GJ

S7.

University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena

Final Examination, Fall 2009

Time: 8.00 - 11.00 am **Friday, December 11, 2009**

Instructions: **Attempt All Questions.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (30 points)

Commercial hot water extraction of bitumen from tar sands involves combining tar sands, water and alkali, and then heating and mixing the slurry until the oil floats to the surface and the sand sinks in the suspension. The process is carried out in large, slightly inclined cylinders that are rotated slowly. Before and after the industrial equipment were designed and constructed, some pilot studies were carried out in tanks.

In one tank, tar sands and dilute alkali solutions are stirred together in a cylindrical steel barrel that is 1.5 m tall and 1 m inside diameter until a paste, to a depth of 1.2 m in the container, has achieved a uniform consistency. Because the suspension at an initial temperature of 20°C is concentrated, it will be assumed that the components do not separate into the phases. That is, settling is hindered and convection currents are absent in the barrel. At time t = 0, the temperature at the bottom of the tank was quickly raised to and maintained at 200°C by a heater. Assume that the side of the barrel is insulated.

It is suggested that when any part of the slurry has been heated to a minimum temperature of 50°C and maintained at or above this temperature for 1 hour, it would be easy to separate the bitumen droplets from the sand and clay particles to which they are stuck.

Use the **integral method** to obtain results for the following:

a) After the heater has been on for a while, it is turned off and the slurry is stirred thoroughly (over a short time period) until the slurry temperature is uniform. Then the bottom and top surfaces of the barrel are immediately covered by insulation. What is the minimum time required (from t = 0) for the operation to effectively separate the bitumen from the solid particles in the barrel?

b) Before stirring the suspension, what fraction of the slurry would have attained or exceeded a temperature of 50°C?

Data: Properties of tar sands slurry - k = 0.286 W/m K ; C_p = 2.386 kJ/kg K ; ρ = 1,264 kg/m^3

Problem #2 (30 points)

Anhydrous ammonia is used as a fertilizer and, when it is mixed with water or other substances, ii finds application in many cleaning products. During a routine delivery of ammonia by a supplier, a farmer discovered that his regular storage tank for anhydrous ammonia leaks. He, therefore, had the ammonia liquid poured into an empty, vertical, cylindrical tank he has in a cold shed maintained at 220K. He reasons that since ammonia boils at -33.4°C, and this is warmer than the temperature in the shed, he would not lose much of the ammonia through evaporation. To his astonishment, the level of the ammonia liquid dropped substantially the next time he checked. He needs an explanation.

The farmer relates to you that the tank diameter is 1.2 m and it is 1.8 m tall. The ammonia was poured to a level of 0.3 m below the top rim of the tank and the top was covered by a wire gauze. There was air flow over the top of the tank and the air is vented so that there is no ammonia smell in the room. The ambient pressure is 0.96 atm.

a) After how long would the liquid level have dropped by 0.5 m below the initial level? Assume there are no convection currents in the tank. State all your assumptions and show your derivations.

b) If the initial liquid level had been 0.8 m below the rim of the tank, how much ammonia would the farmer have lost in the time elapsed for part (a)?

Data:
Molar mass of ammonia = 17.032 g/mol ; density of liquid ammonia at 220K = 705.7 kg/m^3 ; diffusivity of ammonia vapor in air = 1.45 (10^{-5}) m^2/s ; vapor pressure of ammonia at 210 K = 0.176 atm, and at 230K = 0.598 atm. The universal gas constant is 0.08205 (m^3 atm)/(kmol K).

Problem #3 (20 points)

Condensers are essential components of distillation columns and many appliances such as refrigerators and air conditioners. These usually involve pipes within which a vapor condenses and releases heat to another medium such as cooling water or air. A scale-up of a pilot unit requires that a dimensional analysis be conducted.

