109 (2.769)	0.148 (3.759)	0.322 (8.179)	0.500 (12.700)	0.718 (18.237)	0.906 (23.012)	0.875 (22.225)
9	—	9.625 (244.48)	—	—	0.342 (8.687)	0.500 (12.700)	—	—	—

{For carbon, alloy and stainless steel pipes - STD = standard weight; XS or XH = extra strong/heavy; XXS = double extra strong; 10s, 40s or 80s means stainless steel}

ENCH 501 Final Examination Solution Dec. 13, 2010

#1



Let A = water vapor
B = air

The mole fraction of water vapor in the air outside the jar is

$$y_{A\delta} = \frac{3.5}{760} = 0.004605$$

Between z_1 and z_2, it is assumed that the desiccant is saturated with moisture. $\therefore$

at z_1, $C_{A_1} = 0.8$ moles H_2O/ litre dry air.

$$= \frac{0.8}{18} \text{ moles}/m^3 = 0.044 \text{ moles}/m^3$$

For the gas phase, $c = \frac{P}{RT} = \frac{1}{(0.08205)(273.15)}$

$= 4.4619(10^{-2})$ moles/litre or kmols/m^3

$$\therefore \quad y_{A1} = \frac{C_{A1}}{c} = \frac{0.044}{44.619} = 9.8613(10^{-4})$$

But the moisture can only diffuse in the gas through the pore spaces.

Given that $\frac{d(N_A S)}{dz} = 0$, and $S = \underset{\text{porosity}}{\varepsilon} A$

$$\therefore \quad \frac{d(\varepsilon N_A)}{dz} = 0 \quad \text{in} \quad z_1 \leq z < z_2 \qquad (1)$$

In the porous medium, air is not diffusing. Hence

$N_B = 0$ and

$$N_{Az} = -\frac{c \mathcal{D}_{AB}}{1 - y_A} \frac{dy_A}{dz}. \qquad (2)$$

Under pseudo-steady conditions, the rate at which moisture enters the porous medium equals the rate at which saturated desiccant grows. That is

$$\varepsilon N_A = \frac{dz_1}{dt}(1-\varepsilon)\frac{\rho_s}{M_A}(x_{sat}) \quad ; \quad x_{sat} = 0.17 \frac{g\,H_2O}{g\text{ desiccant}}$$

(3)

first we solve eq. (1)

$$\frac{d}{dz}\left(\frac{\varepsilon}{1-y_A}\frac{dy_A}{dz}\right) = 0$$

subject to

$z = z_1 \quad y_A = y_{A1}$

$z = z_2 \quad y_A = y_{A\alpha}$

(4)

Integrate

$$\frac{\varepsilon}{1-y_A}\frac{dy_A}{dz} = C_1$$

$$-d\ln(1-y_A) = \frac{C_1}{\varepsilon}dz$$

and again

$$-\ln(1-y_A) = \frac{C_1}{\varepsilon}z + C_2$$

use b.cs

$$-\ln(1-y_{A1}) = \frac{C_1}{\varepsilon}z_1 + C_2$$

$$-\ln(1-y_{A\alpha}) = \frac{C_1}{\varepsilon}z_2 + C_2$$

Subtract

$$\ln\left(\frac{1-y_{A\alpha}}{1-y_{A1}}\right) = \frac{C_1}{\varepsilon}(z_1-z_2)$$

$$\therefore \quad C_1 = \frac{\varepsilon}{z_1-z_2}\ln\left(\frac{1-y_{A\alpha}}{1-y_{A1}}\right)$$

and $C_2 = -\ln(1-y_{A\alpha}) - \frac{c_1}{\varepsilon} z_2$

$$\therefore \; -\ln(1-y_A) = \frac{c_1}{\varepsilon} z - \ln(1-y_{A\alpha}) - \frac{c_1}{\varepsilon} z_2$$

$$\ln\left(\frac{1-y_{A\alpha}}{1-y_A}\right) = \frac{c_1}{\varepsilon}(z-z_2) = \frac{\cancel{\varepsilon}}{z_1-z_2} \ln\left(\frac{1-y_{A\alpha}}{1-y_{A1}}\right)\frac{1}{\cancel{\varepsilon}}(z-z_2)$$

$$\ln\left(\frac{1-y_{A\alpha}}{1-y_A}\right) = \ln\left(\frac{1-y_{A\alpha}}{1-y_{A1}}\right)\frac{z-z_2}{z_1-z_2}$$

or $$\frac{1-y_{A\alpha}}{1-y_A} = \left(\frac{1-y_{A\alpha}}{1-y_{A1}}\right)^{\frac{z-z_2}{z_1-z_2}}$$

or $$\left(\frac{1-y_A}{1-y_{A\alpha}}\right) = \left(\frac{1-y_{A1}}{1-y_{A\alpha}}\right)^{\frac{z-z_2}{z_1-z_2}} \qquad (5)$$

