University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena

Final Examination, Fall 2008

GJ
JOH

Time: Noon - 3.00 pm **Thursday, December 11, 2008**

Instructions: **Attempt All Questions.**
Use of Electronic Calculators allowed.
Open Notes, Open Book Examination.

Problem #1 (25 points)

Large public spaces can both be kept humidified and odor free by water films as they flow over surfaces in the room. A small fraction of the water evaporates at the exposed liquid-gas interface and impurities at low concentrations in the air are absorbed into the water and removed.

At an airport, a pool of water is located in the hall. Sticking out of the pool are vertical tubes, 2 cm i.d. and 2.6 cm o.d., which are open at the top. Water is pumped, from the bottom, through the inside of each of the vertical tubes at a rate of 0.064 m^3/hr. The water, after it emerges from the top of each tube, forms a film on the outside of the tube and the water runs down towards the pool.

a) Obtain an expression for the velocity distribution in the film as a function of radial position.
b) How thick is the film layer? What is the Reynolds number for the film flow?

Data: Properties of water - $\rho = 999$ kg/m^3 ; $\mu = 1.3$ mPa s

Problem #2 (30 points)

The walls of nuclear power plants are usually thick, over 1 m. In case of accidents, the wall contains radioactive material. Because it is potentially dangerous to inspect the inside of the confines in the case of a fire, and video equipment would be quickly damaged, thermocouples are embedded in the wall at different but known distances from the surface of the inside wall.

A pump malfunctioned and started a fire inside the containment room of a nuclear power plant. The initial temperature of the wall was uniformly 20°C. The heat flux from the fire on the inside surface is given as:

$$q_w = q_o (1 - e^{-\beta t}) \text{ ; t is in minutes and } \beta = 0.2 \text{ min}^{-1}$$

The flames were such that the temperature at a distance of 1 cm from the inside wall was measured to have increased by 15°C in the first 32 minutes after the fire started. Use the **integral method** to determine

a) the heat flux from the flame after a long time, i.e. q_o.
b) Estimate the temperature on the inside wall of the containment room at t equals 32 minutes. How far has heat penetrated into the wall at this instant?

Data:
Properties of cement - $k = 1.37$ W/mK ; $\rho = 1900$ kg/m^3 ; $c_p = 0.88$ kJ/kg K

Problem #3 (20 points)

A centrifugal pump has an impeller diameter (D) equal 35 cm. When pumping water at 20°C and the impeller is rotating at (Ω) 1160 rpm, the volume flow rate (Q) and the pressure rise from pump inlet to outlet (Δp) are related as follows:

Q, litres/min	757	1135.5	1514	1892.5	2271	2649.5
Δp, atm	2.467	2.399	2.330	2.193	1.988	1.576

a) Obtain dimensionless quantities from $\Delta p = F(\rho, D, Q, \Omega)$
b) If the same pump running at 900 rpm is used to displace kerosine at 20°C at a rate of 1514 litres/min, what would the pressure rise be?

Data:

	ρ, kg/m³	μ, mPa s
Water	998	1
Kerosine	804	1.92

Problem #4 (25 points)

Tailings ponds from tar sands extraction operations have been in the news of late. Issues have included birds landing on them and dying as a result and liquids leaking out of the ponds into aquifers. The tailing ponds also contain volatile organic compounds that are released and carried downwind in the air that flows over the surface. This is the problem of current interest.

A square pond (50 X 50 m) is built with a berm (or mound) around it. The top of the berm is flat and 1 m wide all around. The liquid fills up the pond to the level of the berm top. A slight breeze at 0.36 km/hr flows normal to one edge of the pond and the surface of the liquid remains flat, i.e. there are no waves.

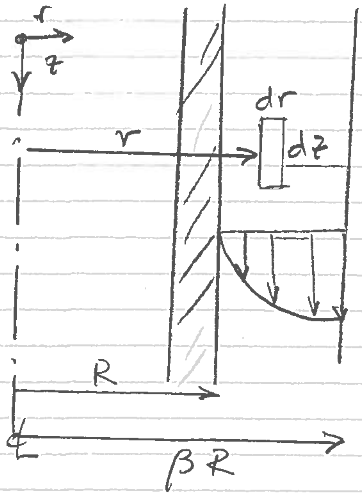
If the concentration of the volatile organic (y_{AO}) is substantial and constant at the gas side of the liquid-air interface, derive an expression for the ***rate*** at which the volatile organic is removed from the pond. Assume the liquid in the pond is already saturated with air and there is no volatile organic substance in the ambient air.