The following variables have been identified as important - the average heat transfer coefficient (**h**, W/m^2 K), the temperature difference between the vapor and the pipe wall (**ΔT**, K), the length of the pipe (**L**, m), the latent heat of vaporization (**ΔH_v**, kJ/kg), the density of the liquid (ρ_L, kg/m^3), the viscosity of the liquid (μ_L, mPa s), the thermal conductivity of the condensate (**k**, W/mK), and the acceleration of gravity (**g**, m/s^2).

Determine the dimensionless groups. Show your steps.

Problem #4 (20 points)

Some crude oils contain substantial fractions of wax, paraffins hydrocarbons in the C18 to C36 range. At certain temperatures, the wax starts to precipitate and settle out of the liquid. The solids accumulate on the wall and ultimately plug the pipeline. This is of concern for an operation.

A 16-in i.d. pipeline transports 6,500 barrels/day of a waxy crude. The pipe is buried to a depth of 2 m. The crude enters the pipeline at a temperature of 50°C. The wax appearance temperature is 9°C. The soil surface temperature is given as -10°C and the effective thermal conductivity of the soil is 1.3 W/mK.

Estimate the distance from the inlet at which solid wax particles are expected to start to appear in suspension. Show your derivations.

Data: Properties of the crude oil - ρ = 845 kg/m^3 ; C_p = 2.45 kJ/kg K ; 1 barrel of oil = 42 US gallons ; 1 US gallon = 3.78 litres.

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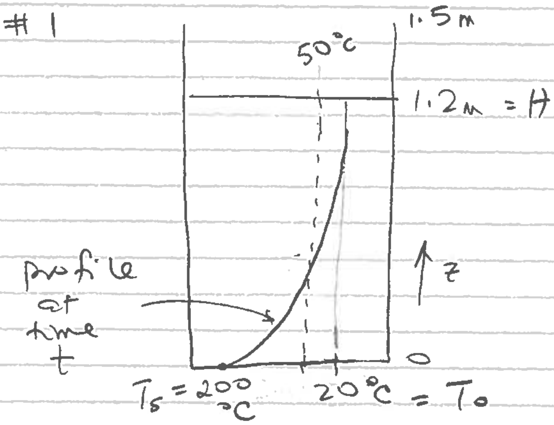
#1



This is heat conduction into a stationary, semi-infinite medium – with constant surface temperature.

The minimum processing time is given by heating the medium until the average temperature is 50 °C. This average temperature is obtained at the time of stirring. Then, 1 hour is allowed for the suspension particles to disengage.

The total amount of energy that must be supplied, $Q = mC_p(50-20)$

where $m = \rho V = \rho(\pi R^2 H)$; $R = 0.5$ m, $H = 1.2$ m

$$Q = (1264)(\pi(0.5)^2 1.2) 2386(30) \text{ J}$$
$$= 8.5273(10^7) \text{ J}$$

This quantity is obtained by adding heat into the medium. The total amount of heat added can be determined in one of two ways –

Integrate the rate of input over time OR

Integrate the temperature profile in the domain over space at the desired time t.

i.e. $Q = \int_0^t q\big|_{z=0} A\, dt$ or $\int_0^H (T-20)\, C_p\, \rho\, A\, dz$ at time t

The integral over space requires trial-and-error.

The set-up for the problem is in the Notes, p 102-6.

The integral energy equation, assuming a semi-infinite domain, is

$$-k\frac{dT}{dz}\bigg|_{z=0} = \frac{d}{dt}\left[\int_0^\delta \rho C_p T\, dz\right] - \rho C_p T_0 \frac{d\delta}{dt}$$