This is the concentration distribution in the gas phase.

from (2) $$N_{Az} = -\frac{c\mathcal{D}_{AB}}{1-y_A}\frac{dy_A}{dz} = \frac{\cancel{\varepsilon}}{z_1-z_2}\ln\left(\frac{1-y_{A1}}{1-y_{A\alpha}}\right)\frac{\cancel{(1-y_A)}}{\cancel{\varepsilon}}\frac{c\mathcal{D}_{AB}}{\cancel{1-y_A}}$$

$$N_{Az} = \frac{c\mathcal{D}_{AB}}{z_1-z_2}\ln\left(\frac{1-y_{A1}}{1-y_{A\alpha}}\right) \qquad (6)$$

Now use eq. (3)

$$\varepsilon\left(\frac{c\,\mathcal{D}_{AB}}{z_1-z_2}\right)\ln\left(\frac{1-y_{A1}}{1-y_{A\alpha}}\right) = \frac{dz_1}{dt}\left[\frac{(1-\varepsilon)\rho_s\,(x_{sat})}{M_A}\right]$$

or $$(z_1-z_2)\frac{dz_1}{dt} = \lambda, \text{ a constant} \; ; \quad \text{i.c. } t=0,\; z_1 = z_2$$

$$\lambda = \frac{\varepsilon}{1-\varepsilon} \frac{M_A c \mathcal{D}_{AB}}{\rho_s (x_{sat})}$$

$$(z_1 - z_2) \frac{dz_1}{dt} = \frac{1}{2} \frac{d(z_1 - z_2)^2}{dt} = \lambda$$

Integrate $(z_1 - z_2)^2 = 2\lambda t + C_0$

use i.c. $C_0 = 0$

$$(z_1 - z_2)^2 = 2\lambda t$$

For time when $z_1 = 0$

$$z_2^2 = 2\lambda t$$

Substitute values

$$\lambda = \frac{0.38}{0.62} \frac{(18)(4.4619)(10^{-2})(2.2)(10^{-7})}{3179\ (0.17)}$$

$$\frac{\cancel{kg}\ water}{\cancel{kmol}} \frac{\cancel{kmol}s}{\cancel{m^3}} \frac{m^2}{s} \frac{\cancel{m^3}}{\cancel{kg}\ salt} \frac{\cancel{kg}\ salt}{kg\ \cancel{H_2O}} = m^2/s$$

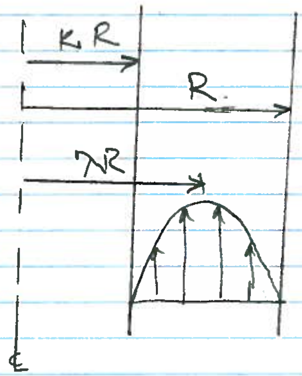
$$= 2.0039(10^{-10})\ m^2/s$$

$$z_2 = 0.05\ m$$

$$t = 6.238(10^6)\ s \quad \text{or} \quad 1732.8\ \text{hrs}$$

or 72.2 days

#2.



Given pipes - use table

4 in schedule 40s — OD = 11.43 cm

$\therefore \kappa_1 R = 5.715$ cm

8 in schedule 80 — OD = 21.908 cm

and the wall thickness = 1.27 cm

$\therefore R = \frac{21.908}{2} - 1.27 = 9.684$ cm.

$\therefore \kappa_1 = \frac{5.715}{9.684} = 0.5901$

The volume rate is

$539.12\ m^3/day = 6.2398(10^{-3})\ m^3/s$

Since $Q = \pi R^2(1-\kappa_1^2)\bar{u}$ - From Notes

$\bar{u} = 0.325\ m/s$ - using above data.

Calculate Re # $= \frac{D_H \bar{u} \rho}{\mu}$, $D_H = 4 r_H$

$D_H = \frac{4\pi R^2(1-\kappa_1^2)}{2\pi R(1+\kappa_1)}$ or $\frac{4(\text{cross sectional flow area})}{\text{Wetted Perimeter}}$

$= 2R(1-\kappa_1) = 0.0794\ m$

$\text{Re}\ \# = \frac{(0.0794)(0.325)(1450)}{18(10^{-3})} = 2{,}079$

Flow is just laminar.

from Notes:

$$\bar{u} = \frac{\chi R^2}{8\mu L}\left[\frac{1-\kappa_1^4}{1-\kappa_1^2} - \frac{1-\kappa_1^2}{\ln(1/\kappa_1)}\right] = \frac{\chi R^2}{8\mu L}(0.1126)$$

Substitute values

$$\Upsilon = P_0 - P_L - \rho g L \sin\beta = 8.8654\,(10^4)$$

Since $\beta = \pi/2$

$$P_0 - P_L = \Upsilon + \rho g L = 8.8654(10^4) + 2.8449(10^7)$$

Pa

$$= 28.538 \text{ MPa} \longrightarrow$$

It is obvious that the hydrostatic head is >> the head to overcome viscous resistance.