Show all important steps and justifications.

Data: Properties of air: density = 1.4128 kg/m³; viscosity = 1.599 (10^{-5}) Pa s

ENCH 501 Final Exam Solution Dec. 11, 2008

#1



Force balance on differential ring dr by dz.

$$-\left(2\pi r\, dz\, \tau_{rz}\right)_r + \left(2\pi r\, dz\, \tau_{rz}\right)_{r+dr} + 2\pi r\, dr\, dz\, \rho g = 0$$

or

$$\frac{d(r\tau_{rz})}{dr} + \rho g r = 0$$

Integrate once

$$r\tau_{rz} = -\frac{\rho g r^2}{2} + C_1$$

or $$\tau_{rz} = -\rho g \frac{r}{2} + \frac{C_1}{r}$$

Use b.c. at $r = \beta R$, $\tau_{rz} = 0$ (free surface)

$$\Rightarrow \quad C_1 = \frac{\rho g \beta^2 R^2}{2}$$

With $\tau_{rz} = \mu \frac{du}{dr}$ (a +ve quantity, since direction is taken care of in force balance)

$$\frac{du}{dr} = \frac{\rho g}{2\mu}\left(\frac{\beta^2 R^2}{r} - r\right)$$

Integrate

$$u = \frac{\rho g}{2\mu}\left(\beta^2 R^2 \ln r - \frac{r^2}{2}\right) + C_2$$

with b.c. $r = R$, $u = 0$ (no slip)

$$C_2 = -\frac{\rho g}{2\mu}\left(\beta^2 R^2 \ln R - \frac{R^2}{2}\right)$$

$$\therefore \quad u = \frac{\rho g}{2\mu}\left(\beta^2 R^2 \ln\frac{r}{R} - \frac{r^2}{2} + \frac{R^2}{2}\right)$$

(a) $$u = \frac{\rho g R^2}{4\mu}\left[1 - \frac{r^2}{R^2} + 2\beta^2 \ln\frac{r}{R}\right]$$

The volume flow rate, Q, is given by

$$Q = 2\pi\int_R^{\beta R} u\,r\,dr = \frac{2\pi\rho g R^2}{4\mu}\int_R^{\beta R}\left(r - \frac{r^3}{R^2} + 2\beta^2 r \ln\frac{r}{R}\right)dr$$

$$Q = \frac{\pi\rho g R^4}{2\mu}\int_1^{\beta}\left(\eta - \eta^3 + 2\beta^2\eta\ln\eta\right)d\eta \quad ; \quad \eta = r/R$$

$$= \frac{\pi\rho g R^4}{2\mu}\left\{\frac{\eta^2}{2} - \frac{\eta^4}{4} + 2\beta^2\left(\frac{\eta^2}{2}\left(\ln\eta - \frac{1}{2}\right)\right)\right\}_1^{\beta}$$

$$= \frac{\pi\rho g R^4}{8\mu}\left\{(4\beta^2 - 1) - \beta^4(3 - 4\ln\beta)\right\}$$

$$\frac{\pi\rho g R^4}{8\mu} = \frac{\pi(999)(9.81)(0.013)^4}{8(1.3)(10^{-3})} = 8.4552(10^{-2})$$

$$Q = 0.064\ \text{m}^3/\text{hr} \quad \text{or} \quad 1.7778(10^{-5})\ \text{m}^3/\text{s}$$

$$\frac{1.7778(10^{-5})}{8.4552(10^{-2})} = 2.1026(10^{-4}) = (4\beta^2-1) - \beta^4(3-4\ln\beta)$$

Solve by excel.

$$\beta = 1.033655$$

(b) The film thickness $= R(\beta-1) = 0.04375$ cm

$$\text{Reynolds \#} = \frac{D_H \bar{u} \rho}{\mu} \quad ; \quad D_H = \frac{4(\text{x-section})}{\text{wetted perimeter}}$$

$$D_H = \frac{4\pi R^2(\beta^2-1)}{2\pi R} = 2R(\beta^2-1)$$

$$\bar{u} = \frac{\int_R^{\beta R} u r\,dr}{\int_R^{\beta R} r\,dr} = \frac{Q}{A} = \frac{Q}{\pi R^2(\beta^2-1)}$$