The conditions are

$z=0 \quad T = T_s$

$z=\delta(t) \quad T = T_0$

$z=\delta(t) \quad \frac{dT}{dz}=0$

Assume $\quad T = a + bz + cz^2$

Apply conditions to obtain the temperature profile as

$$\frac{T-T_0}{T_s-T_0} = \left(1-\frac{z}{\delta}\right)^2$$

Substitute this profile into the integral equation and simplify to get

$$\delta\, d\delta = 6\alpha\, dt \quad ; \text{ with } t=0 \quad \delta=0$$

or $\quad \delta = \sqrt{12\alpha t}$

Since $\quad q\big|_{z=0} = -k\frac{dT}{dz}\bigg|_{z=0} = -k\left[-\frac{2}{\delta}+\frac{2z}{\delta^2}\right]_{z=0}(T_s-T_0)$

$$= +\frac{2k(T_s-T_0)}{\delta}$$

$$A = \pi R^2 = \pi (0.5)^2$$

$$Q = \frac{2k(T_s - T_0)(\pi)(0.5)^2}{\sqrt{12\alpha}} \int_0^t \frac{dt}{t^{\frac{1}{2}}} \quad ; \quad \alpha = \frac{k}{\rho c_p}$$

$$= \frac{2(0.286)(200-20)(\pi)(0.5)^2}{\sqrt{\frac{12(0.286)}{(1264)(2386)}}} \left(2t^{\frac{1}{2}}\right)$$

$$= 1.5161(10^5)\, t^{\frac{1}{2}} = 8.5273(10^7)\ J$$

$$t = 3.1635(10^5)s \quad \text{or} \quad 87.876 \text{ hrs}$$

<u>Check</u>

$$\alpha = \frac{k}{\rho c_p} = \frac{0.286}{(1264)(2386)} = 9.483(10^{-8})\ m^2/s$$

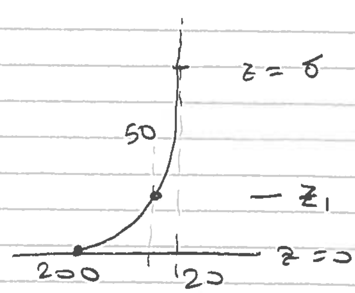
$$\delta = \sqrt{12\alpha t} = 0.6\,m \quad (< 1.2m)$$

$\therefore$ Penetration thickness is less than depth of slurry – the assumption of a semi-infinite domain is justified.

(a) The time required to process the slurry is

$$(87.876 + 1) = 88.876 \text{ hrs}$$

(b) Before stirring, the temp profile is a per sketch where $\delta = 0.6\,m$.



From the temperature profile

$$\frac{T - T_o}{T_s - T_o} = \left(1 - \frac{z}{\delta}\right)^2$$

$T = 50°C$
$T_o = 20°C$
$T_s = 200°C$

$\Bigg\}$ estimate z

$$\frac{50 - 20}{200 - 20} = \left(1 - \frac{z}{0.6}\right)^2 = \frac{30}{180} = \frac{1}{6}$$

$$z = 0.355 \text{ m} \longrightarrow$$

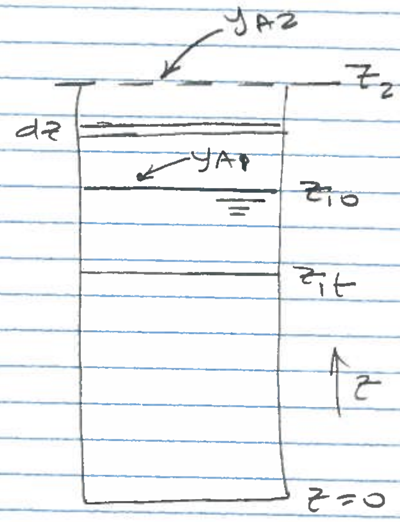
From $z = 0$ to $z = 0.355$ m will be between 200 and 50°C before stirring.

This is $\frac{0.355}{1.2}$ or 0.2959 of the total height.

That is almost 30% of the volume will be at $\geq 50°C$.

$\longrightarrow$

#2



This is a problem of diffusion through a stagnant film.

Assumptions are:

- Liquid ammonia is ~~saturated~~ with air, ∴ zero air flux in gas.
- isothermal system
- ammonia conc at $z_2 = 0$, gas heavily diluted

The pseudo-steady case, with z_1 unchanged, is in the Notes, p. 171-3.