(b) from Notes: The maximum velocity is

$$U_{max} = \frac{\Upsilon R^2}{4\mu L}\left[1 - \left[\frac{1-\kappa^2}{2\ln 1/\kappa}\right]\left[1 - \ln\left[\frac{1-\kappa^2}{2\ln 1/\kappa}\right]\right]\right]$$

$$= \frac{\Upsilon R^2}{4\mu L}(0.084693) = \frac{2\bar{u}}{0.1126}(0.084693)$$

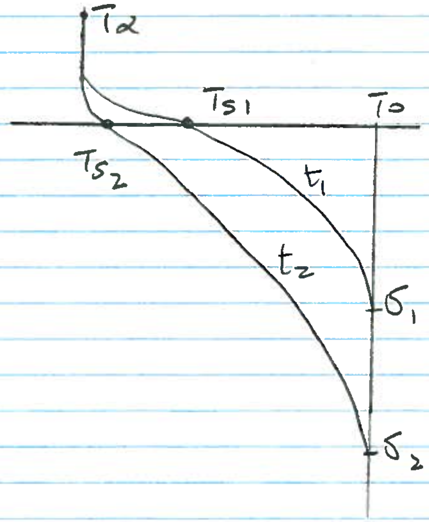
$$= 1.5043\,\bar{u} = 1.5043(0.325)$$

$$= 0.4889 \text{ m/s}$$

$\longrightarrow$

The maximum velocity is 1.5 times the average.

#3



This is a problem of heat transfer into a semi-infinite domain, with a convective boundary condition

$T_\alpha = 7°C$

$T_o = 60°C$

$h = 85\ W/m^2K.$

The temperature profile in the concrete is given by

$$\frac{T - T_o}{T_s - T_o} = \left(1 - \frac{x}{\delta}\right)^2$$

where $T_s(t)$ — from Notes.

The surface temperature may be derived from

$$\frac{T_s - T_o}{T_\alpha - T_o} = \frac{h\delta/2k}{1 + h\delta/2k}$$

where δ is evaluated from

$$6\alpha t = \frac{\delta^2}{2} + \frac{2k\delta}{h} - \frac{4k^2}{h^2}\ln\left(1 + \frac{h\delta}{2k}\right)$$

and

$$\alpha = \frac{k}{\rho c_p} = 5.958(10^{-7})\ m^2/s$$

$$\frac{h}{2k} = 35.124\ m^{-1}$$

(a)

at $t = 10$ mins $= 600$ s

$$2.145(10^{-3}) = 0.5\,\delta^2 + 2.847(10^{-2})\delta - 8.106(10^{-4})\ln(1 + 35.124\,\delta)$$

Solve : $\delta = 0.0542\ m$

$$\therefore \frac{T_s - T_0}{T_\alpha - T_0} = 0.6556$$

$$T_\alpha - T_0 = 7 - 60 = -53^\circ C$$

$$T_s = 0.6556(-53) + 60$$

$$= 25.25\ ^\circ C$$

(b) When $T_s = 15^\circ C$

$$\frac{T_s - T_0}{T_\alpha - T_0} = \frac{15-60}{7-60} = \frac{45}{53} = 0.8491$$

$$0.8491 = \frac{35.124\,\delta}{1 + 35.124\,\delta}$$

$$\therefore \delta = 0.1602\ m$$

Substitute into

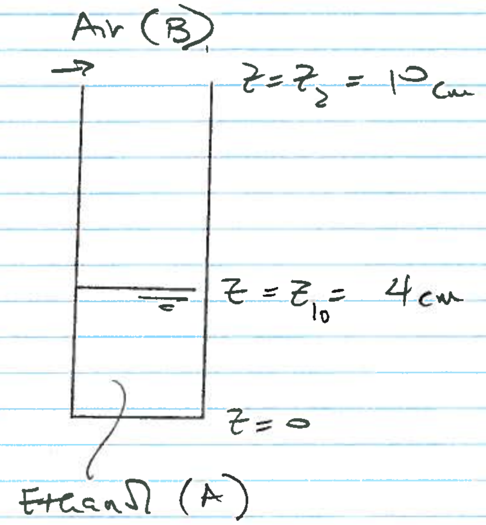
$$6\alpha t = \frac{\delta^2}{2} + \frac{2k\delta}{h} - \frac{4k^2}{h^2} \ln\left(1 + \frac{h\delta}{2k}\right)$$