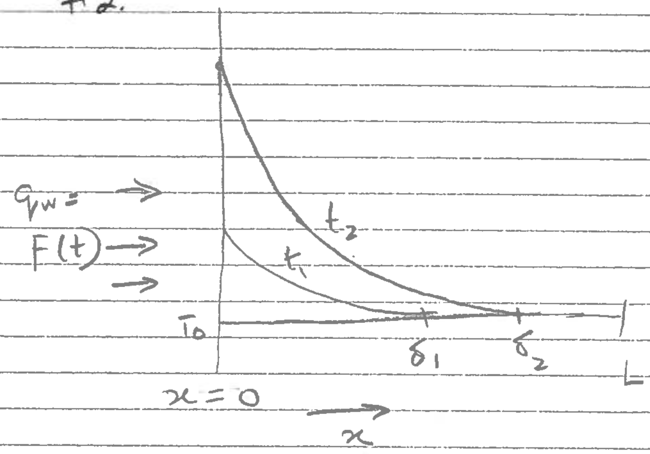
$$\frac{D_H u \rho}{\mu} = 2\cancel{R}\cancel{(\beta^2-1)} \frac{Q}{\pi R^{\cancel{2}}\cancel{(\beta^2-1)}} \frac{\rho}{\mu}$$

$$= \frac{2Q\rho}{\pi R \mu} = \frac{2(1.7778)(10^{-5})(999)}{\pi(0.013)(1.3)(10^{-3})}$$

$$= 669.02 \longrightarrow$$

Flow is expected to be laminar but wavy.

#2.



This is a problem of prescribed heat flux at the wall of a semi-infinite domain. This problem has been worked out in the class Notes. (p 107)

The energy balance equation is (Eq. 5.46)

$$q_{x=0} = \frac{d}{dt}\left[\int_0^L \rho C_p (T - T_0)\,dx\right]$$

Subject to the conditions

$$x = 0 \qquad \frac{dT}{dx} = -\frac{F(t)}{k}$$

$$x = \delta \qquad T = T_0$$

$$x = \delta \qquad \frac{dT}{dx} = 0$$

Assume a profile : $T = a + bx + cx^2$

apply the conditions to obtain (Eq. 5.55)

$$T - T_0 = \frac{F(t)\,\delta}{2k}\left(1 - \frac{x}{\delta}\right)^2$$

Substituting into the energy balance equation and integrating yields

$$\frac{\alpha F(t)}{k} = \frac{d}{dt}\left[\frac{F(t)\,\delta^2}{6k}\right]$$

The latter equation is solved for $\delta(t)$ subject to the initial condition that at $t=0$, $\delta=0$.

This gives (Eq. 5.57)

$$\delta(t) = \sqrt{6\alpha}\left[\frac{1}{F(t)}\int_0^t F(t)\,dt\right]^{1/2}$$

For this problem,

$$F(t) = q_w = q_0\left(1-e^{-\beta t}\right)$$

$$\therefore \quad \delta(t) = \sqrt{6\alpha}\left[\frac{1}{(1-e^{-\beta t})}\int_0^t \left(1-e^{-\beta t}\right)dt\right]^{1/2}$$

$$= \frac{\sqrt{6\alpha}}{(1-e^{-\beta t})}\left\{\left[t+\frac{e^{-\beta t}}{\beta}\right]_0^t\right\}^{1/2}$$

$$= \frac{\sqrt{6\alpha}}{(1-e^{-\beta t})}\left[t+\frac{e^{-\beta t}}{\beta}-\frac{1}{\beta}\right]^{1/2}$$

For this problem

$$t = 32 \text{ mins} \quad \text{or} \quad 1920 \text{ s}$$

$$\alpha = \frac{k}{\rho c_p} = \frac{1.37}{1900(880)} = 8.1938(10^{-7})\ m^2/s$$

$$\beta = 0.2 \ \text{min}^{-1}$$

$$\delta = \frac{0.2173(10^{-3})}{0.99834}\left\{32 + 5\left(1.6616(10^{-3})-1\right)\right\}^{1/2} 60^{1/2}$$

$$= 8.94(10^{-2})\ m \quad \text{or} \quad 8.94 \text{ cm}$$

(a) Now given that $T - T_o = 15°C$ at $x = 1cm$

Substitute into temperature profile

$$15 = \frac{q_o\left(1 - e^{-\beta t}\right)\delta}{2k}\left(1 - \frac{x}{\delta}\right)^2$$

$$= \frac{q_o(0.99834)(8.94)(10^{-2})}{2(1.37)}\left(1 - \frac{1}{8.94}\right)^2$$