A balance on a differential element dz, yeilds that

$$\frac{dN_A}{dz} = 0 \qquad (1)$$

By definition, the flux is

$$N_A = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$$

where $A \equiv$ ammonia & $B \equiv$ air

With $N_{Bz} = 0$ in gas space

$$N_A = -\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dz} \qquad (2)$$

Substitute (2) into (1) and simplify

$$\frac{d}{dz}\left(\frac{1}{1-y_A}\frac{dy_A}{dz}\right) = 0 \qquad (3)$$

with the b.c.

$z = z_1 \quad y_A = y_{A1i} \quad \left(= P_{vp}/P\right)$ by Raoult's Law

$z = z_2 \quad y_A = y_{A2} \quad (= 0)$

Solve to get the concentration profile as:

$$\frac{1-y_A}{1-y_{A1}} = \left(\frac{1-y_{A2}}{1-y_{A1}}\right)^{\frac{z-z_1}{z_2-z_1}}$$

eq. 6.112 Notes

The flux of A, and hence the evaporation rate, is given by

$$N_A\Big|_{z_1} = \left(-\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dz}\right)_{z_1} = \frac{cD_{AB}}{z_2-z_1}\ln\frac{y_{B2}}{y_{B1}}$$

Eq. 6.114 Notes.

Now to relate to the rate of loss of liquid,

$$\frac{cD_{AB}}{z_2-z_1}\ln\frac{y_{B2}}{y_{B1}} = -\frac{\rho_L}{M_A}\frac{dz_1}{dt}$$

Eq. 6.116 Notes.

Re-arrange

$$(z_1-z_2)\frac{dz_1}{dt} = \Gamma = \frac{M_A c D_{AB}}{\rho_L}\ln\frac{(1-y_{A2})}{(1-y_{A1})} = \text{const}$$

Subject to the condition:

$$t=0 \qquad z_1 = z_{10}$$

Solve

$$(z_1-z_2)^2 - (z_{10}-z_2)^2 = 2\Gamma t$$

$\longrightarrow$

To evaluate Γ, find

$$c = \frac{P}{RT} \quad ; \quad P = 0.96\ \text{atm}, \quad T = 220\ \text{K} \quad ; \quad R = 0.08205\ \frac{\text{m}^3\ \text{atm}}{\text{kmol K}}$$

$$= \frac{0.96}{(0.08205)(220)}\ \frac{\text{kmols}}{\text{m}^3} = 5.3183(10^{-2})$$

Need $y_{A1} = \dfrac{P_{vp} \text{ at } 220\text{ K}}{P}$, given

T, K	P_{vp}, atm
210	0.176
230	0.598

Use Clausius - Clapeyron eq.

$$\ln\left(P/P_1\right) = \frac{\Delta H_v}{R}\left(\frac{1}{T_1} - \frac{1}{T}\right)$$

$$\frac{\ln\left(\dfrac{P}{0.598}\right) = \dfrac{\cancel{\Delta H_v}}{\cancel{R}}\left(\dfrac{1}{230} - \dfrac{1}{220}\right)}{\ln\left(\dfrac{0.176}{0.598}\right) = \dfrac{\cancel{\Delta H_v}}{\cancel{R}}\left(\dfrac{1}{230} - \dfrac{1}{210}\right)}$$

$$\ln\left(\frac{P}{0.598}\right) = \frac{-1.9763(10^{-4})}{-4.1408(10^{-4})}(-1.2231)$$

Vapor pressure of Ammonia at 220 K $\quad P_{vp} = 0.598(0.5578) = 0.33356$ atm.