$$6(5.958)(10^{-7})\, t = \frac{(0.1602)^2}{2} + \frac{(0.1602)}{35.124} - \frac{1}{(35.124)^2} \cdot \ln\left(1 + (0.1602)(35.124)\right)$$

$$t = 4.4366(10^3)\, s$$

or 1.2324 hrs

#4



Assumptions:

- Conc profile is rapidly developed in the gas layer
- $N_B = 0$ within the tube, since liquid is saturated with air.
- Initial level of liquid is at z_{10}, but the level changes with time, $z_1(t)$

The system temperature is 25°C.

Find the vapor pressure of ethanol using the Clausius – Clapeyron equation

$$\ln\left(\frac{P}{P_1}\right) = \frac{\Delta H_v}{R}\left(\frac{1}{T_1} - \frac{1}{T}\right) \quad ; \quad T \text{ in absolute scale}$$

Use data at 19°C and 34.9°C

$$\ln\left(\frac{P}{40}\right) = \frac{\Delta H_v}{R}\left[\frac{1}{292.15} - \frac{1}{298.15}\right] \quad ; \text{ P at } 25°C$$

$$\ln\left(\frac{100}{40}\right) = \frac{\Delta H_v}{R}\left[\frac{1}{292.15} - \frac{1}{308.05}\right]$$

$$\ln\left(\frac{P}{40}\right) = \ln(2.5)\left(\frac{6.8883(10^{-5})}{1.7667(10^{-4})}\right)$$

$$\ln P - \ln 40 = (0.91629)(0.38989) = 0.35725$$

The vapor pressure at 25°C,

$$P = 57.176 \text{ mm Hg}$$

For the problem

$$N_A = -cD_{AB}\frac{dx_A}{dz} + x_A(N_A+N_B) \quad ; \quad N_B = 0$$

$$\therefore \quad N_A = -\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dz}$$

For the system: Balance on A in gas phase at steady state gives

$$\frac{dN_{Az}}{dz} = 0 \quad \text{or} \quad \frac{d}{dz}\left(\frac{1}{1-x_A}\frac{dx_A}{dz}\right) = 0$$

subject to $\quad z = z_1 \quad x_A = \frac{P_P}{P_T}$

$$z = z_2 \quad x_A = 0$$

This problem has been solved in the Notes:

$$N_{Az} = \frac{cD_{AB}}{z_2 - z_1}\ln\left(\frac{x_{B2}}{x_{B1}}\right)$$

In pseudo-steady evaporation;

$$N_{Az}\Big|_{Liq} = -\frac{\rho_L}{M_A}\frac{dz_1}{dt}$$

$$\therefore \quad \frac{M_A c D_{AB}}{\rho_L}\ln\frac{x_{B_2}}{x_{B_1}} = \underset{const.}{\Gamma} = (z_1 - z_2)\frac{dz_1}{dt} \quad ; \quad t=0 \quad z_1 = z_{10}$$

Integrate

$$(z_1 - z_2)^2 - (z_{10} - z_2)^2 = 2\Gamma t$$

(a) $\quad z_2 = 0.1\,m \quad , \quad z_{10} = 0.04\,m \quad ; \quad P = 680\,mm = 0.8947\,atm$

$$c = \frac{P}{RT} = \frac{0.8947}{(0.08206)(298.15)} = 0.03657\ \frac{kmol}{m^3}$$

$$\Gamma = \frac{(0.03657)(46.07)(1.45)(10^{-5})}{785.2} \ln\left(\frac{1}{1 - \frac{57.176}{640}}\right)$$

$$= 2.9116(10^{-9})$$

Thus, when $z_1 = 0.03\,m$,

$$(0.03 - 0.1)^2 - (0.04 - 0.1)^2 = 2(2.9116)(10^{-9})\, t$$

$$t = 2.2325(10^5)\,s \quad \text{or} \quad 62.013 \text{ hrs}$$

(b) To completely evaporate the liquid,

$$z_1 = 0$$

$$\Gamma t = z_{10} z_2 - \frac{z_{10}^2}{2}$$

$$t = \frac{1}{\Gamma}\left((0.04)(0.1) - \frac{(0.04)^2}{2}\right)$$

$$= 1.0991(10^6)\,s \quad \text{or} \quad 305.29 \text{ hrs}$$