$\therefore$ The heat flux at long time

$q_o = 583.79 \; W/m^2 \quad \left(\frac{J}{m^2 s}\right)$

$\longrightarrow$

(b) The temperature of the inside wall is at

$x = 0 \quad (t = 32 \text{ mins})$

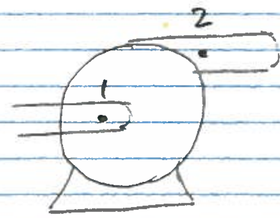
$$\therefore T = 20 + \frac{q_o\left(1 - e^{-\beta t}\right)\delta}{2k}$$

$$= 20 + 19.02$$

$$= 39 \; °C$$

$\longrightarrow$

#3



Variables	Units	Dimensions
ΔP	Pa (N/m^2)	$\frac{M}{Lt^2}$
ρ	kg/m^3	M/L^3
D	m	L
Q	m^3/s	L^3/t
Ω	s^{-1}	$1/t$
Not required [μ	Pa.s	M/Lt]

(a) By inspection, one can deduce the two dimensionless groups

$$\Pi_1 = \frac{\Delta P}{\rho D^2 \Omega^2} \quad ; \quad \Pi_2 = \frac{Q}{D^3 \Omega}$$

The alternative is to use the Buckingham Pi's theorem. (Students to do this.) →

From the data, calculate + plot Π_1 and Π_2

$D = 0.3\,m$, $\Omega = 19.33\,s^{-1}$, $\rho = 998\,kg/m^3$,

For ΔP, 1 atm ≡ 101325 Pa + for Q, 1 litre ≡ $10^{-3}\,m^3$

ΔP atm	Π_1	Q litres/min	Π_2 (use SI)
2.467	5.47	757	0.0152
2.399	5.32	1135.5	0.0228
2.330	5.17	1514	0.0304
2.193	4.86	1892.5	0.0381
1.988	4.41	2271	0.0457
1.576	3.495	2649.5	0.0533

Plot Π_1 vs Π_2

(b) For flow of kerosine

$$\Omega = 900\,\text{rpm} \text{ or } 15\ s^{-1}$$

$$Q = 1514\ \text{litres}/\text{min} \text{ or } 0.0252\ m^3/s$$

$$\Pi_2 = \frac{0.0252}{(0.35)^3(15)} = 0.0392$$

Use the plot to find Π_1

$$\Pi_1 \doteq 4.81 = \frac{\Delta P}{\rho D^2 \Omega^2}$$

$$\Delta P = 4.81(804)(0.35)^2(15)^2 \quad Pa$$

$$= 106,590.8\ Pa$$

$$= 1.052\ atm \longrightarrow$$

Alternate solution to part (b)

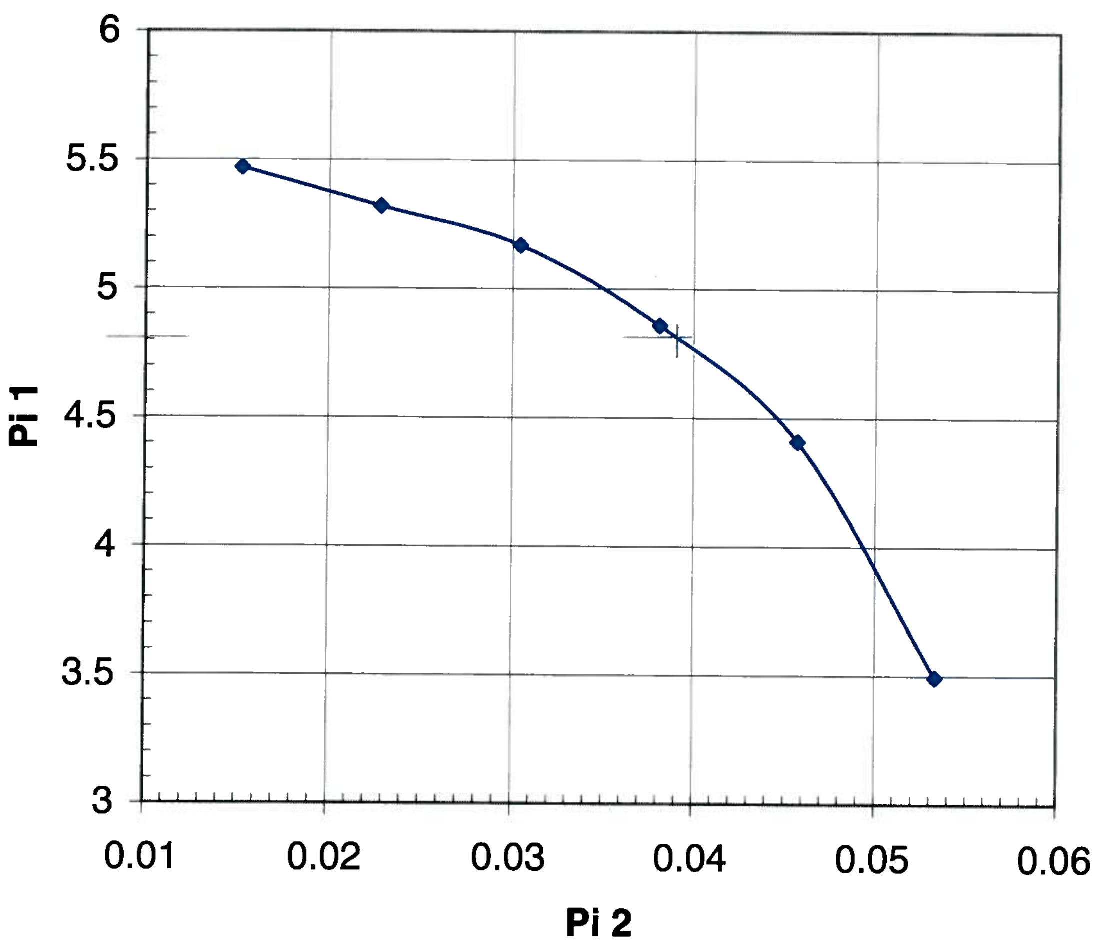
$$\left.\frac{Q}{D^3\Omega}\right|_W = \left.\frac{Q}{D^3\Omega}\right|_K \quad \therefore \text{calc. } Q_W = 1951.38 \text{ and}$$