(0.33467 atm - expt.)

$$\therefore \quad y_{A1} = \frac{0.33356}{0.96} = 0.34746$$

$$\therefore \quad \Gamma = \frac{17.032\,(5.3183)(10^{-2})(1.45)(10^{-5})}{705.7}\ln\left(\frac{1}{1-0.34746}\right)$$

$$\frac{\cancel{kg}}{\cancel{kmol}}\;\frac{\cancel{kmol}}{\cancel{m^3}}\;\frac{m^2}{s}\;\frac{\cancel{m^3}}{\cancel{kg}}$$

$$\Gamma = 7.945\,(10^{-9})\ m^2/s$$

(a) $z_2 = 1.8\,m$; $z_{10} = 1.5\,m$; $z_1(t) = 1.0\,m$

$$\therefore \quad (1-1.8)^2 - (1.5-1.8)^2 = 2\Gamma t$$

$$\therefore \quad t = 3.4612767\,(10^7)\ s$$

or 400.61 days. $\longrightarrow$

(b) $z_2 = 1.8\,m$; $z_{10} = 1.0\,m$; z_1 ?

$$(z_1 - 1.8)^2 - (1.0 - 1.8)^2 = 2\Gamma t = 0.55$$

$$z_1 = 1.8 \pm 1.09087 \quad (\text{use } -ve)$$
$$= 0.70913\ m$$

$\therefore$ Volume of liquid lost

$$= \frac{\pi D^2}{4}(z_{10} - z_1)\ m^3$$

$$= \frac{\pi (1.2)^2}{4}(1.0 - 0.70913) = 0.329\ m^3$$

The mass lost = vol $\times$ density

$$= (0.329)(705.7)$$

$$= 232.15\ kg$$

3

Let $h = f_1(\Delta T, L, \Delta H_v, \rho_L, \mu_L, k, g)$

	h	ΔT	L	ΔH_v	ρ_L	μ_L	k	g
units	$W/m^2 K$	K	m	$\frac{kJ}{kg}$	$\frac{kg}{m^3}$	$Pa.s$	$\frac{W}{mK}$	$\frac{m}{s^2}$
	$\frac{J}{s\, m^2 K}$			$\frac{kg \cdot m \cdot m}{kg\, s^2}$		$\frac{N}{m^2} s$		
	$\frac{kg}{s} \cdot \frac{\cancel{m}}{s^2} \frac{\cancel{m}}{\cancel{m^2} K}$					$\frac{kg}{m^2} \frac{m\, s}{s^2}$		
Dimensions	$\frac{M}{t^3 T}$	T	L	$\frac{L^2}{t^2}$	$\frac{M}{L^3}$	$\frac{M}{Lt}$	$\frac{ML}{t^3 T}$	$\frac{L}{t^2}$

There are 4 dimensions — M, L, t and T

With 8 variables, there are 4 dimensionless gps.

By inspection

$$\Pi_1 = \frac{hL}{k} \quad \text{and} \quad \Pi_2 = \frac{\Delta H_v}{gL}$$

$\therefore$ Remove 2 variables — say ΔH_v and k

$$\therefore \quad h = f_2(\Delta T, L, \rho_L, \mu_L, g)$$

Let $\Pi_3 = \Delta T^a L^b \rho_L^c g^d \mu_L$

and $\Pi_4 = \Delta T^{a'} L^{b'} \rho_L^{c'} g^{d'} h$

For Π_3

$$M^0 L^0 t^0 T^0 = T^a L^b \left(\frac{M}{L^3}\right)^c \left(\frac{L}{t^2}\right)^d \frac{M}{Lt}$$

mass	0	=	$c + 1$
length	0	=	$b - 3c + d - 1$
time	0	=	$-2d - 1$
temp	0	=	a

$\Rightarrow$ $a = 0$, $b = -3/2$, $c = -1$, $d = -1/2$

$$\therefore \quad \Pi_3 = \frac{\mu_L}{(L^3 \rho_L^2 g)^{1/2}}$$

for Π_4

$$M^0 L^0 t^0 T^0 = T^{a'} L^{b'} \left(\frac{M}{L^3}\right)^{c'} \left(\frac{L}{t^2}\right)^{d'} \frac{M}{t^3 T}$$

mass	0	=	$c' + 1$
length	0	=	$b' - 3c' + d'$
time	0	=	$-2d' - 3$
temp	0	=	$a' - 1$