obtain $\Delta P|_W$ from table $\Rightarrow$ $\Delta P_W = 2.17$

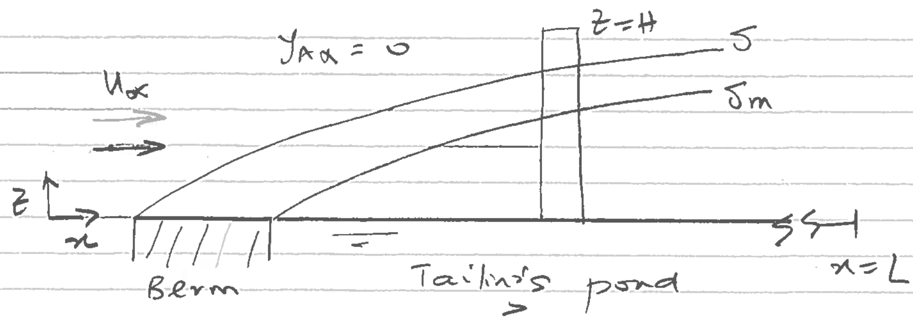
Then use

$$\left.\frac{\Delta P}{\rho D^2 \Omega^2}\right|_W = \left.\frac{\Delta P}{\rho D^2 \Omega^2}\right|_K \Rightarrow \text{calc. } \Delta P_K = 1.053\ atm \longrightarrow$$

3-3



#4



Momentum and concentration boundary layers form.

Let the volatile substance $\equiv$ A and air $\equiv$ B

In the z-direction, the flux is given by

$$N_A = -c D_{AB} \frac{dy_A}{dz} + y_A (N_A + N_B)$$

At the boundary $z = 0$, $N_B = 0$ (B not dissolving)

$$N_A \Big|_{z=0} = -\frac{c D_{AB}}{1 - y_{A0}} \frac{dy_A}{dz} \Big|_{z=0}$$

Consider the differential element - dx by H

Balance on A is the integral equation:

$$-\frac{c D_{AB}}{1 - y_{A0}} \frac{dy_A}{dz} \Big|_{z=0} = \frac{d}{dx} \left[\int_0^{\delta_m} c y_A u \, dz \right]$$

The boundary conditions are:

$z = 0$ $\quad y_A = y_{A0}$

$z = \delta_m$ $\quad y_A = 0$

$z = \delta_m$ $\quad \frac{dy_A}{dz} = 0$

By the integral method, assume a function

$$y_A = a + bz + cz^2$$

Apply the b.c to obtain

$$\frac{y_A}{y_{A0}} = \left(1 - \frac{z}{\delta_m}\right)^2 \quad ; \quad \left.\frac{dy_A}{dz}\right|_{z=0} = -\frac{2y_{A0}}{\delta_m}$$