$\Rightarrow$ $a' = 1$, $b' = -3/2$, $c' = -1$, $d' = -3/2$

$$\therefore \quad \Pi_4 = \frac{(\Delta T) h}{(L^3 \rho_L^2 g^3)^{1/2}}$$

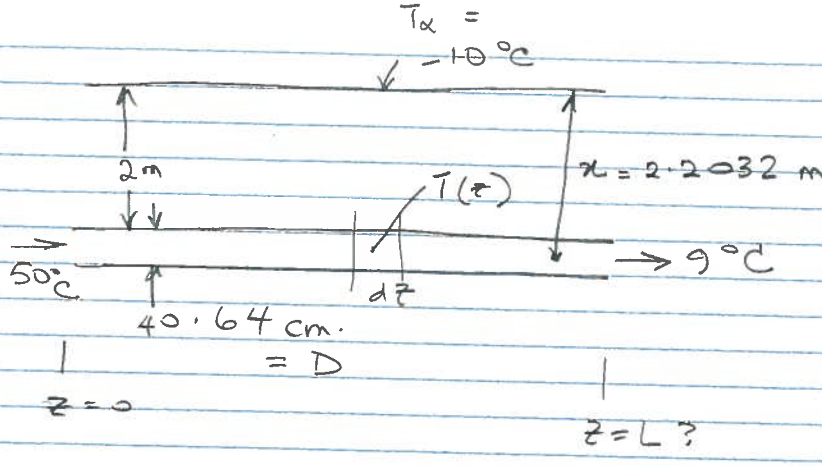
Hence

$$\frac{h(\Delta T)}{(L^3 \rho_L^2 g^3)^{1/2}} = f_3 \left\{ \frac{hL}{k}, \frac{\Delta H_v}{gL}, \frac{\mu_L}{(L^3 \rho_L^2 g)^{1/2}} \right\}$$

$\longrightarrow$

Other dimensionless groups may be formed by combinations of the above. Hence there is a variety of possible correct answers.

#4



The flow rate = 6,500 barrels / day

$$= \frac{6,500\,(42)(3.7854)}{1000\,(24)(3600)} \quad m^3/s$$

$$= 0.01196 \ m^3/s$$

For the differential element, dz, heat loss to the top of the soil is given by

$$dQ = k_{soil}(dS)(T - T_\alpha)$$

From the shape factor table

$$dS = \frac{2\pi\,(dz)}{\cosh^{-1}\left(\frac{2x}{D}\right)}$$

But the heat loss is also given by

$$dQ = -\dot{m}\,C_p\,dT$$

$$\therefore \quad -\dot{m}\,C_p\,dT = k\beta\,(T - T_\alpha)\,dz, \quad \text{where}$$

$$\beta = \frac{2\pi}{\cosh^{-1}\left(\frac{2x}{D}\right)}$$

Re-arrange and integrate

$$-\int_{50}^{9} \frac{dT}{T - T_\alpha} = \frac{k\beta}{\dot{m}\,C_p}\int_0^L dz$$

$$\ln\left(\frac{50-(-10)}{9-(-10)}\right) = \frac{(1\cdot3)\beta}{(0\cdot01196)(845)(2450)} \cdot L$$

Using the identity

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\cosh^{-1}\left(\frac{2(2\cdot2032)}{0\cdot4064}\right) = \cosh^{-1}(10.8425)$$

$$= \ln(21.6388) = 3\cdot07449$$

$$\therefore \beta = \frac{2\pi}{3\cdot07449} = 2\cdot04365$$

$$\therefore L = \frac{1.1499(0\cdot01196)(845)(2450)}{(1\cdot3)(2\cdot04365)} \text{ m}$$

$$= 10{,}717\cdot556 \text{ m}$$

or approximately 10.7 km