Substitute these into the integral equation

$$\frac{2D_{AB}}{1-y_{A0}}\frac{1}{\delta_m} = \frac{d}{dx}\left[U_\alpha \delta \int_0^{\delta_m/\delta} \left(1-\frac{z}{\delta_m}\right)^2 \left(\frac{u}{U_\alpha}\right) d\left(\frac{z}{\delta}\right)\right]$$

Let $\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{z}{\delta}\right) - \frac{1}{2}\left(\frac{z}{\delta}\right)^3$ (From Notes)

and $\delta_m = \xi\delta$ $(\xi < 1)$

Substitute into integral equation

$$\frac{2D_{AB}}{1-y_{A0}}\frac{1}{\delta_m} = \frac{d}{dx}\left[U_\alpha\delta\int_0^{\xi}\left(1-\frac{z}{\xi\delta}\right)\left(\frac{3}{2}\frac{z}{\delta} - \frac{1}{2}\left(\frac{z}{\delta}\right)^3\right)d\left(\frac{z}{\delta}\right)\right)$$

Let $\eta = z/\delta$

$$\frac{2D_{AB}}{1-y_{A0}}\frac{1}{\xi\delta} = \frac{d}{dx}\left[U_\alpha\delta\int_0^{\xi}\left(1-\frac{\eta}{\xi}\right)^2\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)d\eta\right]$$

$$= \frac{d}{dx}\left[U_\alpha\delta\left\{\frac{1}{8}\xi^2 - \frac{1}{120}\xi^4\right\}\right]$$

Neglect 2nd term in brackets of r.h.s. and let

$$\beta = \frac{16 D_{AB}}{(1-y_{A0})U_\alpha}$$

$$\beta = \xi\delta \frac{d}{dx}(\delta\xi^2) = \xi^3\delta\frac{d\delta}{dx} + 2\xi^2\delta^2\frac{d\xi}{dx}$$

From Notes:

$$\delta\frac{d\delta}{dx} = \frac{140}{13}\frac{\nu}{U_\alpha} \quad \text{and} \quad \delta^2 = \frac{280}{13}\frac{\nu x}{U_\alpha}$$

Let $\varepsilon = \beta\frac{13}{140}\frac{U_\alpha}{\nu} = \frac{52\, D_{AB}}{35\nu(1-y_{A0})} = \xi^3 + 4x\xi^2\frac{d\xi}{dx}$

Solution is

$$\xi^3 = Cx^{-3/4} + \varepsilon$$

Given that at $x = x_0$, $\xi = \frac{\delta_m}{\delta} = 0$

$$\xi = \frac{\delta_m}{\delta} = \left(\frac{52}{35}\frac{1}{1-y_{A0}}\frac{D_{AB}}{\nu}\right)^{1/3}\left(1-\left(\frac{x_0}{x}\right)^{3/4}\right)^{1/3}$$

where

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

. This gives δ_m.

The Rate of removal of volatile substance from the pond of width W and length L is given by

$$Q = \int_0^L W N_A\Big|_{z=0} dx = W\int_0^L\left(-\frac{c\,D_{AB}}{1-y_{A0}}\frac{dy_A}{dz}\Big|_{z=0}\right)dx$$

$$= \frac{W c D_{AB}(2y_{A0})}{1-y_{A0}}\int_0^L \frac{1}{\delta_m}dx \qquad \text{moles/s}$$

But

$$\delta_m = \varepsilon^{1/3}\left(1-\left(\frac{x_0}{x}\right)^{3/4}\right)^{1/3}\left[4.64\left(\frac{\nu}{U_\alpha}\right)^{1/2} x^{1/2}\right]$$

Let $\lambda = \left(\frac{\nu}{U_\alpha}\right)^{1/2}(4.64)\,\varepsilon^{1/3}$

$$\therefore \quad Q = \frac{W c D_{AB}(2 y_{A0})}{1-y_{A0}}\ \frac{1}{\lambda}\int_0^L \frac{dx}{x^{1/2}\left(1-\left(\frac{x_0}{x}\right)^{3/4}\right)^{1/3}}$$

$\longrightarrow$

It is necessary to check that the boundary layer on the surface will be laminar.

For $L = 51\,m$, $U_\alpha = 0.36$ km/hr or 0.1 m/s

$$Re_L = \frac{U_\alpha L \rho}{\mu} < 5(10^5)$$ is the required condition

$$Re_L = \frac{(0.1)(51)(1.4128)}{1.599(10^{-5})} = 4.506(10^5)$$

Hence the b.l. is laminar.